



Full length article

Embedded class one analysis of wormhole configurations in $4\mathcal{D}$ Einstein–Gauss–Bonnet gravity with logarithmic redshift functions

S.K. Maurya^a, Jitendra Kumar^b, Sourav Chaudhary^{b,*}, A. Errehymy^{c,d}, G. Mustafa^{e,f}, K. Myrzakulov^{g,*}

^a Department of Mathematical and Physical Sciences, College of Arts and Sciences, University of Nizwa, P.O. Box 33, Nizwa 616, Sultanate of Oman

^b Department of Mathematics, Central University of Haryana, Jant-Pali, Mahendergarh, Haryana 123029, India

^c Astrophysics Research Centre, School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Private Bag, X54001, Durban 4000, South Africa

^d Center for Theoretical Physics, Khazar University, 41 Mehseti Str., Baku AZ1096, Azerbaijan

^e Department of Physics, Zhejiang Normal University, Jinhua 321004, People's Republic of China

^f Research Center of Astrophysics and Cosmology, Khazar University, Baku, AZ1096, 41 Mehseti Str., Azerbaijan

^g Department of General and Theoretical Physics, L.N. Gumilyov Eurasian National University, Astana 010008, Kazakhstan

ARTICLE INFO

Keywords:

Wormhole

Energy condition

Gauss–Bonnet coupling constant

Exact solution

ABSTRACT

This work delves into the characteristics of wormhole solutions in the context of $4\mathcal{D}$ Einstein–Gauss–Bonnet gravity. The redshift function, $\chi(r)$, and the shape function, $\varphi_s(r)$, are key components of the metric and play a crucial role in modeling wormholes. Our exploration reveals the influence of $4\mathcal{D}$ Einstein–Gauss–Bonnet gravity attributes, providing valuable insights into the formation of wormholes. Additionally, we analyze energy conditions and fundamental features of wormhole configurations. The Gauss–Bonnet coupling constant, Υ , is shown to significantly affect the stability of wormhole geometries, especially when the throats are supported by exotic matter. Stability analysis using the Tolman–Oppenheimer–Volkoff equation confirms that these solutions meet equilibrium conditions. Moreover, the deflection angle increases near the throat, where the gravitational field is strongest, and approaches zero at larger distances, indicating minimal light bending in weaker gravitational fields.

1. Introduction

The idea of wormholes (WHs) has drawn interest from both the scientific community and the general public because it presents the alluring prospect of fast travel between far-off areas of the cosmos through spacetime shortcuts. WHs are bridges connecting different space–time points that arise within the solutions to Einstein's field equations of general relativity (GR). Soon after GR was discovered, there was a lot of interest in the first theoretical investigation of WHs, due to work by Flamm [1]. Einstein and Rosen [2] improved this work by introducing the Einstein–Rosen bridge, which at the time was used to characterize WHs. However, most of the early WH research was theoretical and had no applicability to the real world. This was altered when Morris and Thorne's research revealed the possibility of WHs and illustrated how they might be used to facilitate cosmic travel [3].

The concepts of Morris and Thorne [3] have since sparked an uprising of research on WHs in higher-dimensional gravity theories and modified theories of gravity [4–8]. Furthermore, WHs in extended theories of gravity have been studied [9–12], with an emphasis on creating astrophysical methods for their detection [13–15], reconstructing their solutions [16], and confirming their presence in higher dimensions [17]. In Einstein–Gauss–Bonnet (EGB) theory, WH solutions show a remarkable extension of the classical WH concept by including higher-order curvature elements, among many other intriguing situations; see Refs. [18–23]. As a topological invariant, the Gauss–Bonnet (GB) component in $4\mathcal{D}$ is indifferent to gravitational dynamics. Glavan and Lin have proposed a $4\mathcal{D}$ metric theory that avoids Ostrogradsky instability and attains close to Lovelock's theorem [24] as a way to dimensionally regularize the GB equations. Some solutions to the field equations in the context of EGB gravity have been examined in earlier works [25–31]. One of the key criticisms involves the loss of the dynamical character of the Gauss–Bonnet term in $4\mathcal{D}$

* Corresponding authors.

E-mail addresses: sunil@unizwa.edu.om (S.K. Maurya), jitendark@gmail.com (J. Kumar), chaudharysourav192@gmail.com (S. Chaudhary), abdelghani.errehymy@gmail.com (A. Errehymy), gmustafa3828@gmail.com (G. Mustafa), krmmyrzakulov@gmail.com (K. Myrzakulov).

<https://doi.org/10.1016/j.dark.2025.101857>

Received 24 December 2024; Received in revised form 27 January 2025; Accepted 7 February 2025

Available online 20 February 2025

2212-6864/© 2025 Elsevier B.V. All rights are reserved, including those for text and data mining, AI training, and similar technologies.

spacetime, which does not contribute to the equations of motion for most physical scenarios. In $4\mathcal{D}$, the Gauss–Bonnet term is a total derivative and does not affect the field equations in the absence of additional matter fields [32]. This diminishes its potential as a dynamical field, limiting its role in modifying gravitational dynamics [33]. Moreover, modifications to the black hole solutions in $4\mathcal{D}$ EGB gravity do not necessarily yield new observable phenomena, as the theory largely reduces to classical GR in this limit [34]. However, one of the fascinating features of $4\mathcal{D}$ EGB gravity is the presence of nontrivial black hole solutions with altered thermodynamic properties, such as the existence of regular black holes and the appearance of novel critical phenomena in thermodynamic stability [35]. These solutions differ from classical GR, which lacks such black hole configurations, revealing new insights into gravitational collapse and horizon structure [36]. Additionally, the theory predicts modifications to gravitational waves, potentially distinguishing it from classical GR in experimental observations [37]. From the conventional level, it is thought that regular matter should meet energy requirements; however, for WHs, this has become the biggest hurdle. Researchers examined the violation of energy criteria that traversable WHs require in [38], and they came up with an effective measure that encouraged the presence of traversable WHs by a minimal amount of exotic matter. The literature has a large number of examples of traversable WHs, based on work by Morris, Thorne, and Yurtsever. Their work is significant because it is not limited to spherically and symmetric metric scenarios, also possible to reduce the use of exotic matter [39]. Visser and Poisson [40] presented spherically symmetric thin shell WHs in 1995. They investigated the EoS parameter of exotic matter at the WH throat and related stability issues by applying a linearized perturbation technique to some assumed static solutions of Einstein field equations. Many intriguing perspectives of thin shell WHs have been discussed extensively in the literature, e.g., [41–47]. In reference to the existence of traversable wormholes, several noteworthy studies have explored various modified theories of gravity, including $f(R)$ gravity [48–52], $f(R, T)$ gravity [53–55], $f(T)$ gravity [56], non-commutative geometries [57,58], and Rastall–Rainbow gravity [59–61], which are accessible in the literature.

