



Fractional order mathematical model of Ebola virus under Atangana–Baleanu–Caputo operator

Pooja Yadav^{a,1}, Shah Jahan^{a,1,*}, Kottakkaran Sooppy Nisar^{b,c,1}

^a Department of Mathematics, Central University of Haryana, Mohindergarh 123031, India

^b Department of Mathematics, College of Science and Humanities in Alkharj, Prince Sattam Bin Abdulaziz University, Alkharj 11942, Saudi Arabia

^c School of Technology, Woxsen University, Hyderabad 502345, Telangana State, India

ARTICLE INFO

MSC:

34A08

34A12

34A34

Keywords:

Atangana–Baleanu–Caputo operator

Fractional order Ebola virus

Existence and uniqueness

Fixed point theory

ABSTRACT

The aim of this paper is to analyze a fractional model of the Ebola virus. This study is important because it contributes to our understanding of the Ebola virus transmission dynamics using the notion of non-local differential operators. We aim to apply the recently implemented Atangana–Baleanu–Caputo (ABC) fractional derivative with the Mittag-Leffler kernel to study the Ebola virus model closely. The Picard–Lindelöf approach is used to do a comprehensive study of the existence and uniqueness of the model's solutions. The approximate solutions of the fractional order Ebola virus model were obtained using a numerical technique with the ABC operator, a combination of the fundamental theorem of fractional calculus and the two-step Lagrange polynomial interpolation. This innovative approach may offer new insights into the Ebola virus model that were not previously explored. Finally, the numerical simulations illustrate how the control parameters impact specific compartments within the model. The geometrical representation gives significant information about the model's complexity and reliable information about the model. We simulate each model compartment at various fractional orders and compare them with integer-order simulations, highlighting the effectiveness of modern derivatives. The fractional analysis underscores the enhanced accuracy of non-integer order derivatives in capturing the Ebola virus model's dynamics.

1. Introduction

Ebola is a fatal illness named after the Ebola River in the Democratic Republic of the Congo (formerly Zaire) [1]. Rare but deadly outbreaks of Ebola virus illness have been documented since the virus was originally isolated in 1976 along Africa's Ebola River. The illness was named after a hamlet near the Ebola River. The Ebola virus is very harmful. Individuals who have the virus have unusual amounts of bleeding throughout their bodies [2]. Primate species, including monkeys, chimpanzees, gorillas, and humans, are all susceptible to the Ebola virus. Through direct contact with the secretions, blood, organs, or other body fluids of infected animals, Ebola may be spread from infected animals to humans. The Ebola virus has five species: Bundibugyo, Zaire, Reston, Sudan, and Taiforest (TAFV). In its early stages, this virus was harming humans and triggering epidemics in a number of nations throughout Africa. Ebola may be transmitted from infected animals to humans. It was not possible to pinpoint where the Ebola virus first appeared. The treatment of sick forest antelope, monkeys, gorillas, chimpanzees, porcupines, and fruit bats discovered unwell or

* Corresponding author.

E-mail addresses: shahjahan@cuh.ac.in (S. Jahan), n.sooppy@psau.edu.sa (K.S. Nisar).

¹ All the authors have made equal contributions in this article.

<https://doi.org/10.1016/j.rico.2023.100332>

Received 21 June 2023; Received in revised form 15 October 2023; Accepted 2 November 2023

Available online 7 November 2023

2666-7207/© 2023 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Description of parameters**Variable**

S	Susceptible population
E	Exposed class
I_u	Infected undetected population
I_d	Infected detected population
I_s	Infected isolated Population
R	Population recovered
D	Population died

Parameter

Π	Recruitment rate
λ_1	Undetected infected transmission rate among humans
λ_2	Detected infected transmission rate among humans
λ_3	Dead bodies transmission rate among humans
ω_1	Detection rate for infected undetected Individuals
ω_2	Detection rate for Infected detected individuals which are isolated
θ	Endogenous reactivation rate
ζ_s	Isolated infected individuals death rate
ζ_d	Detected individuals death rate
ζ_u	Undetected individuals death rate
a	Isolated individuals recovery rate
μ	Natural death rate of human
ϵ	Progression rate of individuals in exposed stage to active Ebola
ψ	The rate at which exposed individuals are isolated due to transmission
δ	Burial rate

dead in the African rainforest has provided evidence of infection. Humans get infected with this virus first through skin-to-blood contact. According to the most recent 2014 epidemic in West Africa, contact with recently deceased Ebola patients caused about 30% of infections [1,3]. The number of recently deceased corpses from Ebola illness is connected to the infection rate due to the burial procedure [1,3]. This study developed an epidemiological model that included deceased people, sick people who were undiagnosed, confirmed infected people, and people who were in quarantine because of the threat the Ebola virus posed to the human population.

Several cases have been made where non-linear autonomous ordinary differential equations have been utilized to create mathematical models to characterize the dynamics of infectious disease spread utilizing a range of quantitative and qualitative methodologies [4–9]. In this study, we are interested in investigating the transmission of the hemorrhagic form of the Ebola virus using the theory of non-local differential operators.

Fractional calculus and its applications have recently made some excellent strides [10,11]. Many numerical and analytical methods have been developed to address fractional calculus issues [12–18]. Physics, biology, social science, economics, engineering, epidemiology, and other fields have effectively used fractional or non-classical calculus [19–22]. The benefit of using fractional calculus with various differential and integral operators is that, in contrast to normal-order derivatives, it is possible to use fractional operators of any arbitrary order. The author of [23] constructed a mathematical model based on the Ebola epidemic in Liberia, and the World Health Organization's current data was used to replicate the outcomes. The authors of [24] analyzed this model utilizing the fractional derivative of Atangana–Baleanu as well as the presence and distinctiveness of the solutions. The generalized Ebola virus model was examined by Atangana and Goufo [25], who also spoke about the endemic equilibrium locations. One can find further information about the Ebola virus in [26–28]. Researchers may find some new advancements in the field of numerical methods to solve these types of problems in [29,30].

Using Mittag-Leffler functions, this paper examines the unique fractional derivative known as the ABC operator. In the mathematical formulation, Atangana–Baleanu fractional differentiation aims to integrate the influence of non-local fading memory on the dynamical system. Recent advancements in non-local kernel analytical and numerical approaches are gaining additional benefits [31–34]. The development of the ABC operator originated from Caputo and Fabrizio's introduction of a fractional order derivative based on exponential functions. This derivative was initially proposed to address problems involving singular kernels and demonstrated effectiveness in tackling specific classes of physical problems. However, challenges emerged when this derivative was employed for cases with non-singular and non-local kernels. In such situations, it became evident that the associated integral operator did not conform to the characteristics of a fractional operator. To address these issues related to non-singular and non-local kernels, Atangana and Baleanu introduced two fractional derivatives, following the frameworks of Caputo and Riemann–Liouville

[35–42]. The derivatives now have fractional integrals in their outcomes as an anti-derivative of their operators. The authors of [33] explore a generalized Mittag-Leffler function as a non-local and non-singular kernel because the singular kernel of the classical order derivative poses a challenge to adequately explain the non-linear dynamics and crossover impact on a number of biological and physical phenomena. One benefit of using this derivative to represent biological issues is that the physical issue, such as the transmission of Ebola virus diseases into a designated population, follows the Mittag-Leffler rule because the transmission is anticipated to stop at a certain period and is neither non-local nor indefinitely exponential. The Mittag-Leffler guarantees that there will be no singularity, in contrast to the power law kernel given by the traditional Caputo and Riemann–Liouville derivatives. This allows us to clearly identify the beginning and conclusion of the spread's development. The numerical approach described in [33] will be utilized to approximate the Atangana–Baleanu fractional integral. This computational technique combines the two-step Lagrange polynomial interpolation with the principles of fractional calculus. Notably, this method has demonstrated remarkable precision and efficiency. It is also known for its user-friendliness and rapid convergence to the precise solution, even when dealing with substantial discretization steps [33].

The paper is arranged in the following way: First, some fundamental theory of the ABC derivative is presented in Section 2. The mathematical modeling of the spread of the Ebola virus is given in Section 3. The existence and uniqueness analysis is provided in Section 4. Numerical simulation and discussion are determined in Section 5. Section 6 presents a brief conclusion.

2. Preliminaries

Definition 2.1. On the interval $\chi \in [0, 1]$, by taking the function $f \in H^1(a, b)$, $b > a$, so the ABC derivative is given by

$${}^{ABC}D_{\xi}^{\chi} f(\xi) = \frac{M(\chi)}{1-\chi} \int_a^{\xi} f'(\theta) E_{\chi} \left[-\chi \frac{(\xi-\theta)^{\chi}}{1-\chi} \right] d\theta,$$

with $M(0) = M(1) = 1$ [33], here $M(\chi)$ is a normalization function.

Definition 2.2. On the interval $\chi \in [0, 1]$, by taking the function $f \in H^1(a, b)$, $b > a$, which is not differentiable, therefore the Atangana–Baleanu fractional derivative in Riemann–Liouville sense is defined as

$${}^{ABR}D_{\xi}^{\chi} f(\xi) = \frac{M(\chi)}{1-\chi} \frac{d}{d\xi} \int_a^{\xi} f(\theta) E_{\chi} \left[-\chi \frac{(\xi-\theta)^{\chi}}{1-\chi} \right] d\theta,$$

Definition 2.3. The ABC fractional derivative for the fractional integral of order χ is given by

$${}^{AB}I_{\xi}^{\chi} f(\xi) = \frac{1-\chi}{\mathcal{A}(\chi)} f(t) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_a^{\xi} f(y)(\xi-y)^{\chi-1} dy.$$

If $\chi = 0$ and $\chi = 1$, the initial function and ordinary integral are obtained, respectively.

