

Shifted fractional order Gegenbauer wavelets method for solving electrical circuits model of fractional order

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ARTICLE INFO

MSC:

26A33

34A08

65D15

65T60

97M50

Keywords:

Block pulse function

Caputo derivative

Electrical circuits

Shifted fractional-order Gegenbauer wavelets

Shifted fractional-order Gegenbauer

polynomials

ABSTRACT

This study introduces a novel numerical approach based on shifted fractional-order Gegenbauer wavelets. The technique aims to provide approximate solutions for a certain class of extremely significant fractional models of electrical LC , RC , RL , and RLC circuits. We use a collocated approach to generate numerical solutions for these circuits model by making use of the beneficial characteristics of shifted fractional-order Gegenbauer polynomials (SFGBP). We include a parameter that characterizes the presence of fractional structures inside the models in order to retain the dimensional properties of the physical parameters in the electrical circuits. Several particular instances of the various source terms have also been examined. The main findings include the close resemblance of current fractional-order models to classical cases, variations in current with resistance values, and error reduction through additional series terms. Numerical simulations are shown through geometrical interpretation to illustrate the exactness and reliability of our technique.

1. Introduction

The significance and extensive range of applications for fractional models [33–37,41,43,45,48], described by fractional derivatives, have motivated researchers in mathematics and physics to investigate approximate and numerical solutions for fractional differential equations (FDEs) using various techniques [46,47]. These equations find practical use in fluid mechanics [1], image processing [2], biology [3,4], engineering [5], physics [6], electrical circuits [7,8], filters, and other fields [9–11]. The distinctive characteristic of fractional-order models lies in their non-local nature, enabling them to represent physical processes and dynamical systems with greater precision than integer-order models [12,13]. However, finding exact analytic solutions for nonlinear fractional differential equations is challenging. Consequently, several numerical and approximate methods have been developed, including the spectral collocation method [18], homotopy analysis method [17], variational iteration method [16], differential quadrature method [19], adomian decomposition method [15], fractional reduced differential transform method [14], and wavelet methods [20,21,31].

Wavelets have recently gained popularity as efficient numerical techniques for fractional calculus, giving a number of beneficial characteristics such as compact support, multi-resolution representation, orthogonality, and polynomial representation up to a certain degree. The orthogonal basis for $L^2(\mathbb{R})$ is provided by wavelets, such as the Haar wavelet, which also provide compact support in the form of rectangular functions [29,32,44]. There are other wavelet-based techniques that have been proposed, including the cosine and sine wavelet approaches [28], the generalized fractional-order Chebyshev wavelet method [27], the Fibonacci wavelet method [20], Bernoulli wavelet [42] and Gegenbauer wavelet [21]. Legendre polynomials are extended by Gegenbauer wavelets, which are built using Gegenbauer polynomials as their basis functions, depending on the parameter value.

In this study, we presented an effective numerical method to approximate solutions for fractional LC , RC , RL , and RLC electrical circuits using the Caputo fractional derivative. To ensure consistency with boundary conditions, we use shifted orthogonal polynomials as wavelet basis

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<https://doi.org/10.1016/j.asej.2023.102544>

Received 6 July 2023; Received in revised form 22 September 2023; Accepted 14 October 2023

Available online 31 October 2023

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functions. By introducing a suitable transformation, we modify the polynomials, simplifying the method analytically and improving accuracy by avoiding additional algebraic manipulations. In particular, we introduce the SFGBW and establish appropriate normalization factors to ensure their orthonormality. Moreover, based on these SFGBW, we propose a novel class of wavelets known as shifted fractional-order Gegenbauer wavelets (SFGBW). Previous works by Gomez et al. [22], Shah et al. [23], Alsaedi et al. [24], Arora and Chauhan [25], Atangana et al. [7], and Sene et al. [26] have explored solutions for electrical circuits using the Caputo derivative and various fractional calculus techniques. The motivation for this study lies in the development of a novel numerical algorithm utilizing SFGBW to approximate solutions for fractional electrical circuits, building upon previous research in the field and aiming to enhance analytical simplicity and accuracy while avoiding excessive algebraic manipulations. The primary advantage of using numerical methods that employ orthogonal basis functions to solve differential equations numerically is that they transform the problem under consideration into a system of linear or nonlinear algebraic equations. The application of wavelets, on the other hand, results in faster numerical approaches that use a sparse matrix-based system of algebraic equations, which lowers costs and computational complexity. The key feature of SFGBW is that the fractional-order parameter allows us to adjust the shape and properties of the wavelet, which offers a higher degree of flexibility. This approach does not appear to have any significant weaknesses. However, it operates within a limited domain and necessitates transformations for broader applicability.

Electrical circuits involve the flow of electrical charges in the form of electrons and waves, resulting in the creation of an electromagnetic field around the conductor. Different types of circuits include:

- RL circuit: These circuits consist of electrical resistance (R) and inductors (L). Inductors multiply the electromotive force (EMF) generated by the current. RL circuits are commonly used in chokes for luminous tubes.
- RC circuit: These circuits consist of electrical resistance (R) and capacitors (C). They are used in heart pacemakers, camera flashes, and other electrical equipment.
- LC circuit: These comprise resistors (R) and capacitors (C). They are designed to generate signals at specific frequencies.
- RLC circuit: RLC circuits involve the combination of resistors (R), inductors (L), and capacitors (C) in series. They find applications in biological defibrillator circuits, vehicle security systems, and communication system filters. RLC circuits exhibit more complex and appealing responses compared to RC and RL circuits.

The article is organized as follows: Section 2 provides an overview of the fundamental concepts of fractional calculus, including an introduction to the fractional Gegenbauer polynomial and an exploration of its significant properties. Moving on to Section 3, we delve into the investigation of the SFGBW operational matrix of fractional order integration. In Section 4, we concentrate on the practical application aspect, utilizing SFGBW to approximate the solutions of the fractional electrical circuits model. Furthermore, Section 5 presents a comprehensive discussion of error analysis. To demonstrate the efficacy of our proposed method, numerical simulations are presented in Section 6. Lastly, in Section 7, we conclude our study.

2. Fractional calculus

Several methods exist for defining fractional order derivatives, such as Caputo, Riemann-Liouville (R-L), Grünwald-Letnikov, and Baleno fractional. Among these, Caputo's method is particularly notable as it handles initial conditions specified in terms of field coordinates and their integer order. In this article, we adopt the fractional derivative in the sense of Caputo to avoid any confusion. For more in-depth exploration, we recommend referring to the works cited in [12,13].

Definition 2.1. The R-L fractional integration operator of order $\alpha > 0$ for the function $h(\tau)$ is described as follows:

$$I^\alpha h(\tau) = \frac{1}{\Gamma(\alpha)} \int_0^\tau \frac{h(t)}{(\tau-t)^{1-\alpha}} dt, \quad \tau > 0.$$

A few, properties of operator I^α are listed below:

1. $I^\alpha I^\beta h(\tau) = I^{\alpha+\beta} h(\tau), \quad \alpha, \beta > 0,$
2. $I^\alpha I^\beta h(\tau) = I^\beta I^\alpha h(\tau), \quad \alpha, \beta > 0,$
3. $I^\alpha \tau^\beta = \frac{\Gamma(1+\beta)}{\Gamma(1+\alpha+\beta)} \tau^{\alpha+\beta}, \quad \beta > -1.$

Definition 2.2. The Caputo fractional order derivative D^α of the function $h(\tau)$ is defined as follows:

$$D^\alpha h(\tau) = \frac{1}{\Gamma(n-\alpha)} \int_0^\tau \frac{h''(t)}{(\tau-t)^{\alpha-n+1}} dt, \quad n-1 < \alpha \leq n, n \in \mathbb{N},$$

below are some fundamental properties of D^α :

1. $D^\alpha (\gamma h(\tau) + \delta h(\tau)) = \gamma D^\alpha h(\tau) + \delta D^\alpha h(\tau)$, where γ, δ are constants,
2. $D^\alpha \tau^\beta = \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)} \tau^{\beta-\alpha}, \quad 0 < \alpha < \beta+1, \beta > -1;$
3. $I^\alpha D^\alpha h(\tau) = h(\tau) - \sum_{k=0}^{n-1} f^k(0^+) \frac{\tau^k}{k!}, \quad n-1 < \alpha \leq n, n \in \mathbb{N};$
4. $D^\alpha C = 0, \quad C$ is a constant.