Although there have been a number of noteworthy advancements in this sector, there is currently insufficient observational evidence to support the existence of WHs. A noteworthy idea addresses the possibility of white dwarfs existing in the galactic halo regions [62,63]. The authors contend that a traversable WH configuration can explain the rotation curves of the flat galactic halo. Moreover, gravitational lensing has been proposed as a possible means of detecting these white dwarfs [64], further analysis is presented in [65]. When a big object bends the light path, it is known as gravitational lensing. The presence of a white hole may result in distinct lensing effects that are visible to the naked eye. We also want to put here the suggestion made in the work [66] that instead of supermassive black holes, active galactic nuclei (AGNs) are WH mouths. According to certain theories, these WHs could release gamma radiation as a result of accreting flows inside of them colliding. Since this radiation has a unique spectrum that differs greatly from jets or AGN’s accretion discs, its detection would prove that WHs exist. Our goal in this attempt is to study wormhole geometry and physics from an integrated perspective. The notion of spacetime including the equations describing its $4\mathcal{D}$ EGB geometry, including the Einstein field equations, are first reviewed. We examine in detail the field equations for WH configuration and find that there are violated conditions for null energy in the throat neighborhood. More precisely, we introduce $4\mathcal{D}$ EGB gravity and propose novel approaches for spherically symmetric static WHs, including two distinctively deduced shape functions. We explored the shape function criterion under Model I & II. We have examined the energy conditions and equilibrium states to determine the stability of the WH solution.

The outline of the current work is structured as follows: Section 2 introduces the framework of field equations through the action principle. In Section 3.1, we provide a brief discussion of the embedded class one technique. Section 3.2 explores various redshift functions for generating WH solutions. In Section 3.3, we enhance the configuration of shape functions utilizing the embedded class one technique in conjunction with the logarithmic redshift function. Section 4 establishes the energy criteria within the context of $4\mathcal{D}$ EGB gravity. Section 5 engages in a detailed discussion of WH Model-I and Model-II in Sections 5.1 and 5.2, respectively. Section 6 presents the 2D and 3D embeddings of WH geometry within $4\mathcal{D}$ EGB gravity. Section 7 assesses the stability of WHs via the Tolman–Oppenheimer–Volkoff (TOV) equation. Section 8 investigates the deflection angle associated with WH geometry. Finally, Section 9 concludes with closing remarks and a summary of our findings.

2. Basic field equations

Within the background of $\mathcal{D}$ -dimensional space, the EGB theory provides the action in the given form below [67]

$$\mathcal{J}_{\mathcal{G}} = \int d^{\mathcal{D}}x \sqrt{-g} \left[\mathcal{R} + \frac{Y\mathcal{G}}{\mathcal{D}-4} \right] + S_{\text{matter}}, \quad (1)$$

where $\mathcal{R}$ is Ricci scalar and g denotes the determinant of $g_{\alpha\beta}$. The 2nd component in Eq. (1) associated with the GB part of the action in which the invariant of GB is characterized as $\mathcal{G}$ and Y be the GB coupling parameter. Also, the 3rd component incorporates dark matter and baryonic constituents.

The term “GB” is characterized by [68]

$$\mathcal{G} = \mathcal{R}^2 + \mathcal{R}^{\alpha\beta\gamma\delta} \mathcal{R}_{\alpha\beta\gamma\delta} - 4\mathcal{R}^{\alpha\beta} \mathcal{R}_{\alpha\beta}, \quad (2)$$

where $\mathcal{R}_{\alpha\beta}$ is labeled as Ricci tensor and $\mathcal{R}_{\alpha\beta\gamma\delta}$ is referred as Riemann tensor.

On taking the variation of action defined in the Eq. (1) with fundamental tensor $g_{\alpha\beta}$, we acquire the resulting system of field equations:

$$\mathcal{E}_{\alpha\beta} = \frac{Y}{\mathcal{D}-4} \mathcal{H}_{\alpha\beta} + \mathcal{T}_{\alpha\beta}, \quad (3)$$

where $\mathcal{E}_{\alpha\beta}$ symbolizes the Einstein tensor and $\mathcal{H}_{\alpha\beta}$ expresses the Lanczos tensor which can be conveyed as follows

$$\mathcal{H}_{\alpha\beta} = \left[2\mathcal{R}\mathcal{R}_{\alpha\beta} - 4\mathcal{R}_{\alpha\gamma}\mathcal{R}_{\beta}^{\gamma} - 4\mathcal{R}_{\alpha\gamma\beta\delta}\mathcal{R}^{\alpha\beta} - 2\mathcal{R}_{\alpha\gamma\delta\alpha}\mathcal{R}_{\beta}^{\gamma\delta\alpha} \right] - \frac{1}{2}\mathcal{G}g_{\alpha\beta}. \quad (4)$$

Since the energy–momentum tensor complementary to S_{matter} is expressed as

$$\mathcal{T}_{\alpha\beta} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}S_{\text{matter}})}{\delta g^{\alpha\beta}}. \quad (5)$$

Most often, in the realm of $4\mathcal{D}$ continuum, the term GB signifies the total derivative and consequently is unable to impart any support to the field equations. With the re-scaling of the coupling parameter $\frac{Y}{(\mathcal{D}-4)}$, The GB term remains present for $\mathcal{D} = 4$, accordingly, the behavior of the GB component can be further examined.

We have taken into account the $\mathcal{D}$ -dimensional WH structure to examine the WH configuration in $4\mathcal{D}$ EGB theory, given by

$$ds^2 = -e^{2\chi(r)} dt^2 + \left(1 - \frac{\varphi_S(r)}{r}\right)^{-1} dr^2 + r^2 d\Sigma_{\mathcal{D}-2}^2, \quad (6)$$

where

$$d\Sigma_{\mathcal{D}-2}^2 = d\theta_a^2 \sum_{b=2}^{\mathcal{D}-2} \prod_{a=1}^{b-1} \sin^2 \theta_a d\theta_b^2.$$

In Eq. (6), $\chi(r)$ is described as a red-shift function whereas $\varphi_S(r)$ defines the shape of the WH and labeled as shape function.

The stress-energy tensor having non-vanishing components is described in the following way

$$\mathcal{T}_{\alpha\beta} = \begin{bmatrix} -\rho(r) & 0 & 0 & 0 \\ 0 & \mathcal{P}_r(r) & 0 & 0 \\ 0 & 0 & \mathcal{P}_t(r) & 0 \\ 0 & 0 & 0 & \mathcal{P}_l(r) \end{bmatrix}$$

Thus, Eq. (3) yields the matter components of $\mathcal{T}_{\alpha\beta}$ in the following manner.

$$\rho(r) = \frac{Y\varphi_S(r)}{r^6} \left[2r\varphi'_S(r) - 3\varphi_S(r) \right] + \frac{\varphi'_S(r)}{r^2}, \quad (7)$$

$$\mathcal{P}_r(r) = -\frac{\varphi_S(r)}{r^3} + \frac{2}{r^2} \chi'(r) (r - \varphi_S(r)) + \frac{Y\varphi_S(r)}{r^6} \left[4r(r - \varphi_S(r))\chi'(r) + \varphi_S(r) \right], \quad (8)$$

$$\begin{aligned} \mathcal{P}_l(r) = & \frac{1}{2r^6} \left[r(12Y\varphi_S(r) + r^3 - 8Yr) (\varphi_S(r) - r\varphi'_S(r)) \chi'(r) + (r^3 - 2Y\varphi_S(r)) (-r\varphi'_S(r) \right. \\ & \left. + 2r(r - \varphi_S(r))\chi'(r) + \varphi_S(r)) + 2r^2(r - \varphi_S(r)) (4Y\varphi_S(r) + r^3) (\chi''(r) + \chi'(r)^2) - 4Y\varphi_S(r)^2 \right], \end{aligned} \quad (9)$$

where the $\prime$ represents the derivative with respect to r .