In the subsequent sections, we will examine the Laplace transform operators and explore fundamental theorems associated with these derivatives. We will also explore the connection between these operators and the Laplace transform, specifically focusing on its representation in the following form:

$$\mathcal{L} \left\{ {}^{ABR}D_{\xi}^{\chi} [f(\xi)] \right\} (l) = \frac{L(\chi)}{1-\chi} \frac{l^{\chi} \mathcal{L}\{f(\xi)\}(l)}{l^{\chi} + \frac{\chi}{1-\chi}},$$

and

$$\mathcal{L} \left\{ {}^{ABC}D_{\xi}^{\chi} [f(\xi)] \right\} (l) = \frac{M(\chi)}{1-\chi} \frac{l^{\chi} \mathcal{L}\{f(\xi)\}(l) - l^{\chi-1} f(0)}{l^{\chi} + \frac{\chi}{1-\chi}}.$$

Theorem 2.1 ([43]). Take into account a closed interval $[a, b]$ and use g to represent a continuous function defined on it. We can prove the following inequality, which is true for any point inside of $[a, b]$:

$$\left\| {}^{ABR}D_{\xi}^{\chi} [f(\xi)] \right\| < \frac{M(\chi)}{1-\chi} \|f(\theta)\|,$$

where $\|f(\theta)\| = \max_{a \leq \xi \leq b} |f(\theta)|$.

Theorem 2.2 ([43]). The Riemann–Liouville and Caputo types of the Atangana–Baleanu derivative exhibit the Lipschitz condition, which is best defined as follows:

$$\left\| {}^{ABR}D_{\xi}^K [f(\xi)] - {}^{ABR}D_{\xi}^K [g(\xi)] \right\| \leq H \|f(\xi) - g(\xi)\|.$$

and

$$\left\| {}^{ABC}D_{\xi}^{\chi} [f(\xi)] - {}^{ABC}D_{\xi}^{\chi} [g(\xi)] \right\| \leq H \|f(\xi) - g(\xi)\|.$$

3. Mathematical model

The concept from the work of [44] is expanded upon in this section. In this part, the Ebola disease model is built using a dynamical system with ordinary differential equations. To create the mathematical model, the whole population of humans is categorized into susceptible population(S), exposed class(E), the population of infected undetected(I_u), population of infected detected(I_d), population of infected isolated(I_s), recovery population(R) and population died(D). As a result, the model system that has been regulated by the first-order ordinary differential equations in their classical form is shown by (see Fig. 1).

$$\begin{aligned}
 \frac{dS(\xi)}{d\xi} &= \Pi - \lambda_1 S I_u - \lambda_2 S I_d - \lambda_3 S D - \mu S, \\
 \frac{dE(\xi)}{d\xi} &= \lambda_1 S I_u + \lambda_2 S I_d + \lambda_3 S D - (\psi + \epsilon + \mu) E, \\
 \frac{dI_u(\xi)}{d\xi} &= (1 - \theta) \epsilon E - (\omega_1 + \zeta_u + \mu) I_u, \\
 \frac{dI_d(\xi)}{d\xi} &= \theta \epsilon E + \omega_1 I_u - (\omega_2 + \zeta_d + \mu) I_d, \\
 \frac{dI_s(\xi)}{d\xi} &= \psi E + \omega_2 I_d - (\zeta_s + a + \mu) I_s, \\
 \frac{dR(\xi)}{d\xi} &= a I_s - \mu R, \\
 \frac{dD(\xi)}{d\xi} &= \zeta_u I_u + \zeta_d I_d + \zeta_s I_s - \delta D.
 \end{aligned} \tag{1}$$

Since derivatives of fractional order represent epidemiological patterns better than classical order cases. In the present work, let us modify the classical Ebola virus model (1) in terms of the ABC derivative given below:

$$\begin{aligned}
 {}_0^{ABC} D_\xi^\chi S(\xi) &= \Pi - \lambda_1 S I_u - \lambda_2 S I_d - \lambda_3 S D - \mu S, \\
 {}_0^{ABC} D_\xi^\chi E(\xi) &= \lambda_1 S I_u + \lambda_2 S I_d + \lambda_3 S D - (\psi + \epsilon + \mu) E, \\
 {}_0^{ABC} D_\xi^\chi I_u(\xi) &= (1 - \theta) \epsilon E - (\omega_1 + \zeta_u + \mu) I_u, \\
 {}_0^{ABC} D_\xi^\chi I_d(\xi) &= \theta \epsilon E + \omega_1 I_u - (\omega_2 + \zeta_d + \mu) I_d, \\
 {}_0^{ABC} D_\xi^\chi I_s(\xi) &= \psi E + \omega_2 I_d - (\zeta_s + a + \mu) I_s, \\
 {}_0^{ABC} D_\xi^\chi R(\xi) &= a I_s - \mu R, \\
 {}_0^{ABC} D_\xi^\chi D(\xi) &= \zeta_u I_u + \zeta_d I_d + \zeta_s I_s - \delta D.
 \end{aligned} \tag{2}$$

with initial conditions

$$S(0) = S_0(\xi), E(0) = E_0(\xi), I_u(0) = I_{u_0}(\xi), I_d(0) = I_{d_0}(\xi), I_s(0) = I_{s_0}(\xi), R(0) = R_0(\xi), D(0) = D_0(\xi).$$

4. Existence and uniqueness analysis

Solving nonlinear equations is a challenging task within the domain of differential calculus. The fractional order model we are exploring exhibits nonlinearity, making it exceptionally difficult to derive exact solutions for such intricate systems. Consequently, our focus is on examining the issues surrounding the existence and uniqueness of the model described in Eq. (2). To achieve this, the fixed-point theorem will be used.

On the interval q , suppose that $P = K(q) \times K(q)$, where the Banach space $K(q)$ of continuous real value functions is defined with the norm

$$\|S, E, I_u, I_d, I_s, R, D\| = \|S\| + \|E\| + \|I_u\| + \|I_d\| + \|I_s\| + \|R\| + \|D\|,$$

where,

$$\begin{aligned}
 \|S\| &= \sup\{|S(\xi)| : \xi \in q\}, \\
 \|E\| &= \sup\{|E(\xi)| : \xi \in q\}, \\
 \|I_u\| &= \sup\{|I_u(\xi)| : \xi \in q\}, \\
 \|I_d\| &= \sup\{|I_d(\xi)| : \xi \in q\}, \\
 \|I_s\| &= \sup\{|I_s(\xi)| : \xi \in q\}, \\
 \|R\| &= \sup\{|R(\xi)| : \xi \in q\}, \\
 \|D\| &= \sup\{|D(\xi)| : \xi \in q\}.
 \end{aligned}$$

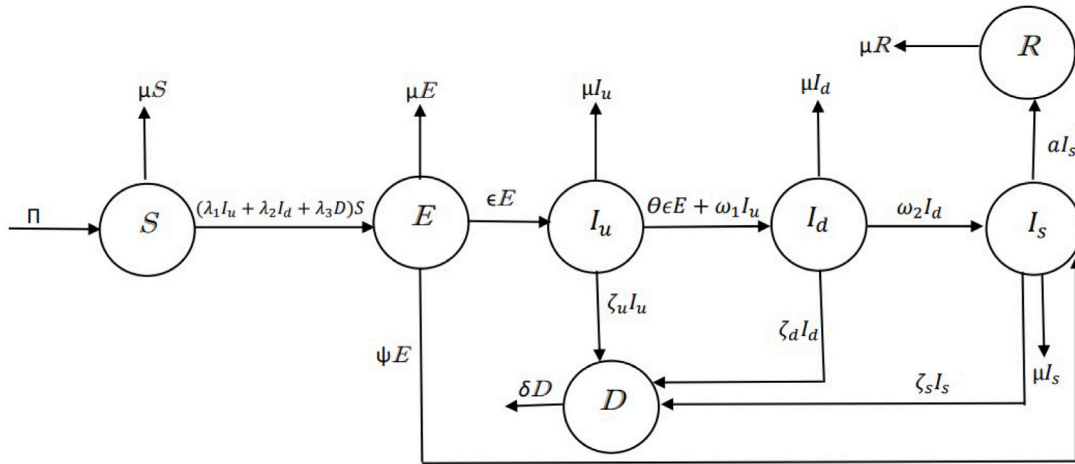


Fig. 1. Flowchart for the Ebola virus model.

The system (2) is transformed into the following equation by taking the Atangana–Baleanu fractional integral into account:

$$\begin{aligned}
 \mathcal{S}(\xi) - \mathcal{S}(0) &= \frac{1-\chi}{\mathcal{A}(\chi)} \{ \Pi - \lambda_1 \mathcal{S} \mathcal{I}_u - \lambda_2 \mathcal{S} \mathcal{I}_d - \lambda_3 \mathcal{S} \mathcal{D} - \mu \mathcal{S} \} \\
 &\quad + \frac{\chi}{\mathcal{A}(\chi) \Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \{ \Pi - \lambda_1 \mathcal{S} \mathcal{I}_u - \lambda_2 \mathcal{S} \mathcal{I}_d - \lambda_3 \mathcal{S} \mathcal{D} - \mu \mathcal{S} \} dy, \\
 \mathcal{E}(\xi) - \mathcal{E}(0) &= \frac{1-\chi}{\mathcal{A}(\chi)} \{ \lambda_1 \mathcal{S} \mathcal{I}_u + \lambda_2 \mathcal{S} \mathcal{I}_d + \lambda_3 \mathcal{S} \mathcal{D} - (\psi + \epsilon + \mu) \mathcal{E} \} \\
 &\quad + \frac{\chi}{\mathcal{A}(\chi) \Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \{ \lambda_1 \mathcal{S} \mathcal{I}_u + \lambda_2 \mathcal{S} \mathcal{I}_d + \lambda_3 \mathcal{S} \mathcal{D} - (\psi + \epsilon + \mu) \mathcal{E} \} dy, \\
 \mathcal{I}_u(\xi) - \mathcal{I}_u(0) &= \frac{1-\chi}{\mathcal{A}(\chi)} \{ (1-\theta) \epsilon \mathcal{E} - (\omega_1 + \zeta_u + \mu) \mathcal{I}_u \} \\
 &\quad + \frac{\chi}{\mathcal{A}(\chi) \Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \{ (1-\theta) \epsilon \mathcal{E} - (\omega_1 + \zeta_u + \mu) \mathcal{I}_u \} dy, \\
 \mathcal{I}_d(\xi) - \mathcal{I}_d(0) &= \frac{1-\chi}{\mathcal{A}(\chi)} \{ \theta \epsilon \mathcal{E} + \omega_1 \mathcal{I}_u - (\omega_2 + \zeta_d + \mu) \mathcal{I}_d \} \\
 &\quad + \frac{\chi}{\mathcal{A}(\chi) \Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \{ \theta \epsilon \mathcal{E} + \omega_1 \mathcal{I}_u - (\omega_2 + \zeta_d + \mu) \mathcal{I}_d \} dy, \\
 \mathcal{I}_s(\xi) - \mathcal{I}_s(0) &= \frac{1-\chi}{\mathcal{A}(\chi)} \{ \psi \mathcal{E} + \omega_2 \mathcal{I}_d - (\zeta_s + a + \mu) \mathcal{I}_s \} \\
 &\quad + \frac{\chi}{\mathcal{A}(\chi) \Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \{ \psi \mathcal{E} + \omega_2 \mathcal{I}_d - (\zeta_s + a + \mu) \mathcal{I}_s \} dy, \\
 \mathcal{R}(\xi) - \mathcal{R}(0) &= \frac{1-\chi}{\mathcal{A}(\chi)} \{ a \mathcal{I}_s - \mu \mathcal{R} \} \\
 &\quad + \frac{\chi}{\mathcal{A}(\chi) \Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \{ a \mathcal{I}_s - \mu \mathcal{R} \} dy, \\
 \mathcal{D}(\xi) - \mathcal{D}(0) &= \frac{1-\chi}{\mathcal{A}(\chi)} \{ \zeta_u \mathcal{I}_u + \zeta_d \mathcal{I}_d + \zeta_s \mathcal{I}_s - \delta \mathcal{D} \} \\
 &\quad + \frac{\chi}{\mathcal{A}(\chi) \Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \{ \zeta_u \mathcal{I}_u + \zeta_d \mathcal{I}_d + \zeta_s \mathcal{I}_s - \delta \mathcal{D} \} dy.
 \end{aligned} \tag{3}$$

We may simplify (3) by writing

$$\begin{aligned}
 K_1(\xi, \mathcal{S}) &= \Pi - \lambda_1 \mathcal{S} \mathcal{I}_u - \lambda_2 \mathcal{S} \mathcal{I}_d - \lambda_3 \mathcal{S} \mathcal{D} - \mu \mathcal{S}, \\
 K_2(\xi, \mathcal{E}) &= \lambda_1 \mathcal{S} \mathcal{I}_u + \lambda_2 \mathcal{S} \mathcal{I}_d + \lambda_3 \mathcal{S} \mathcal{D} - (\psi + \epsilon + \mu) \mathcal{E}, \\
 K_3(\xi, \mathcal{I}_u) &= (1-\theta) \epsilon \mathcal{E} - (\omega_1 + \zeta_u + \mu) \mathcal{I}_u, \\
 K_4(\xi, \mathcal{I}_d) &= \theta \epsilon \mathcal{E} + \omega_1 \mathcal{I}_u - (\omega_2 + \zeta_d + \mu) \mathcal{I}_d, \\
 K_5(\xi, \mathcal{I}_s) &= \psi \mathcal{E} + \omega_2 \mathcal{I}_d - (\zeta_s + a + \mu) \mathcal{I}_s, \\
 K_6(\xi, \mathcal{R}) &= a \mathcal{I}_s - \mu \mathcal{R}, \\
 K_7(\xi, \mathcal{D}) &= \zeta_u \mathcal{I}_u + \zeta_d \mathcal{I}_d + \zeta_s \mathcal{I}_s - \delta \mathcal{D}.
 \end{aligned}$$

Theorem 4.1. *If the aforementioned inequality holds:*

$$\begin{aligned} 0 &\leq \beta_1 < 1, \\ 0 &\leq \beta_2 < 1, \\ 0 &\leq \beta_3 < 1, \\ 0 &\leq \beta_4 < 1, \\ 0 &\leq \beta_5 < 1, \\ 0 &\leq \beta_6 < 1, \\ 0 &\leq \beta_7 < 1. \end{aligned}$$

Then the kernels $\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \mathcal{K}_4, \mathcal{K}_5, \mathcal{K}_6$ and $\mathcal{K}_7$ satisfy the Lipschitz condition and contraction.

Proof. By taking the kernel $\mathcal{K}_1(\xi, \mathcal{S}) = \square - \lambda_1 \mathcal{S} \mathcal{I}_u - \lambda_2 \mathcal{S} \mathcal{I}_d - \lambda_3 \mathcal{S} \mathcal{D} - \mu \mathcal{S}$. Let $\mathcal{S}$ and $\mathcal{S}_1$ be two functions, so we have:

$$\begin{aligned} \left\| \mathcal{K}_1(\xi, \mathcal{S}(\xi)) - \mathcal{K}_1(\xi, \mathcal{S}_1(\xi)) \right\| &= \left\| \square - \lambda_1 \mathcal{S} \mathcal{I}_u - \lambda_2 \mathcal{S} \mathcal{I}_d - \lambda_3 \mathcal{S} \mathcal{D} - \mu \mathcal{S} \right. \\ &\quad \left. - (\square - \lambda_1 \mathcal{S}_1 \mathcal{I}_u - \lambda_2 \mathcal{S}_1 \mathcal{I}_d - \lambda_3 \mathcal{S}_1 \mathcal{D} - \mu \mathcal{S}_1) \right\|, \\ &\leq \left\| -(\lambda_1 \mathcal{I}_u(\xi) + \lambda_2 \mathcal{I}_d(\xi) + \lambda_3 \mathcal{D}(\xi) + \mu) \right\| \left\| \mathcal{S}(\xi) - \mathcal{S}_1(\xi) \right\|, \\ &\leq (\lambda_1 \left\| \mathcal{I}_u(\xi) \right\| + \lambda_2 \left\| \mathcal{I}_d(\xi) \right\| + \lambda_3 \left\| \mathcal{D}(\xi) \right\| + \mu) \left\| \mathcal{S}(\xi) - \mathcal{S}_1(\xi) \right\|, \\ &\leq (\lambda_1 P_3 + \lambda_2 P_4 + \lambda_3 P_7 + \mu) \left\| \mathcal{S}(\xi) - \mathcal{S}_1(\xi) \right\|, \\ &\leq \beta_1 \left\| \mathcal{S}(\xi) - \mathcal{S}_1(\xi) \right\|. \end{aligned}$$

Taking $\beta_1 = \{\lambda_1 P_3 + \lambda_2 P_4 + \lambda_3 P_7 + \mu\}$, where $P_1 = \max_{\xi \in J} \left\| \mathcal{S}(\xi) \right\|$, $P_2 = \max_{\xi \in J} \left\| \mathcal{E}(\xi) \right\|$, $P_3 = \max_{\xi \in J} \left\| \mathcal{I}_u(\xi) \right\|$, $P_4 = \max_{\xi \in J} \left\| \mathcal{I}_d(\xi) \right\|$, $P_5 = \max_{\xi \in J} \left\| \mathcal{I}_s(\xi) \right\|$, $P_6 = \max_{\xi \in J} \left\| \mathcal{R}(\xi) \right\|$, $P_7 = \max_{\xi \in J} \left\| \mathcal{D}(\xi) \right\|$ are bounded function, then we get

$$\left\| \mathcal{K}_1(\xi, \mathcal{S}(\xi)) - \mathcal{K}_1(\xi, \mathcal{S}_1(\xi)) \right\| \leq \beta_1 \left\| \mathcal{S}(\xi) - \mathcal{S}_1(\xi) \right\|.$$

Thus, $\mathcal{K}_1$ satisfied the Lipschitz condition, and if $0 \leq \beta_1 < 1$, then it is also a contraction for $\mathcal{K}_1$. In same manner, the Lipschitz condition is satisfied by other kernels:

$$\begin{aligned} \left\| \mathcal{K}_2(\xi, \mathcal{E}(\xi)) - \mathcal{K}_2(\xi, \mathcal{E}_1(\xi)) \right\| &\leq \beta_2 \left\| \mathcal{E}(\xi) - \mathcal{E}_1(\xi) \right\|, \\ \left\| \mathcal{K}_3(\xi, \mathcal{I}_u(\xi)) - \mathcal{K}_3(\xi, \mathcal{I}_{u_1}(\xi)) \right\| &\leq \beta_3 \left\| \mathcal{I}_u(\xi) - \mathcal{I}_{u_1}(\xi) \right\|, \\ \left\| \mathcal{K}_4(\xi, \mathcal{I}_d(\xi)) - \mathcal{K}_4(\xi, \mathcal{I}_{d_1}(\xi)) \right\| &\leq \beta_4 \left\| \mathcal{I}_d(\xi) - \mathcal{I}_{d_1}(\xi) \right\|, \\ \left\| \mathcal{K}_5(\xi, \mathcal{I}_s(\xi)) - \mathcal{K}_5(\xi, \mathcal{I}_{s_1}(\xi)) \right\| &\leq \beta_5 \left\| \mathcal{I}_s(\xi) - \mathcal{I}_{s_1}(\xi) \right\|, \\ \left\| \mathcal{K}_6(\xi, \mathcal{R}(\xi)) - \mathcal{K}_6(\xi, \mathcal{R}_1(\xi)) \right\| &\leq \beta_6 \left\| \mathcal{R}(\xi) - \mathcal{R}_1(\xi) \right\|, \\ \left\| \mathcal{K}_7(\xi, \mathcal{D}(\xi)) - \mathcal{K}_7(\xi, \mathcal{D}_1(\xi)) \right\| &\leq \beta_7 \left\| \mathcal{D}(\xi) - \mathcal{D}_1(\xi) \right\|. \end{aligned}$$