Shifted fractional-order Gegenbauer polynomials

On the interval $[-1, 1]$, the Sturm-Liouville equation is satisfied by the Gegenbauer polynomial, $\tilde{G}_n^\delta(z)$ of order n as:

$$\frac{d}{dz} \left[(1-z^2)^{\delta+\frac{1}{2}} \frac{d}{dz} \tilde{G}_n^\delta(z) \right] + n(n+2\delta)(1-z^2)^{\delta-\frac{1}{2}} \tilde{G}_n^\delta(z) = 0.$$

The objective of this initial section is to provide a broad introduction to the SFGBP and highlight some of its essential characteristics. The construction of SFGBP involves a variable transformation applied to the classical Gegenbauer polynomials. On the interval $[-1, 1]$, the n th degree of the classical Gegenbauer polynomials $\tilde{G}_n^\delta(z)$ are defined, for $\delta > \frac{-1}{2}, n \in \mathbf{Z}^+$, as:

$$\begin{aligned}\tilde{G}_0^\delta(z) &= 1, \quad \tilde{G}_1^\delta(z) = 2\delta z, \\ \tilde{G}_{n+1}^\delta(z) &= \frac{1}{n+1} \left(2(n+\delta)z\tilde{G}_n^\delta(z) - (n+2\delta-1)\tilde{G}_{n-1}^\delta(z) \right), \quad n = 1, 2, \dots\end{aligned}$$

The definition of the orthogonality relation for Gegenbauer polynomials can be stated as follows:

$$\int_{-1}^1 (1-z^2)^{\delta-\frac{1}{2}} \tilde{G}_n^\delta(z) \tilde{G}_m^\delta(z) dz = J_n^\delta \kappa_{nm},$$

where $J_n^\delta(z) = \frac{\pi 2^{1-2\delta} \Gamma(n+2\delta)}{n!(n+\delta)(\Gamma(\delta))^2}$ is the normalizing factor, $\omega(z) = (1-z^2)^{\delta-\frac{1}{2}}$ is the weight function and κ is the Kronecker delta function.

On interval $[0, \xi]$, from classical Gegenbauer polynomials, we can obtain the SFGBP by using the following transformation $z = 2\frac{t^\beta}{\xi^\beta} - 1$ as

$$SG_n^{\delta,\beta}(t) = \tilde{G}_n^\delta\left(2\frac{t^\beta}{\xi^\beta} - 1\right),$$

$$SG_0^{\delta,\beta}(t) = 1,$$

$$SG_1^{\delta,\beta}(t) = 2\delta \left(2\frac{t^\beta}{\xi^\beta} - 1\right),$$

$$SG_{n+1}^{\delta,\beta}(t) = \frac{1}{n+1} \left(2(n+\delta) \left(2\frac{t^\beta}{\xi^\beta} - 1\right) SG_n^{\delta,\beta}(t) - (n+2\delta-1) SG_{n-1}^{\delta,\beta}(t) \right), n = 1, 2, \dots$$

Theorem 2.1. The SFGBP are orthonormal w.r.t. weight function $\omega_\beta(t) = t^{\beta(\delta+\frac{1}{2})-1} (\xi^\beta - t^\beta)^{\delta-\frac{1}{2}}$ and normalizing factor $J_n^{\delta,\beta} = \frac{\xi^{2\beta\delta} \pi 2^{1-4\delta} \Gamma(n+2\delta)}{\beta n!(n+\delta)(\Gamma(\delta))^2}$.

Proof. Define, orthogonality of Gegenbauer polynomials as:

$$\int_{-1}^1 (1-z^2)^{\delta-\frac{1}{2}} \tilde{G}_n^\delta(x) \tilde{G}_m^\delta(x) dx = J_m^\delta \kappa_{nm}. \quad (1)$$

Utilizing the transformation $z = 2\frac{t^\beta}{\xi^\beta} - 1$, we compute

$$\int_0^\xi \left(1 - \left(2\frac{t^\beta}{\xi^\beta} - 1\right)^2\right)^{\delta-\frac{1}{2}} SG_n^{\delta,\beta}(t) SG_m^{\delta,\beta}(t) d\left(2\frac{t^\beta}{\xi^\beta} - 1\right),$$

or

$$\int_0^\xi \frac{2^{2\delta\beta}}{\xi^{2\beta\delta}} t^{\beta(\delta+\frac{1}{2})-1} (\xi^\beta - t^\beta)^{\delta-\frac{1}{2}} SG_n^{\delta,\beta}(t) SG_m^{\delta,\beta}(t) dt.$$

From Eq. (1), we get

$$\begin{aligned}& \int_0^\xi t^{\beta(\delta+\frac{1}{2})-1} (\xi^\beta - t^\beta)^{\delta-\frac{1}{2}} SG_n^{\delta,\beta}(t) SG_m^{\delta,\beta}(t) dt \\ &= \frac{\xi^{2\beta\delta}}{2^{2\delta\beta}} J_n^\delta \kappa_{nm},\end{aligned}$$

or

$$\int_0^\xi \omega_\beta(t) SG_n^{\delta,\beta}(t) SG_m^{\delta,\beta}(t) dt = J_n^{\delta,\beta} \kappa_{nm},$$

where $\omega_\beta(t) = t^{\beta(\delta+\frac{1}{2})-1} (\xi^\beta - t^\beta)^{\delta-\frac{1}{2}}$ and

$$J_n^{\delta,\beta} = \frac{\xi^{2\beta\delta} \pi 2^{1-4\delta} \Gamma(n+2\delta)}{\beta n!(n+\delta)(\Gamma(\delta))^2}. \quad \square$$

Shifted fractional-order Gegenbauer wavelets

The importance of scaling functions and mother wavelets in wavelet analysis cannot be overstated, as they form the basis for generating a family of functions. The general expression for a scaling function is:

$$\Psi_{n,m}(t) = \frac{1}{\sqrt{|n|}} \psi\left(\frac{t-m}{c}\right), \quad n, m \in \mathbb{R}, n \neq 0,$$

where, m is the translation parameter and n is the scaling parameter. Restrict n, m to discrete values as: $n = n_0^{-k}$, $m = am_0 n_0^{-k}$, where $n_0 > 1, m_0 > 0$ and $k, a \in \mathbb{N}$, we obtain

$$\psi_{k,a}(t) = (n_0)^{\frac{1}{2}} \Psi(n_0^k t - an_0).$$

The collection of wavelets constitutes an orthogonal basis for $L^2(\mathbb{R})$. Specifically, when $n_0 = 2$ and $m_0 = 1$, the wavelet functions $\psi_{k,b}$ form an orthonormal basis, satisfying the following relation:

$$\langle \psi_{k,r}(t), \psi_{s,b}(t) \rangle = \kappa_{ks} \kappa_{rb}.$$

The definition of the discrete wavelet transform is given by:

$$\psi_{k,a}(t) = 2^{\frac{k}{2}} \psi(2^k t - a).$$

On the interval $[0, \xi]$, the SFGBW is defined by

$$\psi_{i,j}^{\delta,\beta}(t) = \begin{cases} \frac{1}{\sqrt{j_i^{\delta,\beta}}} 2^{\frac{k}{2}} S G_i^{\delta,\beta}(2^k t - \xi j + \xi), & \xi^{\frac{j-1}{2^k}} \leq t < \xi^{\frac{j}{2^k}}, \\ 0, & \text{otherwise,} \end{cases}$$

here, the level of resolution is $k = 1, 2, 3, \dots$, the translation parameter is $j = 1, 2, 3, \dots, 2^k$, $i = 0, 1, 2, \dots, N-1$ and order of the SFGBP is $(N-1)\beta$, $N \in \mathbb{Z}^+$.

The SFGBP weight function is scaled and translated for the SFGBW as

$$\omega_{\beta}^{j,k}(t) = \omega_{\beta}(2^k t - \xi j + \xi) = (2^k t - \xi j + \xi)^{\beta(\delta+\frac{1}{2})-1} \left(\xi^{\beta} - (2^k t - \xi j + \xi)^{\beta} \right)^{\delta-\frac{1}{2}}.$$

For $k = p, p \in \mathbb{N}$, we have

$$\omega_{\beta}^{j,p}(t) = \begin{cases} \omega_{\beta}^{1,p}(t), & 0 \leq t < \xi^{\frac{1}{2^p}}, \\ \omega_{\beta}^{2,p}(t), & \xi^{\frac{1}{2^p}} \leq t < \xi^{\frac{2}{2^p}}, \\ \omega_{\beta}^{3,p}(t), & \xi^{\frac{2}{2^p}} \leq t < \xi^{\frac{3}{2^p}}, \\ \vdots & \\ \omega_{\beta}^{2^p,p}(t), & \xi^{\frac{2^p-1}{2^p}} \leq t < \xi. \end{cases}$$

Function approximations of any function $h(t) \in L^2[0, \xi]$ can be expanded into SFGBW series as,

$$\begin{aligned} h(t) &= \sum_{j=1}^{\infty} \sum_{i=0}^{\infty} g_{ij}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{ij}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t), \\ &= \mathbf{g}^T \mathbf{\Psi}(t), \end{aligned}$$

here $\mathbf{g}^{\delta,\beta}$ and $\mathbf{\Psi}^{\delta,\beta}$ are $\hat{i} \times 1$, ($\hat{i} = 2^k N$), matrices, given by

$$\mathbf{g}^{\delta,\beta} = [g_{0,1}^{\delta,\beta}, g_{0,2}^{\delta,\beta}, \dots, g_{N-1,1}^{\delta,\beta}, g_{N-1,2}^{\delta,\beta}, \dots, g_{N-1,2^k}^{\delta,\beta}]^T, \quad (2)$$

$$\mathbf{\Psi}^{\delta,\beta}(t) = [\psi_{0,1}^{\delta,\beta}(t), \dots, \psi_{N-1,1}^{\delta,\beta}(t), \psi_{0,2}^{\delta,\beta}(t), \psi_{1,2}^{\delta,\beta}(t), \dots, \psi_{N-1,2}^{\delta,\beta}(t), \dots, \psi_{0,2^k}^{\delta,\beta}(t), \psi_{1,2^k}^{\delta,\beta}(t), \dots, \psi_{N-1,2^k}^{\delta,\beta}(t)]^T. \quad (3)$$