In the aforementioned system of equations, we encounter five undetermined variables: $\rho(r)$, $\mathcal{P}_r(r)$, $\mathcal{P}_l(r)$, $\chi(r)$, and $\varphi_S(r)$. Therefore, it is tedious to obtain the exact solution. We will present numerous strategies to solve this system of equations in the subsequent sections.

3. Key background infrastructure requirements

3.1. Embedding class one technique

The framework of WH is determined in this part by developing the WH shape function by employing the embedding class one technique. Consequently, we define the static spherical symmetric spacetime metric in the following manner in the curvature coordinates:

$$ds^2 = -e^{\varrho(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Pi^2, \quad (10)$$

where $d\Pi^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $\varrho(r)$ and $\lambda(r)$ are arbitrary functions of radial coordinate r .

One can infer that the spherical symmetric line element Eq. (10) is typically associated with the embedding of Class 2, as it is consistently embedded in the flat spacetime of six dimensions. Alternatively, we can incorporate the spherical line element into five-dimensional flat spacetime, assuming it fulfills the criteria of the embedding class one approach [69]. Karmarkar [69] established a key criterion for a static, spherically symmetric line element to be classified as Class 1. Several researchers have delved into the embedding class one method recently to make predictions on the structures of astrophysical objects exhibiting symmetry [70–73]. By implementing the class one method, one can produce WH geometry and show how embedding theory might serve as the basis for a comprehensive WH solution [74].

It is suggested that a pseudo-Euclidean space can be embedded within an l -dimensional space $\mathcal{V}_l$ as seen in the work of Eisenhart [75] and Eisland [76], has attracted a lot of attention. The metric specified in Eq. (10) usually generates a $:-\mathcal{D}$ spherically symmetric space-time, laying it within Class 2, denoting $m = 2$, and embedding in a six-dimensional pseudo-Euclidean space. In this instance, $\mathcal{V}_l$ is deemed to have an m -Class embedding when an n -dimensional space $\mathcal{V}_l$ is capable of being isometrically immersed in a $(l+m)$ -dimensional space where m denotes the minimum number of extra dimensions. Apart from that, it is important to note that a parametrization may be created that enables the space-time stated in Eq. (10) to be integrated into a five-dimensional pseudo-Euclidean space, corresponding to Class $m = 1$. This process is referred to as embedding Class one [69,75,76]. A spherically symmetric space-time must meet the following necessary and appropriate conditions for it to be categorized as Class-one, regardless of whether it is static or not:

- Codazzi Equation:

$$\mathcal{G}_{m[l;p]} - \Gamma_{[lf]}^g \mathcal{G}_{mg} + \Gamma_m^g [l \mathcal{G}_f]_g = 0.$$

- Gauss equation:

$$\mathcal{R}_{mlfg} = 2\delta \mathcal{G}_m [f \mathcal{G}_g]_l.$$

In this instance, antisymmetrization is indicated by square brackets and $\delta = \pm 1$. $\mathcal{G}_{ml}$ is the representation of the 2nd differential forms with coefficients. We can compute the Karmarkar condition subsequently by using the Gauss equation and Codazzi Equation and following the prescribed mathematical technique.

We can express the non-zero components of the Riemann curvature in the following way

$$\left. \begin{aligned} \mathcal{R}_{1212} &= \mathcal{R}_{2121}, \mathcal{R}_{1221} = \mathcal{R}_{2112}, \mathcal{R}_{1313} = \mathcal{R}_{3131}, \\ \mathcal{R}_{1331} &= \mathcal{R}_{3113}, \mathcal{R}_{2323} = \mathcal{R}_{3232}, \mathcal{R}_{2332} = \mathcal{R}_{3223}, \\ \mathcal{R}_{1414} &= \mathcal{R}_{4141}, \mathcal{R}_{4114} = \mathcal{R}_{1441}, \mathcal{R}_{4242} = \mathcal{R}_{2424}, \\ \mathcal{R}_{4224} &= \mathcal{R}_{2442}, \mathcal{R}_{4343} = \mathcal{R}_{3434}, \mathcal{R}_{4334} = \mathcal{R}_{3443} \end{aligned} \right\} \quad (11)$$

$$\mathcal{R}_{1212} = \frac{e^0 (2\rho'' + \lambda'^2 - \rho' \rho')}{4}, \quad \mathcal{R}_{3434} = \frac{r^2 \sin^2 \theta (e^\lambda - 1)}{e^\lambda}$$

$$\mathcal{R}_{1414} = \frac{r \sin^2 \theta \rho' e^{\rho-\lambda}}{2}, \quad \mathcal{R}_{2323} = \frac{r \lambda'}{2}, \quad \mathcal{R}_{1334} = \mathcal{R}_{1224} \sin^2 \theta$$

The Riemann components mentioned above conform to the well-known embedding class 1 technique as

$$\mathcal{R}_{1414} = \frac{1}{\mathcal{R}_{2323}} \left[\mathcal{R}_{1212} \mathcal{R}_{3434} + \mathcal{R}_{1224} \mathcal{R}_{1334} \right], \quad \mathcal{R}_{2323} \neq 0. \quad (12)$$

We refer to the spacetime as embedding class 1 [69], which satisfies the Karmarkar constraint. Riemann component values are substituted in Eq. (12), we obtain differential equation in the form

$$2 \frac{\rho''}{\rho'} + \rho' = \frac{\lambda' e^\lambda}{e^\lambda - 1}. \quad (13)$$

Thus, the solution of the above equation is given in the form of

$$e^\lambda = 1 + \mathcal{C}_1 e^\rho \rho'^2, \quad (14)$$

where $\mathcal{C}_1$ is the integration constant.

3.2. The role of redshift functions in WH configuration

While exploring WHs, one considers the specific modeling objectives and complexity before deciding on a constant or variable redshift function. Employing a constant redshift function results in simplified solutions and reduces the complexity of the mathematical analysis [77]. In addition, the introduction of a modified type of gravity across WHs simplifies the inherently dynamic and complex nature of real-world WHs. Alternatively, the variable redshift function [78,79] gives a more accurate account of the gravitational field in the wormhole, facilitating a thorough exploration of its fluctuating strength across the entire length of the tunnel. To more effectively capture the nature of gravitational forces along the passage and deliver a more realistic depiction of WHs, utilizing a variable redshift function is more appropriate. In addition, non-static wormhole designs are best studied through changing redshift functions, as the variations in the gravitational field are crucial. Consequently, drawing motivation from these aspects noted in the literature [80–86], we are incorporating a variable redshift function that depends on radial distance to generate WH solutions.