By taking the kernels for the model into consideration, (3) can be presented by:

$$\begin{aligned} \mathcal{S}(\xi) &= \mathcal{S}(0) + \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_1(\xi, \mathcal{S}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_1(y, \mathcal{S}) dy, \\ \mathcal{E}(\xi) &= \mathcal{E}(0) + \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_2(\xi, \mathcal{E}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_2(y, \mathcal{E}) dy, \\ \mathcal{I}_u(\xi) &= \mathcal{I}_u(0) + \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_3(\xi, \mathcal{I}_u) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_3(y, \mathcal{I}_u) dy, \\ \mathcal{I}_d(\xi) &= \mathcal{I}_d(0) + \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_4(\xi, \mathcal{I}_d) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_4(y, \mathcal{I}_d) dy, \\ \mathcal{I}_s(\xi) &= \mathcal{I}_s(0) + \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_5(\xi, \mathcal{I}_s) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_5(y, \mathcal{I}_s) dy, \\ \mathcal{R}(\xi) &= \mathcal{R}(0) + \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_6(\xi, \mathcal{R}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_6(y, \mathcal{R}) dy, \\ \mathcal{D}(\xi) &= \mathcal{D}(0) + \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_7(\xi, \mathcal{D}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_7(y, \mathcal{D}) dy. \end{aligned} \tag{4}$$

Therefore, we can present it recursively as:

$$\begin{aligned}\mathcal{S}_v(\xi) &= \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_1(\xi, \mathcal{S}_{v-1}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_1(y, \mathcal{S}_{v-1}) dy, \\ \mathcal{E}_v(\xi) &= \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_2(\xi, \mathcal{E}_{v-1}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_2(y, \mathcal{E}_{v-1}) dy, \\ \mathcal{I}_{u_v}(\xi) &= \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_3(\xi, \mathcal{I}_{u_{v-1}}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_3(y, \mathcal{I}_{u_{v-1}}) dy, \\ \mathcal{I}_{d_v}(\xi) &= \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_4(\xi, \mathcal{I}_{d_{v-1}}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_4(y, \mathcal{I}_{d_{v-1}}) dy, \\ \mathcal{I}_{s_v}(\xi) &= \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_5(\xi, \mathcal{I}_{s_{v-1}}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_5(y, \mathcal{I}_{s_{v-1}}) dy, \\ \mathcal{R}_v(\xi) &= \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_6(\xi, \mathcal{R}_{v-1}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_6(y, \mathcal{R}_{v-1}) dy, \\ \mathcal{D}_v(\xi) &= \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_7(\xi, \mathcal{D}_{v-1}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} \mathcal{K}_7(y, \mathcal{D}_{v-1}) dy.\end{aligned}$$

With the defined initial conditions as:

$$\mathcal{S}_0(\xi) = \mathcal{S}(0), \mathcal{E}_0(\xi) = \mathcal{E}(0), \mathcal{I}_{u_0}(\xi) = \mathcal{I}_u(0), \mathcal{I}_{d_0}(\xi) = \mathcal{I}_d(0), \mathcal{I}_{s_0}(\xi) = \mathcal{I}_s(0), \mathcal{R}_0(\xi) = \mathcal{R}(0), \mathcal{D}_0(\xi) = \mathcal{D}(0).$$

The following system of equations is formed by using the initial conditions and the difference between the succeeding terms.

$$\begin{aligned}\Delta_v(\xi) &= \mathcal{S}_v(\xi) - \mathcal{S}_{v-1}(\xi) = \frac{1-\chi}{\mathcal{A}(\chi)} (\mathcal{K}_1(\xi, \mathcal{S}_{v-1}) - \mathcal{K}_1(\xi, \mathcal{S}_{v-2})) \\ &\quad + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} (\mathcal{K}_1(y, \mathcal{S}_{v-1}) - \mathcal{K}_1(y, \mathcal{S}_{v-2})) dy, \\ \mathfrak{N}_v(\xi) &= \mathcal{E}_v(\xi) - \mathcal{E}_{v-1}(\xi) = \frac{1-\chi}{\mathcal{A}(\chi)} (\mathcal{K}_2(\xi, \mathcal{E}_{v-1}) - \mathcal{K}_2(\xi, \mathcal{E}_{v-2})) + \\ &\quad \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} (\mathcal{K}_2(y, \mathcal{E}_{v-1}) - \mathcal{K}_2(y, \mathcal{E}_{v-2})) dy, \\ \Upsilon_v(\xi) &= \mathcal{I}_{u_v}(\xi) - \mathcal{I}_{u_{v-1}}(\xi) = \frac{1-\chi}{\mathcal{A}(\chi)} (\mathcal{K}_3(\xi, \mathcal{I}_{u_{v-1}}) - \mathcal{K}_3(\xi, \mathcal{I}_{u_{v-2}})) + \\ &\quad \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} (\mathcal{K}_3(y, \mathcal{I}_{u_{v-1}}) - \mathcal{K}_3(y, \mathcal{I}_{u_{v-2}})) dy, \\ \Xi_v(\xi) &= \mathcal{I}_{d_v}(\xi) - \mathcal{I}_{d_{v-1}}(\xi) = \frac{1-\chi}{\mathcal{A}(\chi)} (\mathcal{K}_4(\xi, \mathcal{I}_{d_{v-1}}) - \mathcal{K}_4(\xi, \mathcal{I}_{d_{v-2}})) + \\ &\quad \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} (\mathcal{K}_4(y, \mathcal{I}_{d_{v-1}}) - \mathcal{K}_4(y, \mathcal{I}_{d_{v-2}})) dy, \\ \varpi_v(\xi) &= \mathcal{I}_{s_v}(\xi) - \mathcal{I}_{s_{v-1}}(\xi) = \frac{1-\chi}{\mathcal{A}(\chi)} (\mathcal{K}_5(\xi, \mathcal{I}_{s_{v-1}}) - \mathcal{K}_5(\xi, \mathcal{I}_{s_{v-2}})) + \\ &\quad \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} (\mathcal{K}_5(y, \mathcal{I}_{s_{v-1}}) - \mathcal{K}_5(y, \mathcal{I}_{s_{v-2}})) dy, \\ \Psi_v(\xi) &= \mathcal{R}_v(\xi) - \mathcal{R}_{v-1}(\xi) = \frac{1-\chi}{\mathcal{A}(\chi)} (\mathcal{K}_6(\xi, \mathcal{R}_{v-1}) - \mathcal{K}_6(\xi, \mathcal{R}_{v-2})) + \\ &\quad \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} (\mathcal{K}_6(y, \mathcal{R}_{v-1}) - \mathcal{K}_6(y, \mathcal{R}_{v-2})) dy, \\ \mathcal{F}_v(\xi) &= \mathcal{D}_v(\xi) - \mathcal{D}_{v-1}(\xi) = \frac{1-\chi}{\mathcal{A}(\chi)} (\mathcal{K}_7(\xi, \mathcal{D}_{v-1}) - \mathcal{K}_7(\xi, \mathcal{D}_{v-2})) + \\ &\quad \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi (\xi-y)^{\chi-1} (\mathcal{K}_7(y, \mathcal{D}_{v-1}) - \mathcal{K}_7(y, \mathcal{D}_{v-2})) dy.\end{aligned}\tag{5}$$

It should be noted that

$$\begin{aligned}
 \mathbb{S}_v(\xi) &= \sum_{i=1}^v \mathbb{A}_i(\xi), \\
 \mathbb{E}_v(\xi) &= \sum_{i=1}^v \mathbb{N}_i(\xi), \\
 \mathbb{I}_{u_v}(\xi) &= \sum_{i=1}^v \mathbb{Y}_i(\xi), \\
 \mathbb{I}_{d_v}(\xi) &= \sum_{i=1}^v \mathbb{X}_i(\xi), \\
 \mathbb{I}_v(\xi) &= \sum_{i=1}^v \mathbb{W}_i(\xi), \\
 \mathbb{R}_v(\xi) &= \sum_{i=1}^v \mathbb{P}_i(\xi), \\
 \mathbb{D}_v(\xi) &= \sum_{i=1}^v \mathbb{F}_i(\xi).
 \end{aligned} \tag{6}$$

By taking (5), the triangular inequality is taken into account. After applying the norm to (6), the equation is transformed into (7).

$$\begin{aligned}
 \|\mathbb{A}_v(\xi)\| &= \|\mathbb{S}_v(\xi) - \mathbb{S}_{v-1}(\xi)\|, \\
 &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \|\mathcal{K}_1(\xi, \mathbb{S}_{v-1}) - \mathcal{K}_1(\xi, \mathbb{S}_{v-2})\| \\
 &\quad + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \left\| \int_0^\xi (\xi-y)^{\chi-1} (\mathcal{K}_1(y, \mathbb{S}_{v-1}) - \mathcal{K}_1(y, \mathbb{S}_{v-2})) dy \right\|.
 \end{aligned} \tag{7}$$

As the Lipschitz condition is satisfied by the kernel, the following equation is obtained:

$$\|\mathbb{S}_v(\xi) - \mathbb{S}_{v-1}(\xi)\| \leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_1 \|\mathbb{S}_{v-1} - \mathbb{S}_{v-2}\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_1 \int_0^\xi (\xi-y)^{\chi-1} \|\mathbb{S}_{v-1} - \mathbb{S}_{v-2}\| dy.$$

also,

$$\|\mathbb{A}_v(\xi)\| \leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_1 \|\mathbb{A}_{v-1}(\xi)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_1 \int_0^\xi (\xi-y)^{\chi-1} \|\mathbb{A}_{v-1}(y)\| dy. \tag{8}$$