We considered the collocation points $t_{\ell} = \xi^{\frac{2^{\ell}-1}{2^{k+1}N}}$, $\ell = 1, 2, \dots, 2^k N$. The SFGBW matrix $\mathbf{\Psi}_{2^k N, 2^k N}^{\delta,\beta}$ is given as

$$\mathbf{\Psi}_{2^k N \times 2^k N}^{\delta,\beta} = \left[\Psi^{\delta,\beta}\left(\xi \frac{1}{2^{k+1}N}\right), \Psi^{\delta,\beta}\left(\xi \frac{3}{2^{k+1}N}\right), \dots, \Psi^{\delta,\beta}\left(\xi \frac{2^{k+1}N-1}{2^{k+1}N}\right) \right]. \quad (4)$$

For $k = 2, N = 3, \delta = 3$ and $\beta = 1.5$, the SFGBW matrix is obtained:

$$\mathbf{\Psi}_{12 \times 12}^{3,1.5} = \begin{pmatrix} 19.7772 & 19.7772 & 19.7772 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 19.7772 & 19.7772 & 19.7772 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 19.7772 & 19.7772 & 19.7772 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.7772 & 19.7772 & 19.7772 \\ -48.3262 & -16.3840 & 29.1692 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -48.3262 & -16.3840 & 29.1692 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -48.3262 & -16.3840 & 29.1692 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -48.3262 & -16.3840 & 29.1692 \\ 54.0184 & -1.1879 & 14.3674 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 54.0184 & -1.1879 & 14.3674 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 54.0184 & -1.1879 & 14.3674 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 54.0184 & -1.1879 & 14.3674 \end{pmatrix}.$$

3. Operational matrix of fractional-order integration of SFGBW

The block-pulse functions (BPFs) represent on $[0, \xi]$ is given by

$$\theta_v(t) = \begin{cases} 1 & \xi^{\frac{v}{\hat{i}}} \leq t < \xi^{\frac{v+1}{\hat{i}}}, \\ 0 & \text{otherwise,} \end{cases} \quad (5)$$

here, with $v = 0, 1, \dots, \hat{i} - 1$, we have the ability to express any function $f(t) \in L_2[0, \xi]$ in the form of BPFs [29,38–40] as:

$$f(t) \simeq \sum_{k=0}^{\hat{i}-1} q_k \theta_k(t) = \mathbf{q}^T \Theta(t), \quad (6)$$

here, the coefficients of the BPFs are $\mathbf{q}^T = [q_0, q_1, q_2, \dots, q_{\hat{i}-1}]$ and $\Theta(t) = [\theta_0(t), \theta_1(t), \dots, \theta_{\hat{i}-1}(t)]^T$. SFGBW can be extended into $\hat{i}$ - set of BPFs as

$$\Psi^{\delta, \beta}(t) = \Psi_{\hat{m} \times \hat{m}}^{\delta, \beta} \Theta(t).$$

The expression for BPFs of the fractional integral is represented by:

$$(I_t^\alpha \Theta)(t) = \mathbf{F}_{\hat{i} \times \hat{i}}^\alpha \Theta(t),$$

where $\mathbf{F}_{\hat{i} \times \hat{i}}^\alpha$ and its entries are given in [29] as

$$\mathbf{F}_{\hat{i} \times \hat{i}}^\alpha = \left(\frac{g}{j}\right)^\alpha \frac{1}{\gamma(\alpha+2)} \begin{pmatrix} 1 & c_1 & c_2 & c_3 & \dots & c_{\hat{i}-1} \\ 0 & 1 & c_1 & c_2 & \dots & c_{\hat{i}-2} \\ 0 & 0 & 1 & c_1 & \dots & c_{\hat{i}-3} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 & 1 & c_1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}, \quad (7)$$

with

$$\mathbf{U}_{\hat{i} \times \hat{i}}^{\delta, \beta, \alpha} = \Psi_{\hat{i} \times \hat{i}}^{\delta, \beta} \mathbf{F}_{\hat{i} \times \hat{i}}^\alpha (\Psi_{\hat{i} \times \hat{i}}^{\delta, \beta})^{-1}.$$

Here, the matrix $\mathbf{U}_{\hat{i} \times \hat{i}}^{\delta, \beta, \alpha}$ denotes the SFGBW operational matrix of integration with order α . This matrix plays a crucial role in solving fractional differential equations.

For $k = 2$, $N = 3$, $\delta = 1.9$ and $\beta = 0.75$, and $\alpha = 0.93$ the following SFGBW matrix is obtained:

$$\mathbf{U}_{12 \times 12}^{1.9, 0.75, 0.93} = \begin{pmatrix} 0.1218 & 0.2673 & 0.2522 & 0.2446 & 0.0614 & -0.0056 & -0.0021 & -0.0013 & 0.0051 & 0.0010 & 0 & -0.0001 \\ 0 & 0.1218 & 0.2673 & 0.2522 & 0 & 0.0614 & -0.0056 & -0.0021 & 0 & 0.0051 & 0.0010 & 0 \\ 0 & 0 & 0.1218 & 0.2673 & 0 & 0 & 0.0614 & -0.0056 & 0 & 0 & 0.0051 & 0.0010 \\ 0 & 0 & 0 & 0.1218 & 0 & 0 & 0 & 0.0614 & 0 & 0 & 0 & 0.0051 \\ -0.0743 & 0.1056 & 0.0953 & 0.0914 & 0.0093 & -0.0047 & -0.0011 & -0.0006 & 0.0659 & 0.0014 & 0.0000 & 0 \\ 0 & -0.0743 & 0.1056 & 0.0953 & 0 & 0.0093 & -0.0047 & -0.0011 & 0 & 0.0659 & 0.0014 & 0 \\ 0 & 0 & -0.0743 & 0.1056 & 0 & 0 & 0.0093 & -0.0047 & 0 & 0 & 0.0659 & 0.0014 \\ 0 & 0 & 0 & -0.0743 & 0 & 0 & 0 & 0.0093 & 0 & 0 & 0 & 0.0659 \\ 0.0233 & 0.1383 & 0.1285 & 0.1243 & 0.0122 & -0.0041 & -0.0012 & -0.0007 & 0.0275 & 0.0011 & 0 & 0 \\ 0 & 0.0233 & 0.1383 & 0.1285 & 0 & 0.0122 & -0.0041 & -0.0012 & 0 & 0.0275 & 0.0011 & 0 \\ 0 & 0 & 0.0233 & 0.1383 & 0 & 0 & 0.0122 & -0.0041 & 0 & 0 & 0.0275 & 0.0011 \\ 0 & 0 & 0 & 0.0233 & 0 & 0 & 0 & 0.0122 & 0 & 0 & 0 & 0.0275 \end{pmatrix}.$$

Since $\Psi_{\hat{i} \times \hat{i}}^{\delta, \alpha}$ and $\mathbf{U}_{\hat{i} \times \hat{i}}^{\delta, \beta, \alpha}$ contain many zero entries which makes the calculation fast. We can derive various SFGBW matrices for different values of δ and β and their corresponding fractional-order operational matrices of integration.

4. Application

Model 1: $\mathcal{LC}$ Electrical Circuit

The classical form of $\mathcal{LC}$ circuit is defined as

$$\frac{d}{dt} \mu(t) + \gamma^2 \mu(t) = \kappa,$$

where $\frac{1}{\mathcal{LC}} = \gamma^2$ and $\frac{F}{\mathcal{L}} = \kappa$.

$\mathcal{LC}$ circuit equation consists of a charged capacitor and an inductor, and the fractional order differential equation (FODE) of the $\mathcal{LC}$ circuit is expressed as follows:

$$D^\alpha [\mu(t)] + \gamma^2 \mu(t) = \kappa, \quad \alpha \in (1, 2], \quad (8)$$

with initial conditions $\mu(0) = \mu_0$, and $D^\alpha [\mu(0)] = 0$.

To find the approximate solution of Eq. (8) by SFGBW, let

$$D^\alpha[\mu(t)] \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t). \quad (9)$$

By integrating Eq. (9) w.r.t. t , we obtain

$$\mu(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + t D^\alpha[\mu(0)] + \mu(0). \quad (10)$$

After incorporating the initial conditions, the resulting outcome is obtained as

$$\mu(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + \mu_0. \quad (11)$$

Now, Eq. (8) is converted into matrix form as follows:

$$\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t) + \gamma^2 \left[\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + \mu_0 \right] = \kappa. \quad (12)$$

The matrix equation mentioned in Eq. (12) can be effectively solved using MATLAB, allowing for the determination of the vector of wavelet coefficients, denoted as $g_{i,j}^{\delta,\beta}$.

Model 2: $\mathcal{RC}$ Electrical Circuit

The classical form of $\mathcal{RC}$ circuit is defined as

$$\mathcal{R}Q + \frac{1}{\mathcal{C}}v = F(t).$$

Now, $Q(t)$ and $v(t)$ are related $Q(t) = \frac{dv(t)}{dt}$.

$$\frac{d}{dt}v(t) + \frac{1}{\mathcal{RC}}v(t) = \frac{F(t)}{\mathcal{R}}.$$

Let us examine the fractional $\mathcal{RC}$ circuit, which includes components such as resistance and a charged capacitor.

The associated FODE is expressed as:

$$D^\alpha[v(t)] + \chi v(t) = \frac{F(t)}{\mathcal{R}}, \quad \text{where } 0 < \alpha \leq 1, \quad (13)$$

with condition $v(0) = v_0$ where $\chi = \frac{1}{\mathcal{RC}}$.