3.3. Shape function constructed via the embedding class-1 technique

The shape function is taken to be described by the Morris–Thorne metric [3], defined in the following manner:

$$ds^2 = -e^{2\chi(r)} dt^2 + \frac{dr^2}{1 - \frac{\varphi_S(r)}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (15)$$

where $\varphi_S(r)$ is known as the shape function, whereas $\chi(r)$ serves as the redshift function. The role of the WH shape function is fundamental in defining its throat's geometry and maintaining its physical viability. It also significantly impacts its stability and the ability to traverse it. Using an embedded class-1 spacetime, Ditta and his colleagues [87] computed the shape function in the context of $f(T, \tau)$ gravity. A new class of embedded WH solutions in the background of GR was introduced by Molla et al. [88] where both newly computed shape functions satisfy all the required conditions. The work of Mustafa et al. [89] involved embedding class one galactic halo WH geometries within the context of torsion and matter coupling gravity, and the computed WH shape functions were found to meet all required specifications. Inspired by the literature reviewed above, we derive novel shape functions that utilize logarithmic redshift functions through the framework of embedded class-1 spacetime.

The WH spatial shape function, represented as $\varphi_S(r)$, characterizes the geometry of the WH in the embedding diagram [90]. At the throat of WH i.e. $r = r_0$, the $\varphi_S(r)$ must satisfies the throat condition given by $\varphi_S(r_0) = r_0$, where $r \in [r_0, \infty)$ in the entire space–time. To meet the flaring-out condition, the shape function must be consistent with the results from the embedding calculation which is defined by $\frac{\varphi_S(r) - r \varphi'_S(r)}{\varphi_S^2(r)} > 0$. This condition is essential to maintain the traversability of the WH geometry. At WH throat, the reduced form of this condition can be presented as $\varphi'_S(r_0) < 1, 1 - \frac{\varphi_S(r)}{r} > 0$. For the sake of simplicity, the redshift function can be taken as zero, a condition that has been addressed by many researchers [91–93]. This work aims to analyze static spherical symmetric WHs with a radial-dependent redshift function within spacetime, reducing to a homogeneous Friedmann–Lemaître–Robertson (FRW) model. The presence of a non-constant redshift function, leading to non-zero tidal effects, connects the WH to a distinct family of WHs. To achieve this, we will consider an asymptotically flat redshift function to ensure the absence of singularities and horizons. The general constraints on the redshift function have been explored by Morris and Thorne. It is important to point out that if both the redshift and shape functions are taken as zero, Eq. (6) becomes a flat FRW metric.

By comparing Eqs. (10) and (15), we get

$$e^{\lambda(r)} = \left[\frac{r}{r - \varphi_S(r)} \right] \quad \& \quad \rho(r) = 2\chi(r). \quad (16)$$

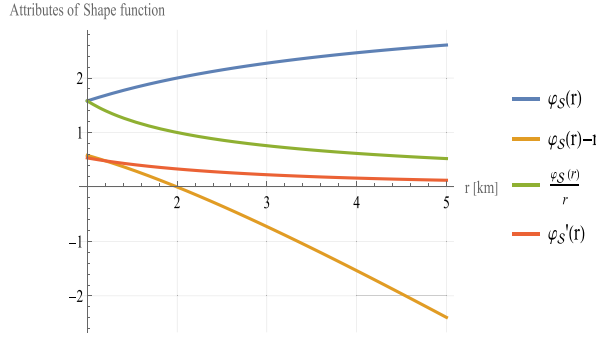


Fig. 1. Behavior of several assumptions regarding the shape function: Model-I, i.e. $0 < \varphi_S(r)$, $\varphi_S(r) = r$ at $r = r_0$, $\frac{\varphi_S(r)}{r} \rightarrow 0$ and $1 > \varphi_S'(r)$ with $r_0 = 2$, $\varpi = -0.5$ and $B_1 = 0.85$.

Now we consider the redshift function $\chi(r) = \varpi \log(\frac{r_0}{r})$ [94], and $\chi(r) = \log(1 + \frac{1}{r})$ [94], to identify the shape function that characterizes the shape of the WH. To determine the shape function, utilizing Eqs. (14) & (16), we compute the shape function in the following manner: $\varphi_S(r) = r - \frac{r^3}{r^2 + 4\varpi^2 \left(\frac{r_0}{r}\right)^{2\varpi} \mathcal{E}_1}$ against the redshift function $\chi(r) = \varpi \log(\frac{r_0}{r})$, and $\varphi_S(r) = r - \frac{r^5}{r^4 + 4\mathcal{E}_1}$ against the redshift function $\chi(r) = \log(1 + \frac{1}{r})$.

Upon using the throat condition $\varphi_S(r) - r = 0$ at $r = r_0$, a trivial solution is obtained. In seeking the non-trivial solution, we introduced new free parameters B_1 and B_2 , again utilizing the throat condition, we obtain the value $\mathcal{E}_1 = \frac{r_0^3}{4\varpi^2 \gamma} - \frac{r_0^2}{4\varpi^2}$ corresponding to $\varphi_S(r) = r - \frac{r^3}{r^2 + 4\varpi^2 \left(\frac{r_0}{r}\right)^{2\varpi} \mathcal{E}_1} + B_1$

and $\mathcal{E}_1 = \frac{r_0^4}{4} \left(\frac{r_0}{\eta} - 1 \right)$ corresponding to $\varphi_S(r) = r - \frac{r^5}{r^4 + 4\mathcal{E}_1} + B_2$ where $0 < B_1 < r_0$ and $0 < B_2 < r_0$.

Therefore, the required shape function

- $\varphi_S(r) = r - \frac{r^3}{r^2 + \left(\frac{r_0^3}{B_1} - r_0^2\right) \left(\frac{r_0}{r}\right)^{2\varpi}} + B_1$ against $\chi(r) = \varpi \log(\frac{r_0}{r})$.
- $\varphi_S(r) = r - \frac{r^5}{r^4 + \frac{r_0^4}{B_2} - r_0^4} + B_2$ against $\chi(r) = \log(1 + \frac{1}{r})$.

4. Energy conditions in WH modeling

Certain inequalities governing energy density, radial pressure as well as tangential pressure associated with the energy-momentum tensor are necessary for the traversable WH to remain stable as well as exist. These inequalities, which are often referred to as energy conditions, are crucial for determining WH solutions. The matter inside the WH usually violates the null energy condition indicating that the matter inside is repellent or under negative pressure. The Raychaudhuri equation incorporates the NEC roots.

$$\frac{d\mathcal{F}}{d\tau} + \frac{1}{2}\mathcal{F}^2 + \mathfrak{U}_{\alpha\beta}\mathfrak{U}^{\alpha\beta} - \mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta} = -\mathcal{R}_{\alpha\beta}y^\alpha y^\beta. \quad (17)$$

Since y^α is the vector field. The shear and the spatial tensor is given by $-\mathcal{R}_{\alpha\beta}y^\alpha k^\beta$ with $\mathfrak{U}^2 = \mathfrak{U}_{\eta\zeta}\mathfrak{U}^{\eta\zeta} \geq 0$ and $\mathcal{E}_{\alpha\beta} \equiv 0$.