In the same manner, we derived the following results:

$$\begin{aligned}
 \|\mathbb{N}_v(\xi)\| &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_2 \|\mathbb{N}_{v-1}(\xi)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_2 \int_0^\xi (\xi-y)^{\chi-1} \|\mathbb{N}_{v-1}(y)\| dy, \\
 \|\mathbb{Y}_v(\xi)\| &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_3 \|\mathbb{Y}_{v-1}(\xi)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_3 \int_0^\xi (\xi-y)^{\chi-1} \|\mathbb{Y}_{v-1}(y)\| dy, \\
 \|\mathbb{X}_v(\xi)\| &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_4 \|\mathbb{X}_{v-1}(\xi)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_4 \int_0^\xi (\xi-y)^{\chi-1} \|\mathbb{X}_{v-1}(y)\| dy, \\
 \|\mathbb{W}_v(\xi)\| &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_5 \|\mathbb{W}_{v-1}(\xi)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_5 \int_0^\xi (\xi-y)^{\chi-1} \|\mathbb{W}_{v-1}(y)\| dy, \\
 \|\mathbb{P}_v(\xi)\| &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_6 \|\mathbb{P}_{v-1}(\xi)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_6 \int_0^\xi (\xi-y)^{\chi-1} \|\mathbb{P}_{v-1}(y)\| dy, \\
 \|\mathbb{F}_v(\xi)\| &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_7 \|\mathbb{F}_{v-1}(\xi)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_7 \int_0^\xi (\xi-y)^{\chi-1} \|\mathbb{F}_{v-1}(y)\| dy. \quad \square
 \end{aligned}$$

Theorem 4.2. A unique solution is possessed by the proposed fractional order Ebola virus model with ABC operator (2) having the following condition that is satisfying the $\xi_{\max}$ as

$$\frac{1-\chi}{\mathcal{A}(\chi)} \beta_i + \frac{\xi_{\max}^\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_i < 1, \quad \text{for } i = 1, 2, \dots, 7.$$

Proof. It is clear that $\mathcal{S}(\xi)$, $\mathcal{E}(\xi)$, $\mathcal{I}_u(\xi)$, $\mathcal{I}_d(\xi)$, $\mathcal{I}_s(\xi)$, $\mathcal{R}(\xi)$ and $\mathcal{D}(\xi)$ are bounded. The kernels of these functions also satisfy the Lipschitz condition. Therefore, employing the succeeding relation with the application of the (8), we derived:

$$\begin{aligned}\|\mathcal{A}_v(\xi)\| &\leq \|\mathcal{S}(0)\| \left[\frac{1-\chi}{\mathcal{A}(\chi)} \beta_1 + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_1 \right]^v, \\ \|\mathcal{B}_v(\xi)\| &\leq \|\mathcal{E}(0)\| \left[\frac{1-\chi}{\mathcal{A}(\chi)} \beta_2 + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_2 \right]^v, \\ \|\mathcal{Y}_v(\xi)\| &\leq \|\mathcal{I}_u(0)\| \left[\frac{1-\chi}{\mathcal{A}(\chi)} \beta_3 + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_3 \right]^v, \\ \|\mathcal{Z}_v(\xi)\| &\leq \|\mathcal{I}_d(0)\| \left[\frac{1-\chi}{\mathcal{A}(\chi)} \beta_4 + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_4 \right]^v, \\ \|\mathcal{W}_v(\xi)\| &\leq \|\mathcal{I}_s(0)\| \left[\frac{1-\chi}{\mathcal{A}(\chi)} \beta_5 + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_5 \right]^v, \\ \|\mathcal{V}_v(\xi)\| &\leq \|\mathcal{R}(0)\| \left[\frac{1-\chi}{\mathcal{A}(\chi)} \beta_6 + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_6 \right]^v, \\ \|\mathcal{F}_v(\xi)\| &\leq \|\mathcal{D}(0)\| \left[\frac{1-\chi}{\mathcal{A}(\chi)} \beta_7 + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_7 \right]^v.\end{aligned}$$

Hence, (6) is a smooth function and exists.

Let us suppose that the aforementioned functions represent the model's solutions.

$$\begin{aligned}\mathcal{S}(\xi) - \mathcal{S}(0) &= \mathcal{S}_v(\xi) - \Theta_{1(v)}(\xi), \\ \mathcal{E}(\xi) - \mathcal{E}(0) &= \mathcal{E}_v(\xi) - \Theta_{2(v)}(\xi), \\ \mathcal{I}_u(\xi) - \mathcal{I}_u(0) &= \mathcal{I}_{u_v}(\xi) - \Theta_{3(v)}(\xi), \\ \mathcal{I}_d(\xi) - \mathcal{I}_d(0) &= \mathcal{I}_{d_v}(\xi) - \Theta_{4(v)}(\xi), \\ \mathcal{I}_s(\xi) - \mathcal{I}_s(0) &= \mathcal{I}_{s_v}(\xi) - \Theta_{5(v)}(\xi), \\ \mathcal{R}(\xi) - \mathcal{R}(0) &= \mathcal{R}_v(\xi) - \Theta_{6(v)}(\xi), \\ \mathcal{D}(\xi) - \mathcal{D}(0) &= \mathcal{D}_v(\xi) - \Theta_{7(v)}(\xi).\end{aligned}$$

The term $\|\Theta_{\infty}(\xi)\| \rightarrow 0$ at infinity. To show this, the following is taken:

$$\begin{aligned}\|\Theta_{1(v)}(\xi)\| &\leq \left\| \frac{1-\chi}{\mathcal{A}(\chi)} \mathcal{K}_1(\xi, \mathcal{S}) - \mathcal{K}_1(\xi, \mathcal{S}_{v-1}) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^{\xi} (\xi-y)^{\chi-1} (\mathcal{K}_1(\xi, \mathcal{S}) - \mathcal{K}_1(\xi, \mathcal{S}_{v-1})) dy \right\|, \\ \|\Theta_{1(v)}(\xi)\| &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \|\mathcal{K}_1(\xi, \mathcal{S}) - \mathcal{K}_1(\xi, \mathcal{S}_{v-1})\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^{\xi} (\xi-y)^{\chi-1} \|\mathcal{K}_1(\xi, \mathcal{S}) - \mathcal{K}_1(\xi, \mathcal{S}_{v-1})\| dy, \\ &\leq \frac{1-\chi}{\mathcal{A}(\chi)} \beta_1 \|\mathcal{S} - \mathcal{S}_{v-1}\| + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \beta_1 \|\mathcal{S} - \mathcal{S}_{v-1}\|.\end{aligned}$$

By recursively repeating the process, we get

$$\|\Theta_{1(v)}(\xi)\| \leq \left[\frac{1-\chi}{\mathcal{A}(\chi)} + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \right]^{v+1} \beta_1^v M.$$

So, for $\xi_{\max}$ we get

$$\|\Theta_{1(v)}(\xi)\| \leq \left[\frac{1-\chi}{\mathcal{A}(\chi)} + \frac{\xi_{\max}^{\chi}}{\mathcal{A}(\chi)\Gamma(\chi)} \right]^{v+1} \beta_1^v M.$$

By taking the limit on both sides as $v \rightarrow \infty$, we get $\|\Theta_{\infty}(\xi)\| \rightarrow 0$. $\square$

Uniqueness of the solution

The ability to demonstrate the system's uniqueness is an important application. So, via contraction, we suppose that there is another system of solutions to (2), $\mathcal{S}_1(\xi)$, $\mathcal{E}_1(\xi)$, $\mathcal{I}_{u_1}(\xi)$, $\mathcal{I}_{d_1}(\xi)$, $\mathcal{I}_{s_1}(\xi)$, $\mathcal{R}_1(\xi)$ and $\mathcal{D}_1(\xi)$. Then

$$\|\mathcal{S}(\xi) - \mathcal{S}_1(\xi)\| \leq \frac{1-\chi}{\mathcal{A}(\chi)} \|\mathcal{K}_1(\xi, \mathcal{S}) - \mathcal{K}_1(\xi, \mathcal{S}_1)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^{\xi} (\xi-y)^{\chi-1} \|\mathcal{K}_1(\xi, \mathcal{S}) - \mathcal{K}_1(\xi, \mathcal{S}_1)\| dy, \quad (9)$$

Now applying the norm to (9), we compute

$$\|\mathcal{S}(\xi) - \mathcal{S}_1(\xi)\| \leq \frac{1-\chi}{\mathcal{A}(\chi)} \|\mathcal{K}_1(\xi, \mathcal{S}) - \mathcal{K}_1(\xi, \mathcal{S}_1)\| + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^{\xi} (\xi-y)^{\chi-1} \|\mathcal{K}_1(\xi, \mathcal{S}) - \mathcal{K}_1(\xi, \mathcal{S}_1)\| dy,$$

By utilizing the kernel's Lipschitz conditional properties, we obtain

$$\|\mathcal{S}(\xi) - \mathcal{S}_1(\xi)\| \leq \frac{1-\chi}{\mathcal{A}(\chi)} \|\mathcal{S}(\xi) - \mathcal{S}_1(\xi)\| \beta_1 + \frac{\beta_1 \xi^\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \|\mathcal{S}(\xi) - \mathcal{S}_1(\xi)\|,$$

which gives

$$\begin{aligned} \|\mathcal{S}(\xi) - \mathcal{S}_1(\xi)\| \left(1 - \beta_1 \frac{1-\chi}{\mathcal{A}(\chi)} + \frac{\xi^\chi \beta_1}{\mathcal{A}(\chi)\Gamma(\chi)} \right) &\leq 0, \\ \|\mathcal{S}(\xi) - \mathcal{S}_1(\xi)\| = 0 &\rightarrow \mathcal{S}(\xi) = \mathcal{S}_1(\xi). \end{aligned}$$

So the equation has a unique solution. In the same manner, one can obtain the same outcomes for other solutions of $\mathcal{E}(\xi)$, $\mathcal{I}_u(\xi)$, $\mathcal{I}_v(\xi)$, $\mathcal{I}_s(\xi)$, $\mathcal{R}(\xi)$ and $\mathcal{D}(\xi)$.