To find the approximate solution of Eq. (13) by SFGBW, let

$$D^\alpha[v(t)] \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t). \quad (14)$$

By integrating Eq. (14) w.r.t. t , we obtain

$$v(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + t D^\alpha[v(0)] + v(0). \quad (15)$$

After applying the provided conditions, the result obtained is as follows:

$$v(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + v_0. \quad (16)$$

Now, Eq. (13) is converted into matrix form as follows:

$$\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t) + \chi \left[\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + v_0 \right] = \frac{F(t)}{\mathcal{R}}. \quad (17)$$

The matrix equation mentioned in Eq. (17) can be effectively solved using MATLAB, allowing for the determination of the vector of wavelet coefficients, denoted as $g_{i,j}^{\delta,\beta}$.

Model 3: $\mathcal{RL}$ Electrical Circuit

The classical form of $\mathcal{RL}$ circuit is defined as

$$\mathcal{L} \frac{dQ(t)}{dt} + \mathcal{R}Q(t) = F(t).$$

Let us consider a fractional $\mathcal{RL}$ circuit that comprises a resistor and a charged capacitor.

The associated FODE is expressed as follows:

$$D^\alpha[v(t)] + \rho v(t) = \sigma, \quad \text{where } 0 < \alpha \leq 1, \quad (18)$$

with condition $v(0) = v_0$ where $\rho = \frac{R}{L}$, $\sigma = \frac{F(t)}{L}$.

In order to approximate the solution of Eq. (18) using SFGBW, let us consider the following:

$$D^\alpha[v(t)] \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t). \quad (19)$$

By integrating Eq. (19) to α order w.r.t. t , we obtain

$$v(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + t D^\alpha[v(0)] + v(0). \quad (20)$$

After applying the provided conditions, the result obtained is as follows:

$$v(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + v_0. \quad (21)$$

Now, Eq. (18) is converted into matrix form as follows:

$$\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t) + \rho \left[\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) + v_0 \right] = \sigma. \quad (22)$$

The matrix equation mentioned in Eq. (22) can be effectively solved using MATLAB, allowing for the determination of the vector of wavelet coefficients, denoted as $g_{i,j}^{\delta,\beta}$.

Model 4: RLC Electrical Circuit

The classical form of RLC circuit is defined as

$$L \frac{d^2 v}{dt^2} + R \frac{dv}{dt} + \frac{1}{C} v(t) = F(t).$$

Let us examine the fractional RLC circuit that consists of components such as resistance, inductance, and a charged capacitor.

The associated FODE is expressed as follows:

$$D^{2\alpha}[v(t)] + \varpi D[v^\alpha(t)] + \rho v(t) = \frac{F(t)}{L}, \quad \text{where } 0 \leq \alpha \leq 1, \quad (23)$$

with conditions $v(0) = v_0$ and $D^\alpha[v(0)] = 0$ where $\varpi = \frac{R}{L}$ and $\rho = \frac{1}{LC}$.

Applying the SFGBW wavelet method to Eq. (23), let

$$D^{2\alpha}[v(t)] \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t). \quad (24)$$

By integrating Eq. (24) w.r.t. t and using given condition, we obtain

$$D^\alpha[v(t)] \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t). \quad (25)$$

By integrating Eq. (25) w.r.t. t , we obtain

$$v(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,2\alpha} \Psi_{i,j}^{\delta,\beta}(t) + t v^\alpha(0) + v(0). \quad (26)$$

After using the conditions in equation (26), we get

$$v(t) \approx \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,2\alpha} \Psi_{i,j}^{\delta,\beta}(t) + v_0. \quad (27)$$

Now, Eq. (23) is converted into matrix form as follows:

$$\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} \Psi_{i,j}^{\delta,\beta}(t) + \varpi \left[\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,\alpha} \Psi_{i,j}^{\delta,\beta}(t) \right] + \rho \left[\sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta,\beta} U_{i,j}^{\delta,\beta,2\alpha} \Psi_{i,j}^{\delta,\beta}(t) + v_0 \right] = \frac{F(t)}{L}. \quad (28)$$

The matrix equation mentioned in Eq. (28) can be effectively solved using MATLAB, allowing for the determination of the vector of wavelet coefficients, denoted as $g_{i,j}^{\delta,\beta}$.

5. Error analysis

Theorem 5.1. [30] Suppose that ${}^c D_u^{m\alpha} \mu(t) \in C[u, v]$ for $m = 0, 1, 2, \dots, N$, where $0 < \alpha \leq 1$, we have

$$\mu(t) = \sum_{m=0}^{N-1} \frac{(t-u)^{m\alpha}}{\Gamma(m\alpha+1)} ({}^c D_u^{m\alpha} \mu)(u) + \frac{({}^c D_u^{N\alpha} \mu)(\zeta)}{\Gamma(N\alpha+1)} (t-u)^{N\alpha},$$

with $u \leq \zeta \leq t, \forall t \in (u, v]$. Also

$$\left| \mu(t) - \sum_{m=0}^{N-1} \frac{(t-u)^{m\alpha}}{\Gamma(m\alpha+1)} ({}^c D_u^{m\alpha} \mu)(u) \right| \leq R \frac{(t-u)^{N\alpha}}{\Gamma(N\alpha+1)},$$

where $R \geq |({}^c D_u^{N\alpha} \mu)(\zeta)|$.

Theorem 5.2. Suppose that ${}^c D_u^{m\alpha} \mu(t) \in C[0, \eta]$ ($m = 0, 1, \dots, N$), and $\mathfrak{N}^{\delta, \beta} = \text{Span}\{\psi_{i,j}^{\delta, \beta}(t), j = 1, \dots, 2^k, i = 0, 1, \dots, N-1\}$ be the subspace of $L^2[0, \eta]$. If the best approximation of $\mu(t)$ is $\hat{\mu}(t)$ from $\mathfrak{N}^{\delta, \eta}(t)$, therefore

$$\|\mu(t) - \hat{\mu}(t)\| \leq \frac{R}{\Gamma(N\alpha+1)} \frac{\pi^{\frac{1}{4}} \eta^{\alpha(\delta+N)}}{2^{k\alpha(\delta+N)}} \sqrt{\frac{\Gamma\left(\frac{\delta+\frac{1}{2}+2N}{1-2\delta}\right)}{\alpha(1-2\delta)\Gamma\left(\frac{2+4N}{2(1-2\delta)}\right)}}.$$

Proof. Let $\hat{y}(t) = \sum_{m=0}^{N-1} \frac{(t-u)^{m\alpha}}{\Gamma(m\alpha+1)} ({}^c D_u^{m\alpha} \mu)(u)$, then from Theorem 5.1, we have

$$|\mu(t) - \hat{y}(t)| \leq R \frac{(t-u)^{N\alpha}}{\Gamma(N\alpha+1)}.$$

As, the best approximation of $\mu(t)$ is $\hat{\mu}(t) = \sum_{j=1}^{2^k} \sum_{i=0}^{N-1} g_{i,j}^{\delta, \beta} \psi_{i,j}^{\delta, \beta}(t) = \mathbf{b}^{\delta, \beta} \mathbf{\Psi}^{\delta, \beta}(t)$ from $\mathfrak{N}^{\delta, \beta}(t)$, and $\hat{y}(t) \in \mathfrak{N}^{\delta, \beta}(t)$, then

$$\begin{aligned} \|\mu(t) - \hat{\mu}(t)\|^2 &\leq \|\mu(t) - \hat{y}(t)\|^2, \\ &\leq \frac{R^2}{(\Gamma(N\alpha+1))^2} \int_u^v \frac{(t-u)^{\alpha\left(\delta+\frac{1}{2}+2N\right)-1}}{[(v-u)^\alpha - (t-u)^\alpha]^{\frac{1}{2}-\delta}} dt, \\ &\leq \frac{R^2}{(\Gamma(N\alpha+1))^2} \frac{\eta^{\alpha(2\delta+2N)}}{2^{2k\alpha(\delta+N)} \alpha(1-2\delta)} \frac{\sqrt{\pi} \Gamma\left(\frac{\delta+\frac{1}{2}+2N}{1-2\delta}\right)}{\Gamma\left(\frac{2+4N}{2(1-2\delta)}\right)}, \end{aligned}$$

or

$$\|\mu(t) - \hat{\mu}(t)\| \leq \frac{R}{\Gamma(N\alpha+1)} \frac{\pi^{\frac{1}{4}} \eta^{\alpha(\delta+N)}}{2^{k\alpha(\delta+N)}} \sqrt{\frac{\Gamma\left(\frac{\delta+\frac{1}{2}+2N}{1-2\delta}\right)}{\alpha(1-2\delta)\Gamma\left(\frac{2+4N}{2(1-2\delta)}\right)}}.$$

It depicts that error by SFGWB approach reduces while polynomial order increases, αN , or level of resolution, k , or both. $\square$

6. Numerical simulation

Currently, we employ the suggested approach to address the four models obtained from equations (8), (13), (18) and (23) in Section 4, which are based on the voltage source $F(t)$. All the numerical simulations are performed in the MATLAB(R2022a) software.