1. Within the context of modified gravity field equations, nonnegativity constraint $\mathcal{R}_{\alpha\beta}y^\alpha y^\beta \geq 0$ in the Raychaudhuri equation gives the Null Energy Condition (NEC), $T_{\alpha\beta}^{eff}y^\alpha y^\beta \geq 0$ [95]. Since $T_{\alpha\beta}^{eff}$ is the effective stress-energy tensor. The NEC $T_{\alpha\beta}y^\alpha y^\beta \geq 0 \Leftrightarrow$ NEC, one can describe this formulation as $\forall i, \rho(r) + P_i \geq 0 \Leftrightarrow$ NEC, in terms of principal pressures, defines the non-negativity of pressure as perceived by an observer.
2. The constraint $T_{\alpha\beta}y^\alpha y^\beta \geq 0 \Leftrightarrow$ WEC is known as the weak energy condition, in relation to a timelike vector. With principal pressures, this requirement is formulated as $\rho(r) \geq 0$ and $\forall i, \rho(r) + P_i \geq 0 \Leftrightarrow$ WEC. It clearly signifies that, for a timelike object, the energy density must be positive locally.
3. The invalidation of strong energy condition (SEC) $\left(T_{\alpha\beta} - \frac{T}{2}g_{\alpha\beta}\right)y^\alpha y^\beta \geq 0 \Leftrightarrow$ WEC, where T denotes the trace of energy-momentum tensor. The violation of the SEC reflects the inflationary phase of the universe. In terms of principal pressures SEC is given as $T = -\rho(r) + \sum_j P_j$ and $\forall j, \rho(r) + P_j \geq 0, \rho(r) + \sum_j P_j \geq 0 \Leftrightarrow$ SEC.
4. The dominant energy condition (DEC) $\Leftrightarrow T_{\alpha\beta}y^\alpha y^\beta \geq 0$ where $T_{\alpha\beta}y^\alpha$ is not space-like $\rho(r) \geq 0$ and $\forall i, P_i \in [-\rho(r), \rho(r)] \Leftrightarrow$ DEC, it imposes a restriction on the flow of energy and mass, ensuring that it does not exceed the speed of light. Additionally, it indicates that energy density must be positive locally.

5. Wormhole modeling in 4D EGB gravity

5.1. Wormhole modeling with shape function

$$\varphi_S(r) = r - \frac{r^3}{r^2 + \left(\frac{r_0^3}{B_1} - r_0^2\right) \left(\frac{r_0}{r}\right)^{2\varpi}} + B_1$$

The properties of the shape function are satisfied only for $0 < B_1 < 2$ and $\varpi < 1.755$. Furthermore, for $\varpi \geq 1.755$, the throat constraint is violated. It can be observe from Fig. 1, with $r_0 = 2$, $\varpi = -0.5$ and $B_1 = 0.85$, the shape function $\varphi_S(r)$ display a positively increasing nature against the radial distance r . The throat constraint $\varphi_S(r) = r_0$ is validated at $r_0 = 2$. Moreover, the flare-up condition, i.e. $\varphi_S'(r) < 1$ is adequately fulfilled, along with the condition that $\frac{\varphi_S(r)}{r} \rightarrow 0$ as $r \rightarrow 0$. The significant characteristics of the shape function are represented in Fig. 1.

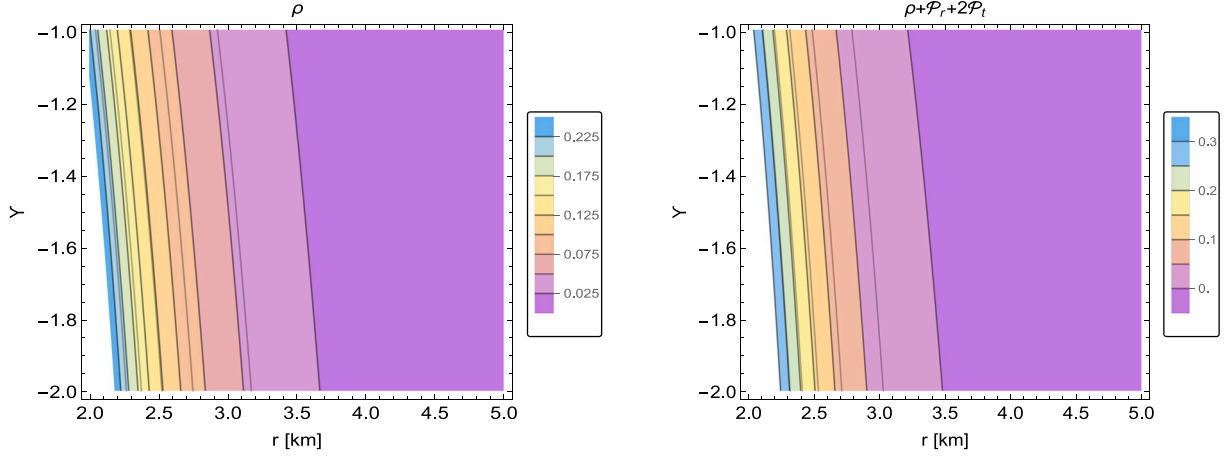


Fig. 2. Development of the ρ (left) and $\rho + P_r + 2P_t$ (right) with $r_0 = 2$, $\varpi = -0.5$, and $B_1 = 0.85$ - Model-I.

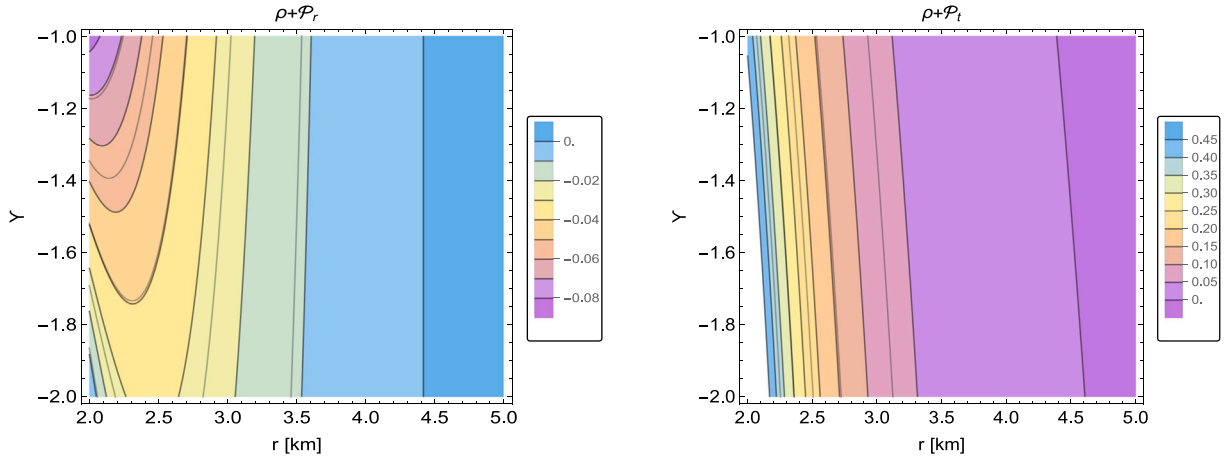


Fig. 3. Development of the $\rho + P_r$ (left) and $\rho + P_t$ (right) with $r_0 = 2$, $\varpi = -0.5$, and $B_1 = 0.85$ - Model-I.