5. Numerical scheme

A unique numerical approach to solve several particular problems with non-singular, non-local kernels of fractional derivatives has been proposed by Toufik and Atangana [33]. It is apparent from their study that their technique is not only remarkably precise but also rapidly converges to exact solutions. As other well-known methods such as Eulers, Adams–Bashforth, etc. cannot efficiently and convergently solve the fractional derivatives of non-singular, non-local kernels, this method has the benefit of enhancing the limitations of those methods. By taking the aforementioned non-linear fractional ordinary equation to illustrate their methodology:

$$\begin{cases} {}_0^{ABC} D_\xi^\chi \kappa(\xi) = f(\xi, \kappa(\xi)), \\ \kappa(0) = \kappa_0. \end{cases} \quad (10)$$

Using the core concepts of fractional calculus, the (10) is converted into a fractional integral equation.

$$\kappa(\xi) - \kappa(0) = \frac{1-\chi}{\mathcal{A}(\chi)} f(\xi, \kappa(\xi)) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^\xi f(y, \kappa(y)) (\xi - y)^{\chi-1} dy. \quad (11)$$

At the point $\xi = \xi_{v+1}$, $v = 0, 1, 2, \dots$ the above (11) can be written as

$$\kappa(\xi_{v+1}) - \kappa(0) = \frac{1-\chi}{\mathcal{A}(\chi)} f(\xi_v, \kappa(\xi_v)) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \int_0^{\xi_{v+1}} f(y, \kappa(y)) (\xi_{v+1} - y)^{\chi-1} dy. \quad (12)$$

Let us assume $f(y, \kappa(y))$ in the interval $[\xi_l, \xi_{l+1}]$ and applying Lagrange interpolation, we get:

$$\begin{aligned} p_l(y) &= f(y, \kappa(y)), \\ &= \frac{y - \xi_{l-1}}{\xi_l - \xi_{l-1}} f(\xi_l, \kappa(\xi_l)) - \frac{y - \xi_l}{\xi_l - \xi_{l-1}} f(\xi_{l-1}, \kappa(\xi_{l-1})), \\ &= \frac{f(\xi_l, \kappa(\xi_l))}{h} (y - \xi_{l-1}) - \frac{f(\xi_{l-1}, \kappa(\xi_{l-1}))}{h} (y - \xi_l), \\ &\simeq \frac{f(\xi_l, \kappa_l)}{h} (y - \xi_{l-1}) - \frac{f(\xi_{l-1}, \kappa_{l-1})}{h} (y - \xi_l). \end{aligned}$$

By assuming (12) again for $f(y, \kappa(y))$ and applying Lagrange interpolation where h is step length, one gets:

$$\begin{aligned} \kappa_{v+1} &= \frac{1-\chi}{\mathcal{A}(\chi)} f(\xi_v, \kappa(\xi_v)) + \frac{\chi}{\mathcal{A}(\chi)\Gamma(\chi)} \sum_{l=0}^v \left(\frac{f(\xi_l, \kappa_l)}{h} \int_{\xi_l}^{\xi_{l+1}} (y - \xi_{l-1}) (\xi_{v+1} - y)^{\chi-1} dy \right. \\ &\quad \left. - \frac{f(\xi_{l-1}, \kappa_{l-1})}{h} \int_{\xi_l}^{\xi_{l+1}} (y - \xi_l) (\xi_{v+1} - y)^{\chi-1} dy \right) + \kappa_0. \end{aligned} \quad (13)$$

Let us take the following equation without losing its generality:

$$P_{\chi,l,1} = \int_{\xi_l}^{\xi_{l+1}} (y - \xi_{l-1}) (\xi_{v+1} - y)^{\chi-1} dy, \quad (14)$$

and

$$P_{\chi,l,2} = \int_{\xi_l}^{\xi_{l+1}} (y - \xi_l) (\xi_{v+1} - y)^{\chi-1} dy, \quad (15)$$

so that

$$P_{\chi,l,1} = h^{\chi+1} \frac{(v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)}{\chi(\chi+1)}, \quad (16)$$

and

$$P_{\chi,l,2} = h^{\chi+1} \frac{(v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)}{\chi(\chi+1)}. \quad (17)$$

By integrating (15), (16), (17), and replacing them into (13), we compute

$$\begin{cases} x_{v+1} = \frac{(1-\chi)}{\mathcal{A}(\chi)} f(\xi_v, \kappa(\xi_v)) + \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \left(\frac{h^\chi f(\xi_l, \kappa_l)}{\Gamma(\chi+2)} \right) \\ \left(((v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)) \right) - \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \\ \left(\frac{h^\chi f(\xi_{l-1}, \kappa_{l-1})}{\Gamma(\chi+2)} ((v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)) \right) + \kappa(0). \end{cases} \quad (18)$$

The fractional derivative of ABC is numerically represented in (18). By applying this approach to the suggested system (2), the following results are obtained:

$$\begin{cases} \mathcal{S}_{v+1} = \frac{(1-\chi)}{\mathcal{A}(\chi)} \mathcal{K}_1(\xi_n, \mathcal{S}(\xi_n)) + \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_1(\xi_l, \mathcal{S}_l)}{\Gamma(\chi+2)} \right) \\ \left(((v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)) \right) - \frac{\chi}{\mathcal{A}(\chi)} \\ \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_1(\xi_{l-1}, \mathcal{S}_{l-1})}{\Gamma(\chi+2)} ((v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)) \right) + \mathcal{S}(0), \\ \mathcal{E}_{v+1} = \frac{(1-\chi)}{\mathcal{A}(\chi)} \mathcal{K}_2(\xi_n, \mathcal{E}(\xi_n)) + \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_2(\xi_l, \mathcal{E}_l)}{\Gamma(\chi+2)} \right) \\ \left(((v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)) \right) - \frac{\chi}{\mathcal{A}(\chi)} \\ \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_2(\xi_{l-1}, \mathcal{E}_{l-1})}{\Gamma(\chi+2)} ((v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)) \right) + \mathcal{E}(0), \\ \mathcal{I}_{u_v+1} = \frac{(1-\chi)}{\mathcal{A}(\chi)} \mathcal{K}_3(\xi_n, \mathcal{I}_u(\xi_n)) + \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_3(\xi_l, \mathcal{I}_{u_l})}{\Gamma(\chi+2)} \right) \\ \left(((v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)) \right) - \frac{\chi}{\mathcal{A}(\chi)} \\ \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_3(\xi_{l-1}, \mathcal{I}_{u_{l-1}})}{\Gamma(\chi+2)} ((v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)) \right) + \mathcal{I}_u(0), \\ \mathcal{I}_{d_v+1} = \frac{(1-\chi)}{\mathcal{A}(\chi)} \mathcal{K}_4(\xi_n, \mathcal{I}_d(\xi_n)) + \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_4(\xi_l, \mathcal{I}_{d_l})}{\Gamma(\chi+2)} \right) \\ \left(((v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)) \right) - \frac{\chi}{\mathcal{A}(\chi)} \\ \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_4(\xi_{l-1}, \mathcal{I}_{d_{l-1}})}{\Gamma(\chi+2)} ((v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)) \right) + \mathcal{I}_d(0), \\ \mathcal{I}_{s_v+1} = \frac{(1-\chi)}{\mathcal{A}(\chi)} \mathcal{K}_5(\xi_n, \mathcal{I}_s(\xi_n)) + \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_5(\xi_l, \mathcal{I}_{s_l})}{\Gamma(\chi+2)} \right) \\ \left(((v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)) \right) - \frac{\chi}{\mathcal{A}(\chi)} \\ \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_5(\xi_{l-1}, \mathcal{I}_{s_{l-1}})}{\Gamma(\chi+2)} ((v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)) \right) + \mathcal{I}_s(0), \\ \mathcal{R}_{v+1} = \frac{(1-\chi)}{\mathcal{A}(\chi)} \mathcal{K}_6(\xi_n, \mathcal{R}(\xi_n)) + \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_6(\xi_l, \mathcal{R}_l)}{\Gamma(\chi+2)} \right) \\ \left(((v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)) \right) - \frac{\chi}{\mathcal{A}(\chi)} \\ \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_6(\xi_{l-1}, \mathcal{R}_{l-1})}{\Gamma(\chi+2)} ((v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)) \right) + \mathcal{R}(0), \\ \mathcal{D}_{v+1} = \frac{(1-\chi)}{\mathcal{A}(\chi)} \mathcal{K}_7(\xi_n, \mathcal{D}(\xi_n)) + \frac{\chi}{\mathcal{A}(\chi)} \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_7(\xi_l, \mathcal{D}_l)}{\Gamma(\chi+2)} \right) \\ \left(((v-l+1)^\chi (v-l+\chi+2) - (v-l)^\chi (v-l+2+2\chi)) \right) - \frac{\chi}{\mathcal{A}(\chi)} \\ \sum_{l=0}^v \left(\frac{h^\chi \mathcal{K}_7(\xi_{l-1}, \mathcal{D}_{l-1})}{\Gamma(\chi+2)} ((v-l+1)^{\chi+1} - (v-l)^\chi (v+1-l+\chi)) \right) + \mathcal{D}(0). \end{cases}$$

6. Numerical simulation

In the current study, we deliberated the model developed by [44] for observing the dynamic spread of the disease in the population and obtained some results for the considered model. The purpose of this extensive model is to observe what happens when fractional order χ changes in the model (2). Hence, we will present multiple numerical simulations of this model employing a computational method recently advanced by Toufik and Atangana [33]. Here, the numerical outcomes are presented for model (2) employing the parameters' values defined in Table 1, which are compatible with Ebola [3,45] for different χ . Now, by taking

Table 1
Parameter and values.