Model 1: By taking the constant voltage source $F(F=0)$ in the $\mathcal{LC}$ circuit Eq. (8) with parameters $\delta, \beta, \mathcal{L}, \mathcal{C}, \alpha$. The exact solution to Eq. (8) at $\alpha=2$ is given by

$$\mu(t)_{\mathcal{LC}} = \mu_0 \cos(\gamma_0 t),$$

where $\gamma_0 = \sqrt{\frac{1}{\mathcal{LC}}}$.

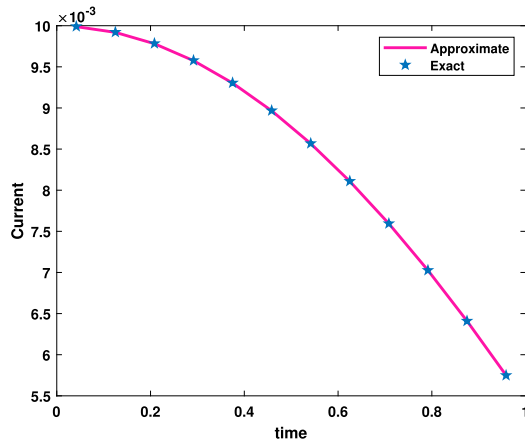
Considering the values of $\delta=3, \beta=1.5, \mathcal{L}=1, \mathcal{C}=1, \mu_0=0.01$ in the proposed numerical approach (12), we conduct a numerical simulation of the examined model and depict the results in Fig. 1 as illustrated below:

As we take $\alpha=2$, Fig. 1(a) demonstrates that the current's behavior with a fractional-order derivative closely resembles the exact solution. Fig. 1(b) and Table 1 provide a comprehensive analysis of the error. The behavior of the approximate solutions for fractional order $\alpha = 1.80, 1.85, 1.90, 1.95$, and 2 is graphically shown in Fig. 1(c).

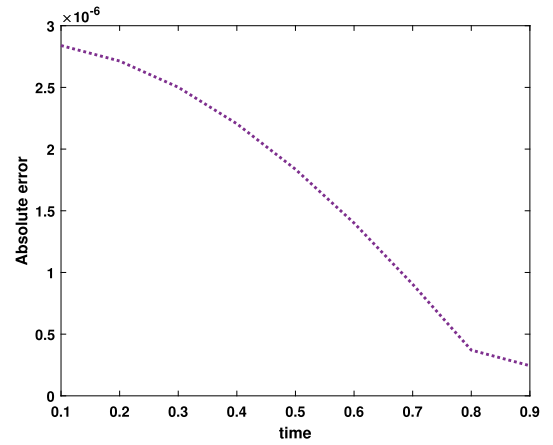
Model 2: Considering the fractional model (13), with parameters $\delta, \beta, \mathcal{R}, \mathcal{C}, \alpha$. In Model 2 Case 1, by taking the constant voltage source $F(F=0)$, the exact solution to Eq. (13) at $\alpha=1$ is given by

$$v(t) = v_0 e^{-\chi t},$$

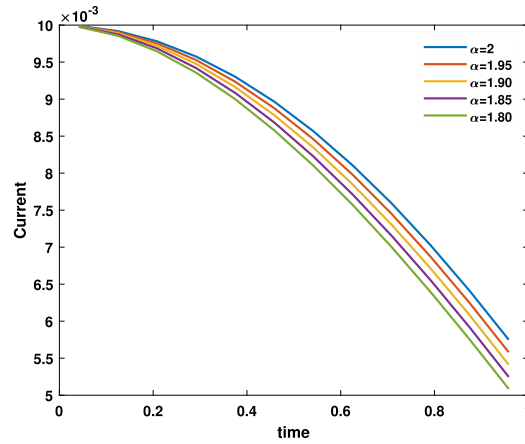
where $\chi = \frac{1}{\mathcal{RC}}$.



(a)



(b)



(c)

Fig. 1. LC circuit approximate solutions for different parameters.

Table 1

Absolute error of fractional LC , RC , RL models.

t	Model 1	Model 2	Model 3 Case 1	Model 3 Case 2
0.1	2.8394e-06	9.6041e-05	6.7269e-03	2.3036e-03
0.2	2.7139e-06	9.4448e-05	2.0349e-03	7.5984e-04
0.3	2.5012e-06	9.2876e-05	2.4925e-03	4.3732e-04
0.4	2.2052e-06	9.1326e-05	1.5161e-03	1.0298e-04
0.5	1.8365e-06	8.9799e-05	7.8094e-04	2.8890e-05
0.6	1.4007e-06	8.8291e-05	3.5824e-04	5.0578e-05
0.7	9.0455e-07	8.6806e-05	1.6141e-04	3.8523e-05
0.8	3.7069e-07	8.5342e-05	6.8899e-05	2.3099e-05
0.9	2.4410e-08	8.3897e-05	2.8411e-05	1.2538e-05

With parameters $\delta=3$, $\beta=1.5$, $R=10$, $C=1$, $v_0=20$, numerical simulations are presented in Fig. 2 and Table 1 by applying the proposed method.

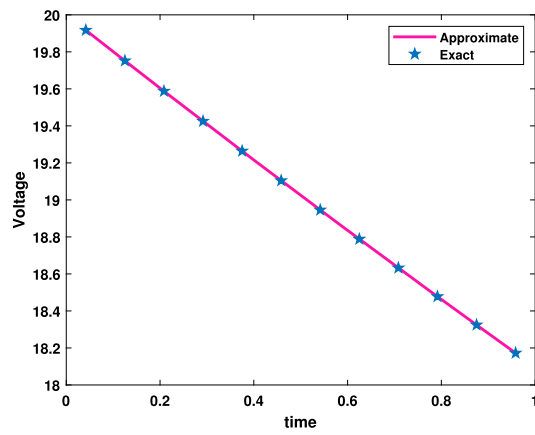
In Model 2 Case 2, by taking the variant voltage source F ($F = e^{-t\chi}$), the exact solution to Eq. (13) at $\alpha = 1$ is given by

$$v(t) = \left(\frac{t}{R} + v_0 \right) e^{-\chi t},$$

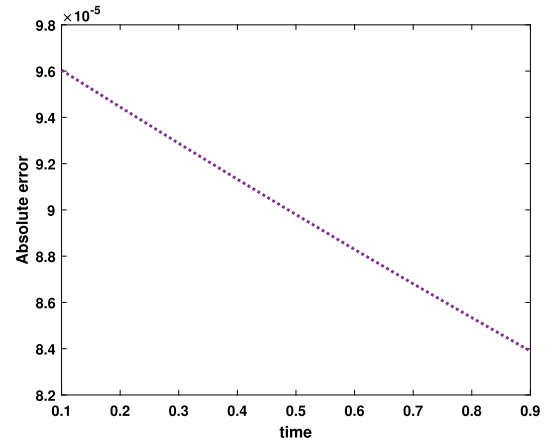
where $\chi = \frac{1}{RC}$. With parameters $\delta=3$, $\beta=1.5$, $R=5$, $C=2$, $v_0=0.01$, some simulations are provided through the Fig. 3 of this model (13).

Model 3: Considering the fractional model (18), with parameters δ , β , R , L , α . In case 1 of model 3, by taking the constant voltage source F ($F = 10$), the exact solution to Eq. (18) at $\alpha = 1$ is given by

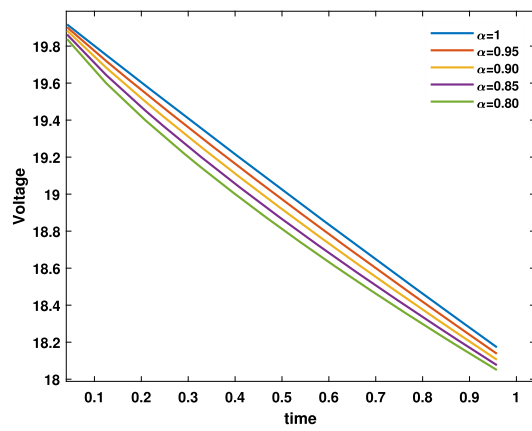
$$v(t) = \left[v_0 - \frac{FL}{R} \right] e^{-\rho t} + \frac{FL}{R},$$



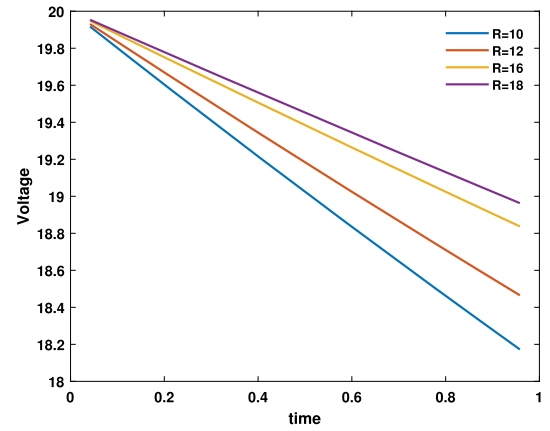
(a)



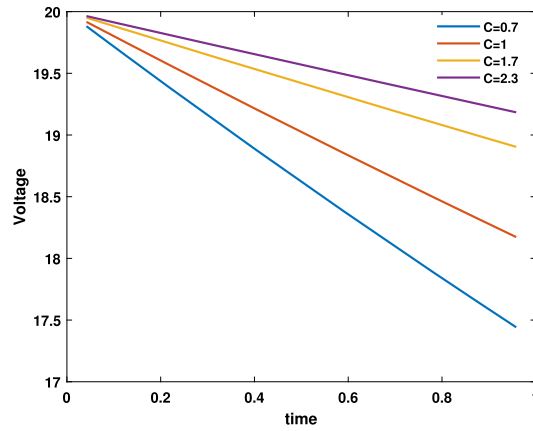
(b)



(c)



(d)



(e)

Fig. 2. RC circuit case 1 approximate solutions for different parameters.

where $\rho = \frac{R}{L}$. With parameters $\delta=3$, $\beta=1.5$, $R=10$, $L=1$, $v_0=0.01$, numerical simulations are presented in Fig. 4 and Table 1 by applying the proposed method. It is clearly shown by the Figs. 4(d, e) that the length of conductor L or the resistance R leads to a decrease in the current by increasing in their values.