With the execution of $\varphi_S(r) = r - \frac{r^3}{r^2 + (\frac{r_0^3}{B_1} - r_0^2) \left(\frac{r_0}{r}\right)^{2\varpi}} + B_1$ and $\chi(r) = \beta \log\left(\frac{r_0}{r}\right)$ in Eqs. (7)–(9), we leftover with the stress energy matter components ρ , P_r and P_t ,

$$\rho = \frac{1}{r^6} \left[r^4 - r^2 Y - \frac{4r^8 B_1^3 (\varpi + 1) Y}{\left(r^2 B_1 + r_0^2 (r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^3} - 4r B_1 Y - 3B_1^2 Y - \frac{r^3 B_1 (r^3 (2\varpi + 3) + 2r(2\varpi Y + Y) + 4B_1 \varpi Y)}{r^2 B_1 + r_0^2 (r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi}} + \frac{r^5 B_1^2 (2r^3 (\varpi + 1) + r(8\varpi + 7)Y + 4B_1 (\varpi + 1)Y)}{\left(r^2 B_1 + r_0^2 (r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^2} \right], \quad (18)$$

$$P_r = \frac{B_1^2 (4\varpi + 1) Y}{\left(r^2 B_1 + r_0^2 (r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^2} - \frac{B_1 (r^3 (2\varpi - 1) + 2r(2\varpi Y + Y) + 2B_1 (4\varpi + 1)Y)}{r^5 B_1 + r^3 r_0^2 (r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi}} + \frac{-r^4 + r^3 B_1 (2\varpi - 1) + r^2 Y + 2r B_1 (2\varpi + 1)Y + B_1^2 (4\varpi + 1)Y}{r^6}, \quad (19)$$

$$P_t = -\frac{1}{2r^6} \left[-2r^3 B_1 \varpi^2 + 5r^3 B_1 \varpi - r^3 B_1 + 2r^2 Y - 4r B_1 \varpi^2 Y + 2r B_1 \varpi Y + 6r B_1 Y - 4B_1^2 \varpi^2 Y - 2B_1^2 \varpi Y + 4B_1^2 Y - \frac{4r^8 B_1^3 (\varpi^2 - 1) Y}{\left(r^2 B_1 + r_0^2 (r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^3} - \frac{2r^5 B_1^2 (r^3 (\varpi^2 - 1) + r(2\varpi(\varpi + 2) + 3)Y - 2B_1 (\varpi^2 - 1)Y)}{\left(r^2 B_1 + r_0^2 (r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^2} + \frac{2r^3 B_1 (r^3 (2(\varpi - 1)\varpi - 1) + 4r\varpi(\varpi + 1)Y + B_1 (2\varpi^2 + \varpi - 1)Y)}{r^2 B_1 + r_0^2 (r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi}} \right]. \quad (20)$$

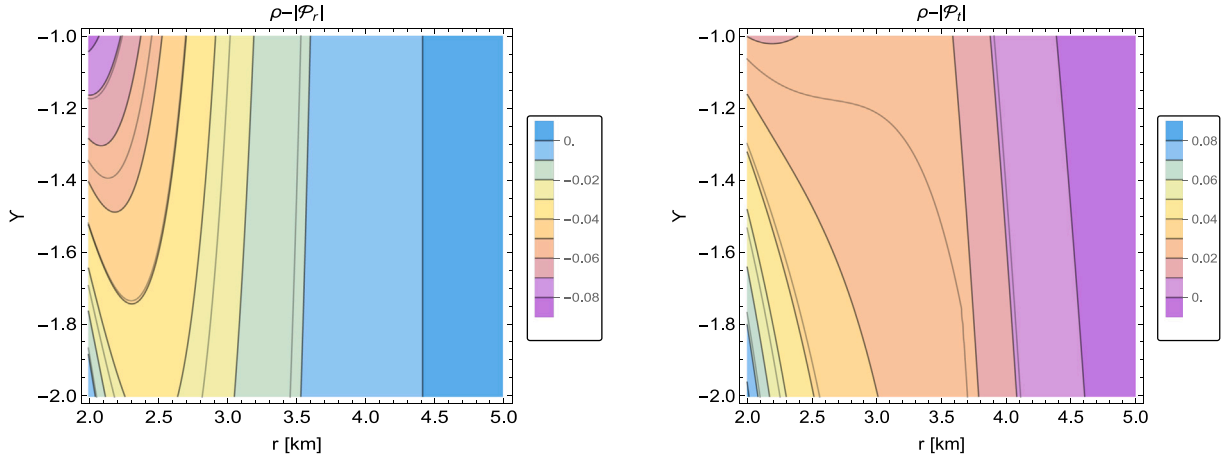


Fig. 4. Development of the $\rho - |P_r|$ (left) and $\rho - |P_t|$ (right) with $r_0 = 2$, $\varpi = -0.5$, and $B_1 = 0.85$ - Model-I.

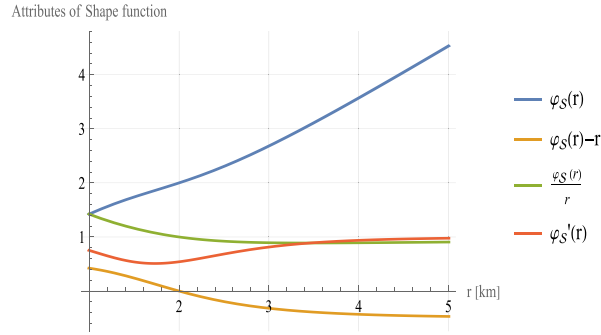


Fig. 5. Behavior of several assumptions regarding the shape function: Model-II, i.e. $0 < \varphi_S(r)$, $\varphi_S(r) = r$ at $r = r_0$, $\frac{\varphi_S(r)}{r} \rightarrow 0$ and $\varphi_S'(r) < 1$ with $r_0 = 2$, and $B_2 = 0.5$.

On analyzing the matter components, it is observed that the energy density depicts negative behavior for GB coupling parameter Y with $Y \geq -1$ and positive for $Y < -1$. We choose the GB coupling parameter Y with range $-2 \leq Y < -1$ in the current investigation. Still, one can also pick any other range of Y provided the constraint $Y < -1$. With $r_0 = 2$, $\varpi = -0.5$, $B_1 = 0.85$, in Fig. 2 (left), the energy density ρ illustrates positive dynamics and shows decreasing behavior with an increase in radial distance r . Since the tangential NEC Fig. 3 (right) is invalidated with the radial distance r , the radial NEC Fig. 3 (left) is violated with the variation of GB coupling parameter Y . Also, the tangential DEC Fig. 4 (right) is validated for the radial distance r and approaches zero as r approaches ∞ , the radial DEC Fig. 4 (left) is violated with the variation of Y and goes to zero as r approaches to ∞ . Furthermore, in Fig. 2 (right), the validation of SEC can be observed within the given range of Y .

5.2. WH modeling with shape function $\varphi_S(r) = r - \frac{r^5}{r^4 + \frac{r_0^5}{B_2} - r_0^4} + B_2$

With $r_0 = 2$, $B_2 = 0.5$, the shape function $\varphi_S(r)$ shows positive and growing nature with the radial distance r . The throat constraint $\varphi_S(r) = r$ is validated at $r_0 = 2$. Moreover, the flare up criterion, i.e. $1 > \varphi_S'(r)$ is also maintained and as $r \rightarrow 0$, $\frac{\varphi_S(r)}{r} \rightarrow 0$. The development of the traversability criterion of the shape function is described in Fig. 5.