Parameters	Values	References
Π	500	[44]
λ_1	0.000118	[44]
λ_2	0.000118	[44]
λ_3	0.000118	[44]
μ	0.02	[44]
ψ	0.2	[44]
ϵ	0.6	[44]
θ	0.03	[44]
ω_1	0.12	[44]
ω_2	0.12	[44]
ζ_u	0.937	[44]
ζ_d	0.937	[44]
ζ_s	0.937	[44]
a	0.225	[44]
δ	0.8	[44]

initial values as

$$\begin{aligned} S(0) &= 1000, & I_s(0) &= 150, \\ E(0) &= 500, & R(0) &= 100, \\ I_u(0) &= 300, & D(0) &= 20, \\ I_d(0) &= 250, \end{aligned}$$

Fig. 2 exemplifies the effect of derivative order and behavior on achieved outcomes by the proposed solution procedure for $S(\xi)$, $E(\xi)$, $I_u(\xi)$, $I_d(\xi)$, $I_s(\xi)$, $R(\xi)$ and $D(\xi)$. The outcomes have been plotted for different fractional orders $\chi = 1, 0.95, 0.9, 0.85$, and 0.8 in Fig. 2 of the model (2). From Fig. 2(a), it is observed that the group of susceptible people $S(\xi)$ varies continuously with time derivative. From Fig. 2(f) it is clear that the group of recovered people increases with time. As there is inclined in the fractional order, it is observed that the recovery rate R increases with the decline in the death population D . In Fig. 2(c, d, e), we have plotted the population of infected undetected, detected, and isolated w.r.t. time for different values of fractional order time derivatives. From Fig. 2(c, d, e), it is observed that the group of infected population varies continuously with time derivative. In Fig. 3(a) we analyze the rate of exposed individuals which are isolated at $\psi = 0.2, 0.35$, and 0.5 on $\chi = 0.95$. Fig. 3(b) shows the detection rate at $\omega_1 = 0.12, 0.42$, and 0.62 on $\chi = 0.95$ for infected and undetected individuals. In brief, a low detection rate in contact tracing leads to a substantial increase in undetected infections and, consequently, a higher total infection count in the population. Fig. 3(c) analyzes the rate of infected detected individuals by setting $\omega_2 = 0.12, 0.42$, and 0.62 at $\chi = 0.95$. The results indicate that an increased rate of transition from infected detected individuals to isolated individuals, as a consequence of isolation techniques, correlates with a reduction in the number of infected detected individuals within the population. And the recovery rate at $a = 0.225, 0.255$, and 0.275 on $\chi = 0.95$ of isolated individuals is given by Fig. 3(d). As the isolated individual's recovery rate increases, the recovery population also increases. From the figures, it is apparent that the model (2) exhibits a substantial degree of order dependence and flexibility. Additionally, compared to the integer order model [44], the fractional order model produces more intriguing findings and allows for a better examination of the outcomes. MATLAB (R2020a) software is used for graphical simulations.

7. Conclusion

The fractional mathematical model of Ebola virus diseases has been examined using the idea of fractional differential. In this paper, a nonsingular kernel in the sense of fractional ABC derivatives is used to investigate the model. First, the classical form of the Ebola virus model is modified in terms of the ABC derivative. Second, fixed point theory establishes the existence and uniqueness of a system of solutions under certain circumstances. Figures indicate that the results are quite intriguing, demonstrating the continuous behavior of the approximation solution for fractional order. Fig. 3 represents the behavior of parameters ψ , ω_1 , ω_2 , and a for exposed, infected undetected, infected detected, and recovery populations, respectively. We presented different graphical findings for various values of χ for the solution of the model. Based on the analysis of the numerical solution, it becomes evident that the dynamics of the Ebola virus model within a fixed total population are intricately linked to its parameters. These parameter values play a pivotal role in influencing the increase in the number of individuals who recover from the virus within society. This paper exemplifies the applications of the proposed algorithm and analyzes the real-world problem by taking fractional operators into consideration and modeling relevant effects. In the future, the proposed algorithm can be extended to solve diverse biological and physical models. The ABC fractional order derivative can be extended to solve more complex types of equations and to analyze their stability. As far as future research work is concerned, in order to get more insights into the dynamics of the spread of the Ebola virus, a fractal fractional operator can be used for further elaboration of the model. Also, one may apply the fractional optimal control technique for the said model.

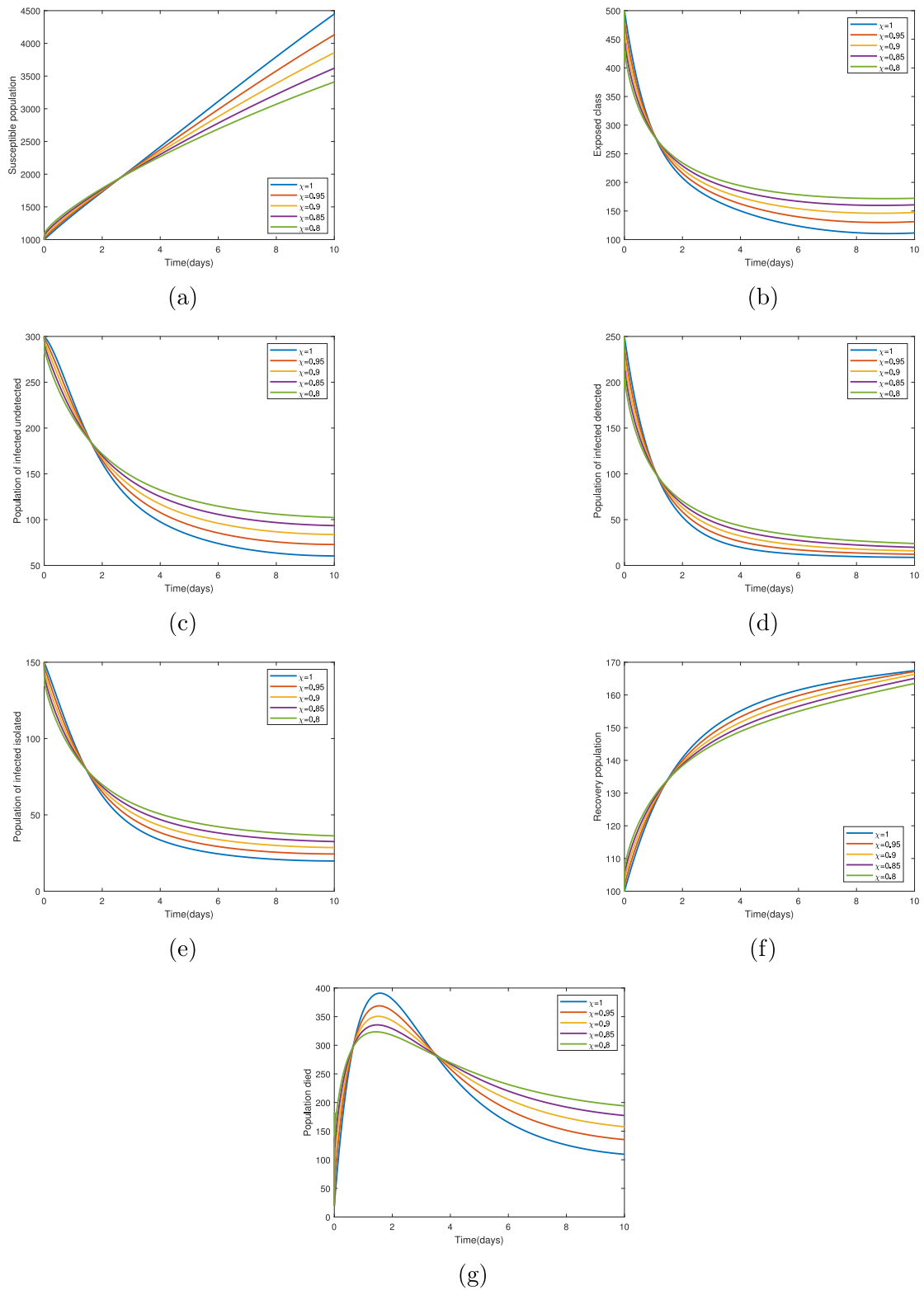


Fig. 2. Numerical results at different values of χ .

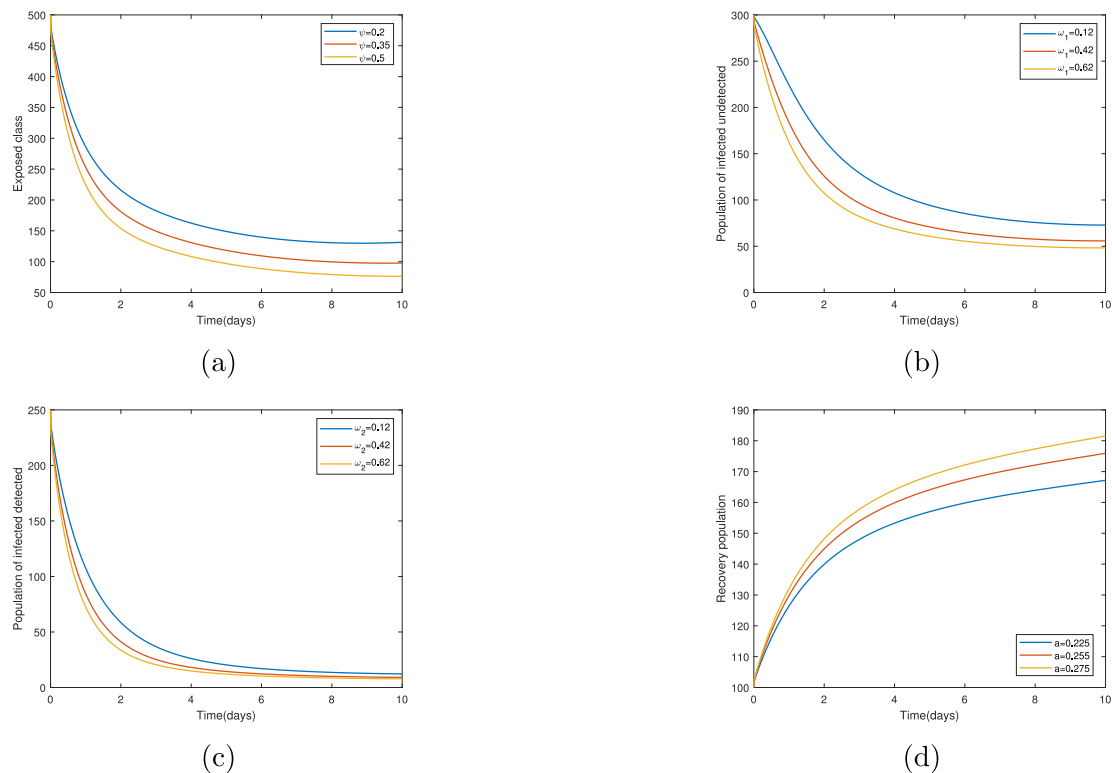


Fig. 3. Plot (a), (b), (c), and (d) shows the behavior of exposed class, population of infected undetected, population of infected detected and recovery population for different values of ψ , ω_1 , ω_2 and a respectively.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgments

We would like to thank Central University of Haryana for providing necessary facilities to carry out this research. This study was supported via funding from Prince Sattam bin Abdulaziz University, Saudi Arabia project number (PSAU/2023/R/1444). All authors read and approved the final manuscript.