In case 2 of model 3, by taking the variant voltage source F ($F = e^{-\rho t}$), the exact solution to Eq. (18) at $\alpha = 1$ is given by

$$v(t) = \left(\frac{t}{L} + v_0 \right) e^{-\rho t},$$

where $\rho = \frac{R}{L}$. With parameters $\delta=3$, $\beta=1.5$, $R=10$, $L=1$, $v_0=0.01$, numerical simulations are presented in Fig. 5 and Table 1 by applying the proposed method.

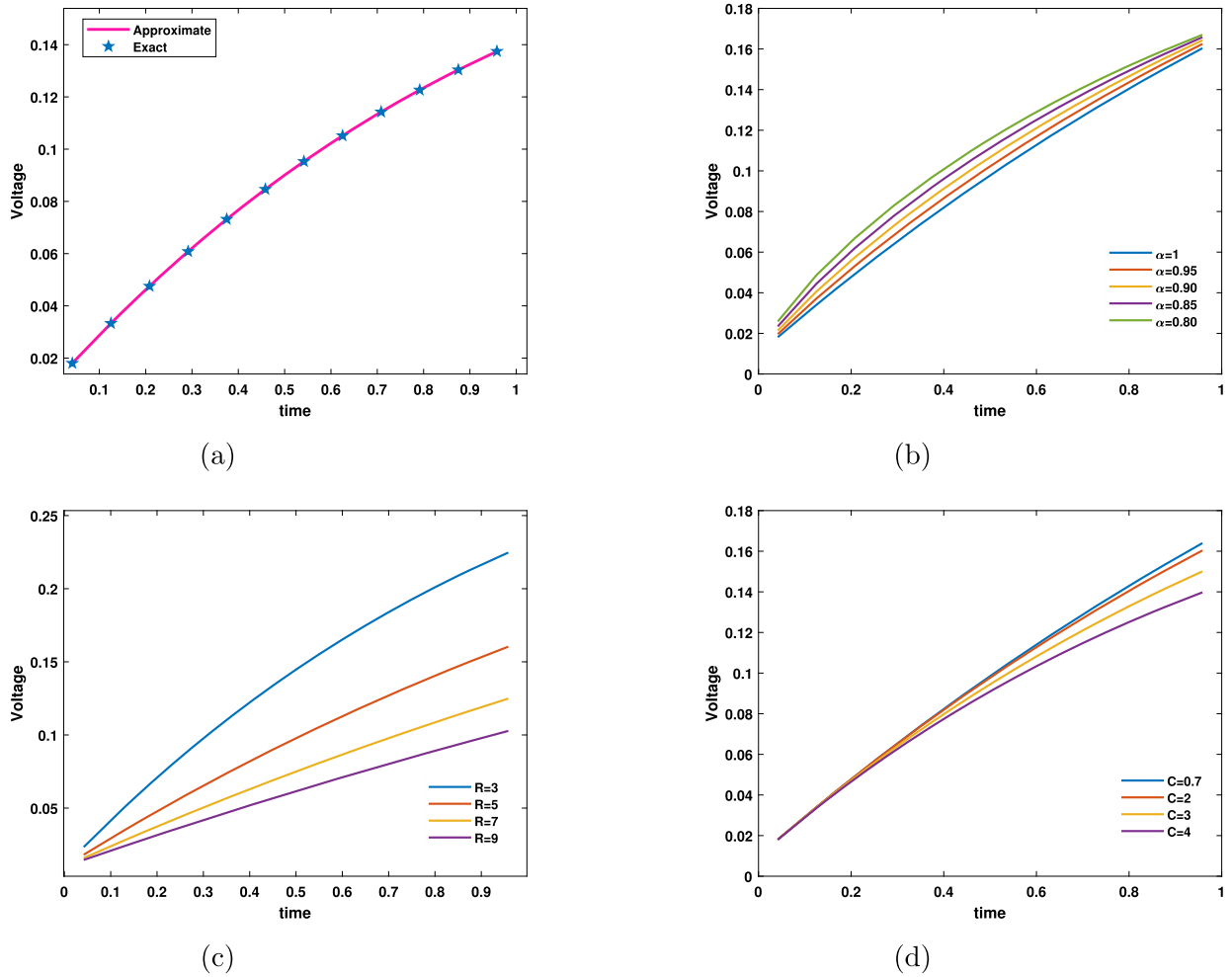


Fig. 3. RC circuit case 2 approximate solutions for different parameters.

Table 2
Approximate solutions CPU time(s) of LC , RC , RL and RLC models.

CPU time	Model 1	Model 2 Case 1	Model 2 Case 2	Model 3 Case 1	Model 3 Case 2	Model 4
s	0.584	0.731	0.720	0.683	0.708	0.720

Model 4: Considering the fractional model (23), with parameters δ , β , R , L , C , α . By taking the constant voltage source F ($F = 0$), the exact solution to Eq. (18) at $\alpha = 1$ is given by

$$v(t) = v_0 e^{-\varpi t/2} \cos\left(\sqrt{\rho - \frac{\varpi^2}{4}} t\right),$$

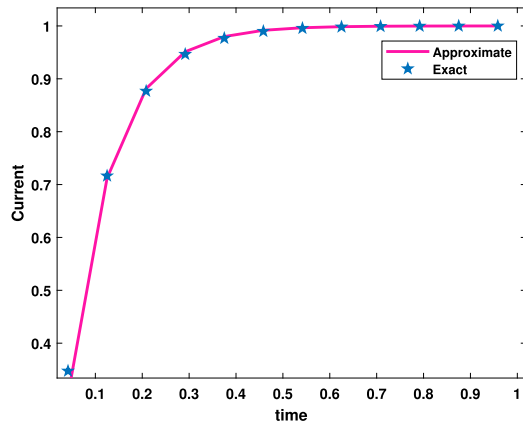
where $\varpi = \frac{R}{L}$ and $\rho = \frac{1}{LC}$. With parameters $\delta = 1.9$, $\beta = 0.75$, $R = 10$, $L = 10$, $C = 10$, $v_0 = 0.01$, numerical simulations are presented in Fig. 6 by applying the proposed method.

We have examined the fractional LC , RC , RL , and RLC models and conducted numerical simulations to illustrate their behavior graphically. It is evident that the approximate solutions are affected by the various values of $\alpha = 0.80, 0.85, 0.90, 0.95$, and 1 (Figs. 1(c), 2(c), 3(b), 4(c), and 5(c)). The accuracy of our numerical approach is validated by evaluating the absolute error, as demonstrated in our results. In conclusion, we have successfully implemented the proposed technique, resulting in an accurate representation of the physical behavior exhibited by the models (Table 2).

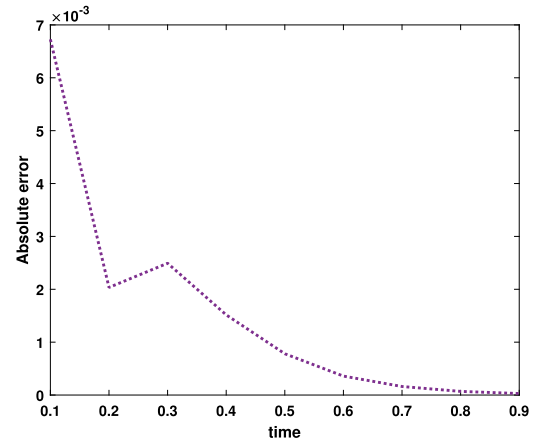
7. Conclusion

The paper introduces the SFGBW method, which offers enhanced efficiency and simplicity compared to existing solutions for approximating solutions of fractional electrical circuits model (such as fractional LC , RC , RL , and RLC circuits). To support the proposed method, we have derived and constructed the operational matrices of SFGWs, including $\psi_{i \times i}^{\delta, \beta}$ and $U^{\delta, \beta, \alpha}$ for fractional-order integration. These matrices play a crucial role in solving the fractional electrical model.