On employing $\varphi_S(r) = r - \frac{r^5}{r^4 + \frac{r_0^5}{B_2} - r_0^4} + B_2$ with $\chi(r) = \log(1 + \frac{1}{r})$ in Eqs. (7)–(9), the matter components of the stress–energy tensor are derived as ρ , P_r and P_t which are given by

$$\rho = \frac{1}{r^6 (r^3(r+1)B_2 - (r_0^4(B_2-1)) - r_0^3(B_2-1))} \left[(B_2-1) B_2 (r^6(r+1)^2 B_2 - r^3 r_0^4 (2rB_2 + 3r + 2B_2 + 2)) \right. \\ \left. - (r_0^4(B_2-1) (r^3 + 2rY + 2B_2Y)) - r_0^3(B_2-1) (r^3 + 2rY + 2B_2Y) + r^3(r+1)B_2 (r^3 + 2rY + 2(B_2-1)Y) \right. \\ \left. \times r^3 r_0^3 (2rB_2 + 3r + 2B_2 + 2) + r_0^8(B_2-1) + 2r_0^7(B_2-1) + r_0^6(B_2-1) \right] + \frac{(1-A_2)(r-A_1)}{r^3}, \quad (21)$$

$$P_r = \frac{(1-A_2)(A_1-r) - \frac{2A_1(2A_2+1)}{r+1}}{r^3}, \quad (22)$$

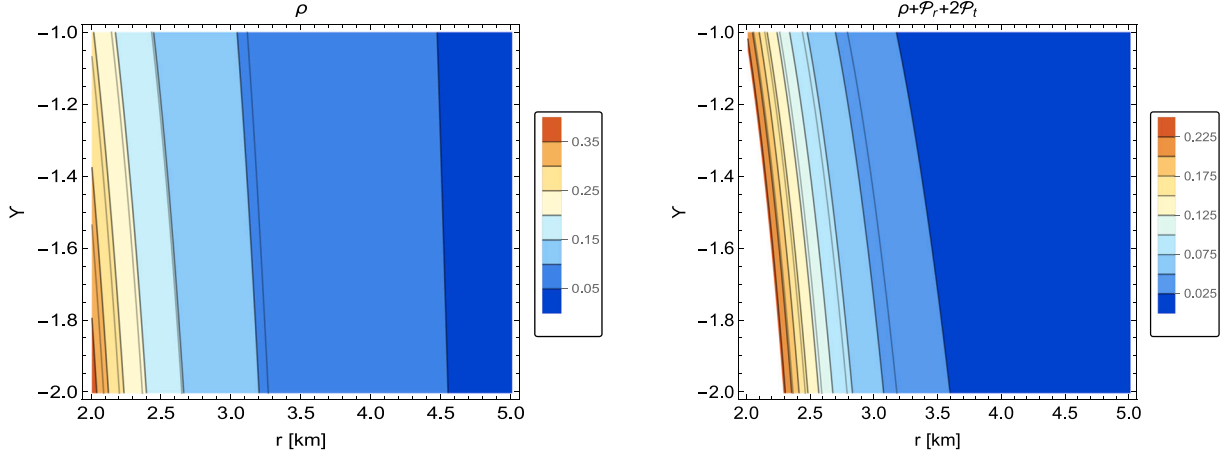


Fig. 6. Development of the ρ (left) and $\rho + 2P_t + P_r$ (right) with $r_0 = 2$, and $B_2 = 0.5$ - Model-II.

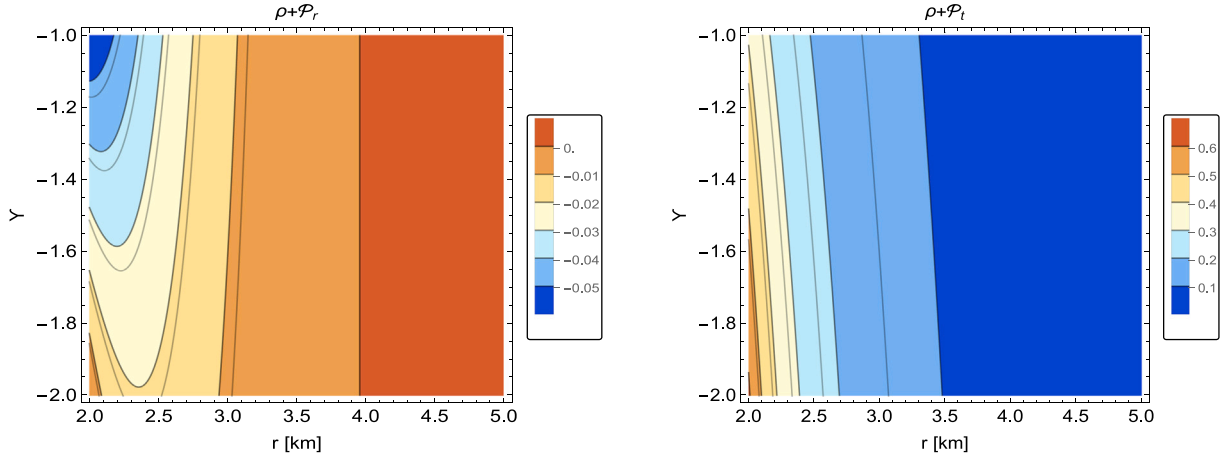


Fig. 7. Development of the $\rho + P_r$ (left) and $\rho + P_t$ (right) with $r_0 = 2$, and $B_2 = 0.5$ - Model-II.

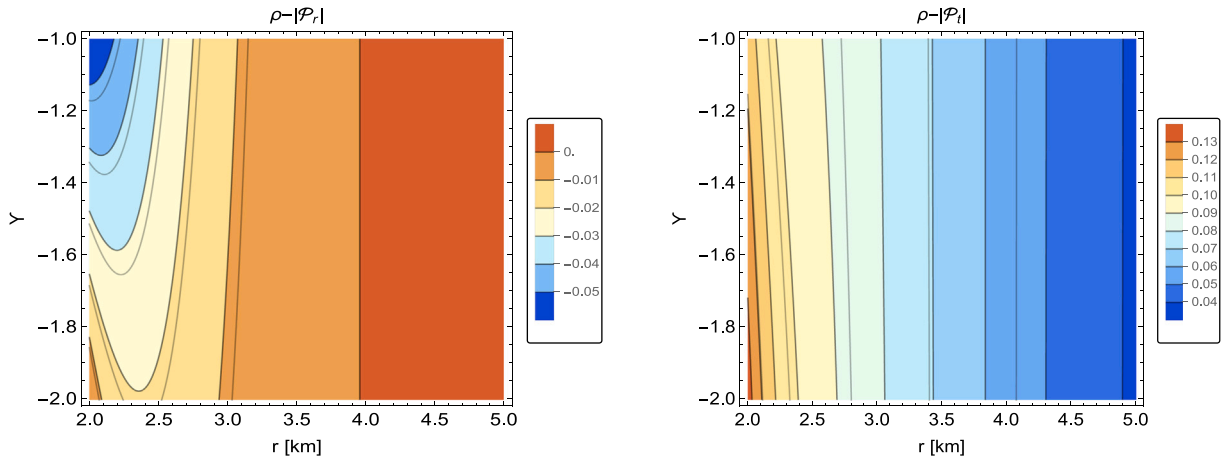


Fig. 8. Development of the $\rho - |P_r|$ (left) and $\rho - |P_t|$ (right) with $r_0 = 2$, and $B_2 = 0.5$ - Model-II.