Funding

No funding.

References

- [1] CDC website: Ebola – QAs on transmission. 2015, Available:<http://www.cdc.gov/vhf/ebola/transmission/qas.html>.
- [2] Bausch DG, Feldmann H, Geisbert TW, Bray M, Sprecher AG, Boumandouki P, Rollin PE, Roth C. And winnipeg filovirus clinical working group, outbreaks of filovirus hemorrhagic fever: time to refocus on the patient. *J Infect Dis* 2007;196:S136–41.
- [3] Samba TJM, Aldila D, Soewono E. Epidemic model for ebola disease. In: Proc. of the second international conference on advances in applied science and environmental technology. 2015.
- [4] Cooper I, Mondal A, Antonopoulos CG. A SIR model assumption for the spread of COVID-19 in different communities. *Chaos Solitons Fractals* 2020;139:110057.
- [5] Bekiros S, Kouloumpou D. Sbdien: a new mathematical model of infectious disease dynamics. *Chaos Solitons Fractals* 2020;136:109828.

- [6] Masoumnezhad M, Rajabi M, Chapnevis A, Dorofeev A, Shateyi S, Kargar NS, Nik HS. An approach for the global stability of mathematical model of an infectious disease. *Symmetry* 2020;12(11):1778.
- [7] Yüzbaşı Ş, İzadi M. Bessel-quasilinearization technique to solve the fractional-order HIV-1 infection of CD4+ T-cells considering the impact of antiviral drug treatment. *Appl Math Comput* 2022;431:127319.
- [8] Singh R, Mishra J, Gupta VK. The dynamical analysis of a tumor growth model under the effect of fractal fractional Caputo–fabrizio derivative. *Int J Math Comput Eng* 2023;1(1):115–26.
- [9] Sabir Z, Umar M. Levenberg–Marquardt backpropagation neural network procedures for the consumption of hard water-based kidney function. *Int J Math Comput Eng* 2023;1(1):127–38.
- [10] Baba IA, Ghanbari B. Existence and uniqueness of solution of a fractional order tuberculosis model. *Eur Phys J Plus* 2019;134:1–10.
- [11] Özdemir N, Agrawal OP, Iskender BB, Karadeniz D. Fractional optimal control of a 2-dimensional distributed system using eigenfunctions. *Nonlinear Dynam* 2009;55:251–60.
- [12] Cattani C, Srivastava HM, Yang XJ. Fractional dynamics, walter de gruyter gmbh and co KG. 2015.
- [13] Singh H, Srivastava HM, Kumar D. A reliable numerical algorithm for the fractional vibration equation. *Chaos Solitons Fractals* 2017;103:131–8.
- [14] Tohidi E, Bhrawy AH, Erfani K. A collocation method based on Bernoulli operational matrix for numerical solution of generalized pantograph equation. *Appl Math Model* 2013;37(6):4283–94.
- [15] Khan H, Ahmad F, Tunç O, Idrees M. On fractal-fractional Covid-19 mathematical model. *Chaos Solitons Fractals* 2022;157:111937.
- [16] Baleanu D, Khan H, Jafari H, Khan RA. On the exact solution of wave equations on cantor sets. *Entropy* 2015;17(9):6229–37.
- [17] Shah A, Khan RA, Khan A, Khan H, Gómez-Aguilar JF. Investigation of a system of nonlinear fractional order hybrid differential equations under usual boundary conditions for existence of solution. *Math Methods Appl Sci* 2021;44(2):1628–38.
- [18] Hussain S, Madi EN, Khan H, Gulzar H, Etemad S, Rezapour S, Kaabar MK. On the stochastic modeling of COVID-19 under the environmental white noise. *J Funct Spaces* 2022;2022:1–9.
- [19] Veerasha P, Prakasha DG, Baleanu D. An efficient technique for fractional coupled system arisen in magnetothermoelasticity with rotation using Mittag–Leffler kernel. *J Comput Nonlinear Dyn* 2021;16(1).
- [20] Podlubny I. Fractional differential equations: An introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications. Amsterdam: Elsevier; 1998.
- [21] Alkahtani BST, Atangana A. Analysis of non-homogeneous heat model with new trend of derivative with fractional order. *Chaos Solitons Fractals* 2016;89:566–71.
- [22] Yadav P, Jahan S, Shah K, Peter OJ, Abdeljawad T. Fractional-order modelling and analysis of diabetes mellitus: Utilizing the Atangana-Baleanu Caputo (ABC) operator. *Alex Eng J* 2023;81:200–9.
- [23] Rachah A, Torres DF. Mathematical modelling, simulation, and optimal control of the 2014 Ebola outbreak in West Africa, *Discrete dynamics in nature and society*. vol. 2015, 2015.
- [24] Koca I. Modelling the spread of ebola virus with Atangana-Baleanu fractional operators. *Eur Phys J Plus* 2018;133:1–11.
- [25] Atangana A, Goufo EFD. On the mathematical analysis of ebola hemorrhagic fever: deathly infection disease in west african countries. *BioMed Res Int* 2014;2014.
- [26] Bonyah E, Badu K, Asiedu-Addo SK. Optimal control application to an ebola model. *Asian Pacif J Trop Biomed* 2016;6(4):283–9.
- [27] Dokuyucu MA, Dutta H. A fractional order model for ebola virus with the new Caputo fractional derivative without singular kernel. *Chaos Solitons Fractals* 2020;134:109717.
- [28] Emond RT, Evans B, Bowen ET, Lloyd G. A case of ebola virus infection. *Br Med J* 1977;2(6086):541–4.
- [29] Latha VP, Rihan FA, Rakkiyappan R, Velmurugan G. A fractional-order delay differential model for ebola infection and CD8+ T-cells response: stability analysis and hopf bifurcation. *Int J Biomath* 2017;10(08):1750111.
- [30] Akgül A. A novel method for a fractional derivative with non-local and non-singular kernel. *Chaos Solitons Fractals* 2018;114:478–82.
- [31] Coronel-Escamilla A, Gómez-Aguilar JF, Torres L, Escobar-Jiménez RF. A numerical solution for a variable-order reaction–diffusion model by using fractional derivatives with non-local and non-singular kernel. *Physica A* 2018;491:406–24.
- [32] Yavuz M, Özdemir N. European vanilla option pricing model of fractional order without singular kernel. *Fractal Fract* 2018;2(1):3.
- [33] Toufik M, Atangana A. New numerical approximation of fractional derivative with non-local and non-singular kernel: application to chaotic models. *Eur Phys J Plus* 2017;132:1–16.
- [34] Izadi M, Sene N, Adel W, El-Mesady A. The layla and majnun mathematical model of fractional order: Stability analysis and numerical study. *Results Phys* 2023;106650.
- [35] Khan H, Alam K, Gulzar H, Etemad S, Rezapour S. A case study of fractal-fractional tuberculosis model in China: Existence and stability theories along with numerical simulations. *Math Comput Simulation* 2022;198:455–73.
- [36] Baleanu D, Jafari H, Khan H, Johnston SJ. Results for mild solution of fractional coupled hybrid boundary value problems. *Open Math* 2015;13(1):000010151520150055.
- [37] Begum R, Tunç O, Khan H, Gulzar H, Khan A. A fractional order zika virus model with Mittag–Leffler kernel. *Chaos Solitons Fractals* 2021;146:110898.
- [38] Tajadodi H, Khan A, Francisco Gómez-Aguilar J, Khan H. Optimal control problems with Atangana-Baleanu fractional derivative. *Optim Control Appl Methods* 2021;42(1):96–109.
- [39] Omame A, Isah ME, Abbas M, Abdel-Aty AH, Onyenegecha CP. A fractional order model for dual variants of COVID-19 and HIV co-infection via Atangana-Baleanu derivative. *Alex Eng J* 2022;61(12):9715–31.
- [40] Omame A, Abbas M, Abdel-Aty AH. Assessing the impact of SARS-CoV-2 infection on the dynamics of dengue and HIV via fractional derivatives. *Chaos Solitons Fractals* 2022;162:112427.
- [41] Omame A, Onyenegecha IP, Raetz AA, Rihan FA. Co-dynamics of COVID-19 and viral hepatitis b using a mathematical model of non-integer order: Impact of vaccination. *Fractal Fract* 2023;7(7):544.
- [42] Omame A, Abbas M, Onyenegecha CP. A fractional order model for the co-interaction of COVID-19 and hepatitis b virus. *Results Phys* 2022;37:105498.
- [43] Atangana A, Baleanu D. New fractional derivatives with non-local and non-singular kernel: theory and applications to heat transfer model. *Therm Sci* 2016;20:763–9.
- [44] Ojo M, Akinpelu F. Sensitivity analysis of ebola virus model. *Asian Res J Math* 2017;1(10).
- [45] Adewale SO, Olapade IA, Adeniran GA, Mohammed IT, Ajao SO. Mathematical analysis of effects of isolation on ebola transmission dynamics. *Res J Math* 2015;2(2):1–21.