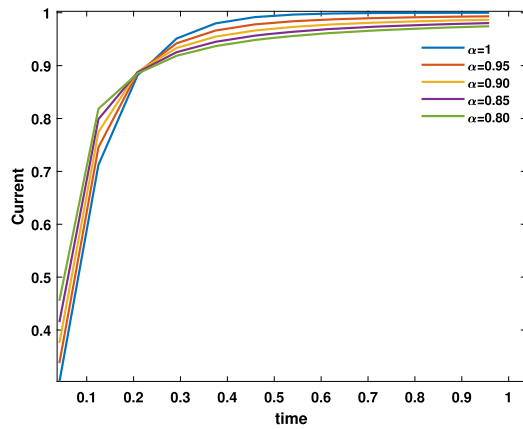
By conducting numerical experiments and comparing the obtained results with exact solutions under various parameter values, we observed a remarkable agreement with existing solutions. This demonstrates the effectiveness of the SFGBW method in efficiently addressing the models



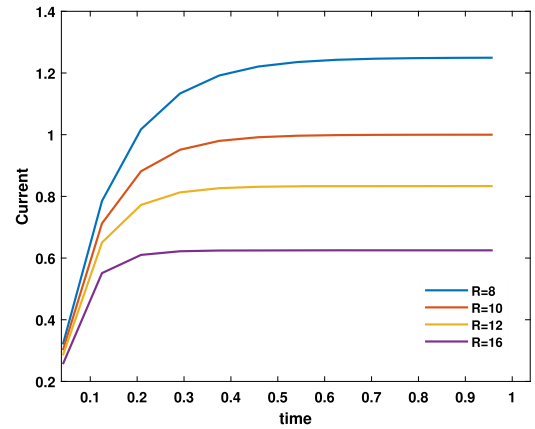
(a)



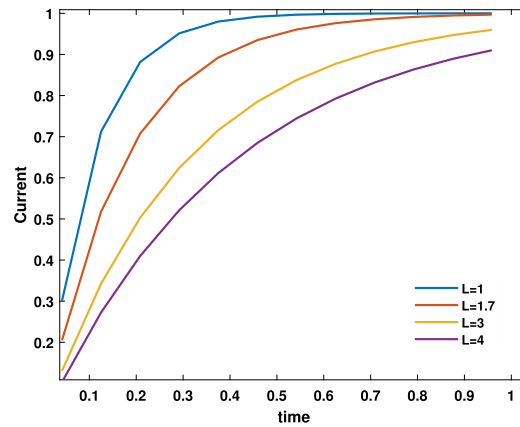
(b)



(c)



(d)

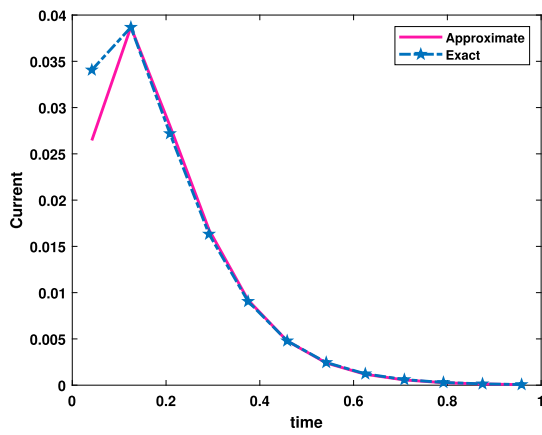


(e)

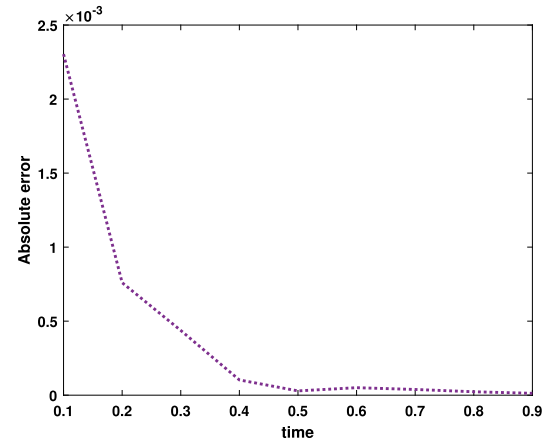
Fig. 4. RL circuit case 1 current vs time graphs.

considered and various related issues. Specifically, when the fractional order α is less than 1, the fractional differentiation with respect to time signifies energy dissipation (internal friction) associated with the electrical circuits in fractal geometries.

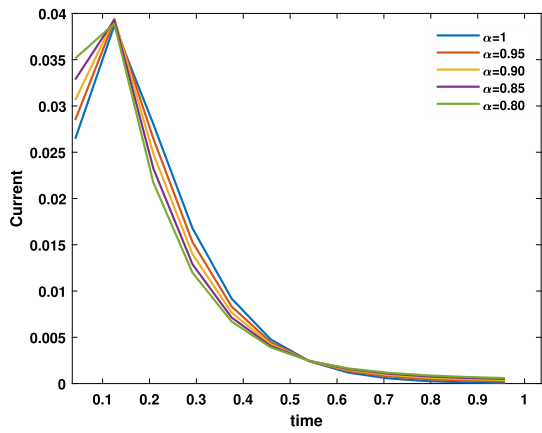
The main findings of the study can be summarized as follows: The behavior of the current fractional-order model closely resembles the classical case with $\alpha = 1$ for RC , RL , and RLC (in Figs. 2(a), 3(a), 4(a), 5(a), and 6(a)) and $\alpha = 2$ for LC circuit (in Fig. 1(a)). Additionally, an increase in resistance (R) leads to a decrease in current, confirming the physical relationship between the two, and this is evident from Figs. 4 and 5. Furthermore, by including additional terms from the solution's series, the errors are reduced. Table 1 presents the absolute error up to eight orders magnitude in model 1 and up to five orders of magnitude in model 2, model 3 case 1, and model 3 case 2. These comparisons highlight the stability and effectiveness of the presented technique.



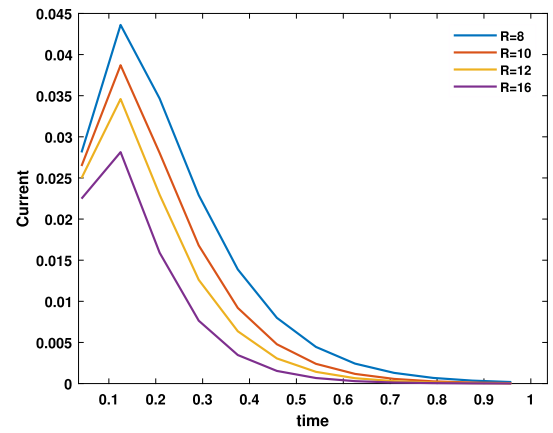
(a)



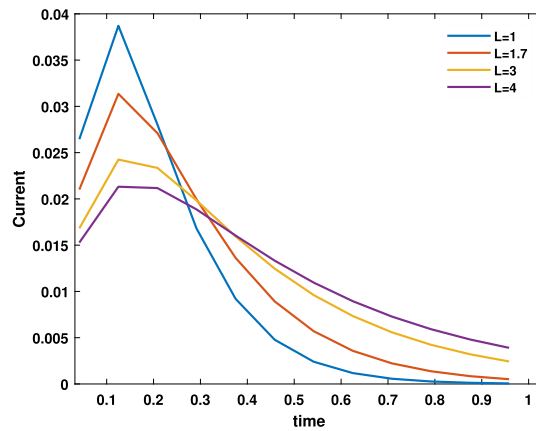
(b)



(c)



(d)



(e)

Fig. 5. RL circuit case 2 current vs time graphs.

Looking ahead, we anticipate that these numerical approaches will provide new insights into the application of FDEs. The obtained results can be extended to the study of fractional linear electrical circuits, linear systems with disturbed parameters, and nonlinear electrical circuits. Moreover, in case a new definition of fractional derivative emerges, we intend to apply the presented technique to solve similar models or those with analogous characteristics.

The outcomes derived from this research can be extrapolated to encompass a wide range of applications, including the analysis of fractional linear electrical circuits, linear systems with perturbed parameters, and nonlinear electrical circuits. Additionally, this approach can be applied to the examination of electromagnetic transients within electrical systems, control theory, transmission lines, machine windings, and power electronics.

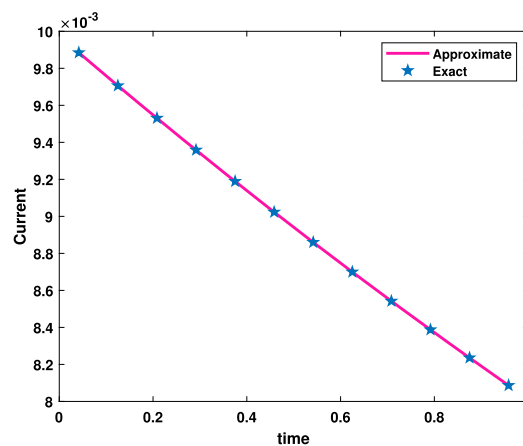


Fig. 6. RLC circuit exact and approximate comparison.

Funding

No funding.

Declaration of competing interest

We declared that there is no conflict of interest financial or non-financial among the authors.

Acknowledgements

We would like to thank Central University of Haryana and Prince Sattam bin Abdulaziz University for providing necessary facilities to carry out this research. This study was supported by Prince Sattam bin Abdulaziz University project number (PSAU/2023/R/1444).

References

- [1] Inc M, Yusuf A, Aliyu AI, Baleanu D. Time-fractional Cahn–Allen and time-fractional Klein–Gordon equations: Lie symmetry analysis, explicit solutions and convergence analysis. *Phys A, Stat Mech Appl* 2018;493:94–106.
- [2] Viera-Martin E, Gómez-Aguilar JF, Solís-Pérez JE, Hernández-Pérez JA, Escobar-Jiménez RF. Artificial neural networks: a practical review of applications involving fractional calculus. *Eur Phys J Spec Top* 2022;231(10):2059–95.
- [3] Korpınar Z, Inc M, Hınçal E, Baleanu D. Residual power series algorithm for fractional cancer tumor models. *Alex Eng J* 2020;59(3):1405–12.
- [4] Gao W, Veeresha P, Baskonus HM, Prakasha DG, Kumar P. A new study of unreported cases of 2019-nCoV epidemic outbreaks. *Chaos Solitons Fractals* 2020;138:109929.
- [5] Gao W, Veeresha P, Prakasha DG, Baskonus HM. New numerical simulation for fractional Benney–Lin equation arising in falling film problems using two novel techniques. *Numer Methods Partial Differ Equ* 2021;37(1):210–43.
- [6] Sun H, Zhang Y, Baleanu D, Chen W, Chen Y. A new collection of real world applications of fractional calculus in science and engineering. *Commun Nonlinear Sci Numer Simul* 2018;64:213–31.
- [7] Atangana A, Nieto JJ. Numerical solution for the model of RLC circuit via the fractional derivative without singular kernel. *Adv Mech Eng* 2015;7(10):1–7.
- [8] Srivastava S, Dwivedi AK, Nagaria D. Optimal design of symmetric fractional delay filter using firefly algorithm. *Int J Circuit Theory Appl* 2020;48(9):1511–22.
- [9] Srivastava HM, Irfan M, Shah FA. A Fibonacci wavelet method for solving dual-phase-lag heat transfer model in multi-layer skin tissue during hyperthermia treatment. *Energies* 2021;14(8):2254.
- [10] Gupta V, Jarad F, Valliammal N, Ravichandran C, Nisar KS. Existence and uniqueness of solutions for fractional nonlinear hybrid impulsive system. *Numer Methods Partial Differ Equ* 2022;38(3):359–71.
- [11] Ravichandran C, Logeswari K, Panda SK, Nisar KS. On new approach of fractional derivative by Mittag-Leffler kernel to neutral integro-differential systems with impulsive conditions. *Chaos Solitons Fractals* 2020;139:110012.
- [12] Podlubny I. Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications. Elsevier; 1998.
- [13] Miller KS, Ross B. An introduction to the fractional calculus and fractional differential equations. New York: Wiley; 1993.
- [14] Abusad S, Hashim I, Karim SAA. Modified fractional reduced differential transform method for the solution of multiterm time-fractional diffusion equations. *Adv Math Phys* 2019;2019.
- [15] Duan J, An J, Xu M. Solution of system of fractional differential equations by Adomian decomposition method. *Appl Math J Chin Univ* 2007;22(1):7–12.
- [16] Prakasha A, Goyal M, Gupta S. Fractional variational iteration method for solving time-fractional Newell–Whitehead–Segel equation. *Nonlinear Eng* 2019;8(1):164–71.
- [17] Maitama S, Zhao W. Local fractional homotopy analysis method for solving non-differentiable problems on Cantor sets. *Adv Differ Equ* 2019;2019:127.
- [18] Azin H, Mohammadi F, Tenreiro Machado JA. A piecewise spectral-collocation method for solving fractional Riccati differential equation in large domains. *Comput Appl Math* 2019;38:1–13.
- [19] Saeed U, Rehman M, Din Q. Differential quadrature method for nonlinear fractional partial differential equations. *Eng Comput* 2018;35(6):2349–66.
- [20] Yadav P, Jahan S, Nisar KS. Solving fractional Bagley–Torvik equation by fractional order Fibonacci wavelet arising in fluid mechanics. *Ain Shams Eng J* 2023;102299.
- [21] Shah FA, Irfan M, Nisar KS. Gegenbauer wavelet quasi-linearization method for solving fractional population growth model in a closed system. *Math Methods Appl Sci* 2022;45(7):3605–23.
- [22] Gómez JF, Astorga CM, Escobar RF, Medina MA, Guzmán R, González A, et al. Overview of simulation of fractional differential equation using numerical Laplace transform. *Cent Eur J Phys* 2012;1–14.
- [23] Shah PV, Patel AD, Salehkhani IA, Shukla AK. Analytic solution for the electric circuit model in fractional order. *Abstract and applied analysis*, vol. 2014. Hindawi; 2014.
- [24] Alsaedi A, Nieto JJ, Venkatesh V. Fractional electrical circuits. *Adv Mech Eng* 2015;7(12):1687814015618127.
- [25] Arora R, Chauhan NS. An application of Legendre wavelet in fractional electrical circuits. *Glob J Pure Appl Math* 2017;13(2):183–202.
- [26] Sene N, Gómez-Aguilar JF. Analytical solutions of electrical circuits considering certain generalized fractional derivatives. *Eur Phys J Plus* 2019;134(6):260.
- [27] Do QH, Ngo HTB, Razzaghi M. A generalized fractional-order Chebyshev wavelet method for two-dimensional distributed-order fractional differential equations. *Commun Nonlinear Sci Numer Simul* 2021;95:105597.

- [28] Idrees S, Saeed U. Generalized sine-cosine wavelet method for Caputo–Hadamard fractional differential equations. *Math Methods Appl Sci* 2022;45(16):9602–21.
- [29] Kilicman A, Zhou ZAA. Kronecker operational matrices for fractional calculus and some applications. *Appl Math Comput* 2007;187(1):250–65.
- [30] Odibat ZM, Shawagfeh NT. Generalized Taylor's formula. *Appl Math Comput* 2007;186(1):286–93.
- [31] Ahmed S, Jahan S, Nisar KS. Hybrid Fibonacci wavelet method to solve fractional-order logistic growth model. *Math Methods Appl Sci* 2023;1–14.
- [32] Hussain B, Afroz A, Jahan S. Approximate solution for proportional-delay Riccati differential equations by Haar wavelet method. *Poincare J Anal Appl* 2021;8(2):155–68.
- [33] Khan H, Alzabut J, Shah A, He ZY, Etemad S, Rezapour S, et al. On fractal-fractional waterborne disease model: a study on theoretical and numerical aspects of solutions via simulations. *Fractals* 2023;2340055.
- [34] Khan H, Alzabut J, Tunç O, Kaabar MK. A fractal–fractional COVID-19 model with a negative impact of quarantine on the diabetic patients. *Results Control Optim* 2023;10:100199.
- [35] Shah A, Khan H, De la Sen M, Alzabut J, Etemad S, Deressa CT, et al. On non-symmetric fractal-fractional modeling for ice smoking: mathematical analysis of solutions. *Symmetry* 2022;15(1):87.
- [36] Khan A, Khan ZA, Abdeljawad T, Khan H. Analytical analysis of fractional-order sequential hybrid system with numerical application. *Adv Cont Discr Mod* 2022;2022(1):1–9.
- [37] Khan A, Abdeljawad T, Khan H. A numerical scheme for the generalized ABC fractional derivative based on Lagrange interpolation polynomial. *Fractals* 2022;30(05):2240180.
- [38] Mirzaee F, Hadadiyan E. Applying the modified block-pulse functions to solve the three-dimensional Volterra–Fredholm integral equations. *Appl Math Comput* 2015;265:759–67.
- [39] Mirzaee F, Hadadiyan E, Bimesl S. Numerical solution for three-dimensional nonlinear mixed Volterra–Fredholm integral equations via three-dimensional block-pulse functions. *Appl Math Comput* 2014;237:168–75.
- [40] Mirzaee F, Hoseini SF. Hybrid functions of Bernstein polynomials and block-pulse functions for solving optimal control of the nonlinear Volterra integral equations. *Indag Math* 2016;27(3):835–49.
- [41] Samadyar N, Ordokhani Y, Mirzaee F. Hybrid Taylor and block-pulse functions operational matrix algorithm and its application to obtain the approximate solution of stochastic evolution equation driven by fractional Brownian motion. *Commun Nonlinear Sci Numer Simul* 2020;90:105346.
- [42] Mirzaee F, Samadyar N. Application of Bernoulli wavelet method for estimating a solution of linear stochastic Itô–Volterra integral equations. *Multidiscip Model Mater Struct* 2019;15(3):575–98.
- [43] Mirzaee F, Alipour S. Numerical solution of nonlinear partial quadratic integro-differential equations of fractional order via hybrid of block-pulse and parabolic functions. *Numer Methods Partial Differ Equ* 2019;35(3):1134–51.
- [44] Shah K, Khan ZA, Ali A, Amin R, Khan H, Khan A. Haar wavelet collocation approach for the solution of fractional order COVID-19 model using Caputo derivative. *Alex Eng J* 2020;59(5):3221–31.
- [45] Thirithar AA, Abboubakar H, Khan A, Abdeljawad T. Mathematical modeling of the COVID-19 epidemic with fear impact. *AIMS Math* 2023;8(3):6447–65.
- [46] Khan A, Alshehri HM, Gómez-Aguilar JF, Khan ZA, Fernández-Anaya G. A predator–prey model involving variable-order fractional differential equations with Mittag-Leffler kernel. *Adv Differ Equ* 2021;2021:183.
- [47] Alkhazzan A, Jiang P, Baleanu D, Khan H, Khan A. Stability and existence results for a class of nonlinear fractional differential equations with singularity. *Math Methods Appl Sci* 2018;41(18):9321–34.
- [48] Yadav P, Jahan S, Shah K, Peter OJ, Abdeljawad T. Fractional-order modelling and analysis of diabetes mellitus: utilizing the Atangana–Baleanu Caputo (ABC) operator. *Alex Eng J* 2023;81:200–9.

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