$$P_t = \frac{4(r^4 + r^3 - r_0^3(r_0 + 1))(B_2 - 1)^2 B_2^2 Y((-r^3(r + 1) + r_0^4 + r_0^3)^2 B_2 - r_0^3(r_0 + 1)(r^3(3r + 2) + r_0^4 + r_0^3))}{2r^6(r + 1)(r^3(r + 1)B_2 - (r_0^4(B_2 - 1)) - r_0^3(B_2 - 1))^3} + \frac{(B_2 - 1)B_2((r - 1)(r^4 + r^3 - r_0^3(r_0 + 1))^2 B_2 - r_0^3(r_0 + 1)(r^3(r + 1)(3r + 4) + (r - 1)r_0^4 + (r - 1)r_0^3))}{(r + 1)(r^3(r + 1)B_2 - (r_0^4(B_2 - 1)) - r_0^3(B_2 - 1))^3}$$

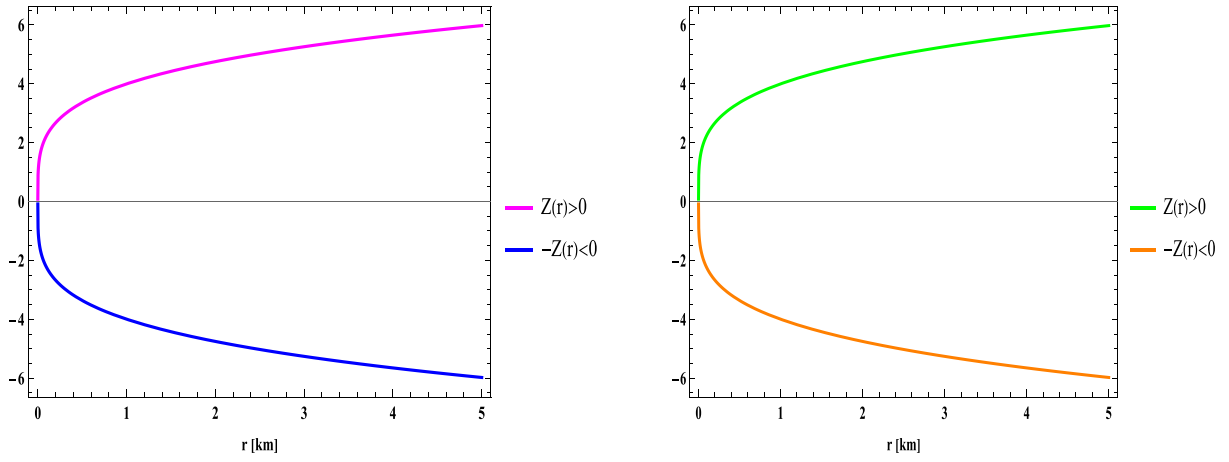


Fig. 9. 2-D WH embedding surface diagram: Model-I (left) & Model-II (right).

$$\begin{aligned}
 & \times \frac{2(-r^3(r+1) + r_0^4 + r_0^3)B_2^2Y - (-r^3(r+1) + r_0^4 + r_0^3)B_2(r^3 - 2rY + 2Y) + r(r_0+1)r_0^3(r^2 - 2Y)}{(r+1)(r^3(r+1)B_2 - (r_0^4(B_2-1)) - r_0^3(B_2-1))^3} - \\
 & \frac{(B_2-1)B_2((5r+1)(r^4 + r^3 - r_0^3(r_0+1))^2B_2 - r_0^3(r_0+1)(-r^5 + r^4 + 2r^3 + 5r_0^3(r_0+1)r + r_0^3(r_0+1)))}{(r+1)^2(r^3(r+1)B_2 - (r_0^4(B_2-1)) - r_0^3(B_2-1))^3} \\
 & \times \frac{-r_0^4(B_2-1)(r^3 + 2rY + 2B_2Y) - r_0^3(B_2-1)(r^3 + 2rY + 2B_2Y) + r^3(r+1)B_2(r^3 + 2rY + 2(B_2-1)Y)}{(r+1)^2(r^3(r+1)B_2 - (r_0^4(B_2-1)) - r_0^3(B_2-1))^3} \\
 & - \frac{Y(r - A_1^2)}{r^6}, \tag{23}
 \end{aligned}$$

$$\text{where } A_1 = \frac{r^3(r+1)}{r^3(r+1) - \frac{r_0^3(r_0+1)(B_2-1)}{B_2}} - B_2, \quad A_2 = \frac{\frac{r^3(r+1)}{r_0^3(r_0+1)(B_2-1)} - B_2}{r^3(r+1) - \frac{r_0^3(r_0+1)(B_2-1)}{B_2}}.$$

It is prominent that the energy density ρ shows a positive nature on the entire spacetime corresponding to the GB coupling parameter Y provided $Y < -1$. In the present inspection, we opt the GB coupling parameter Y with range $-2 \leq Y < -1$. Alternatively, one may select any other range of Y . With $r_0 = 2$, $B_2 = 0.5$, the energy density ρ display the positive development in Fig. 6 (left). A variation of the GB coupling parameter Y results in violations of the radial NEC Fig. 7 (left) while the tangential NEC Fig. 7 (right) validate for the given range of Y . There is a violation of the radial DEC Fig. 8 (left), tangential DEC Fig. 8 (right) is satisfied. Additionally, the behavior of the SEC, as shown in Fig. 6 (right panel), is also violated for the selected range of the GB coupling parameter.

6. Embedded WH geometry in 4D EGB gravity

To comprehend how gravity operates in the universe, we rely on an embedding diagram, which is used to illustrate and derive key insights into the structure of WHs. Many geometric properties are preserved when the equatorial slice is embedded within a spherically symmetric spacetime aligned with an equatorial plane $\theta = \frac{\pi}{2}$ and $t = \text{constant}$. Based on the assumptions in Eq. (15), we get

$$ds^2 = \left(1 - \frac{\varphi_S(r)}{r}\right)^{-1} dr^2 + r^2 d\phi^2. \tag{24}$$

The above equation can also be expressed using a three-dimensional cylindrical coordinate system (r, z, ϕ) as shown below:

$$\begin{aligned}
 ds^2 &= dr^2 + dz^2 + r^2 d\phi^2, \\
 ds^2 &= dr^2 \left[1 + \left(\frac{dz}{dr}\right)^2\right] + r^2 d\phi^2. \tag{25}
 \end{aligned}$$

On equating Eq. (24) with Eq. (25), we get

$$\frac{dz}{dr} = \pm \frac{1}{\sqrt{\frac{r}{\varphi_S(r)} - 1}}. \tag{26}$$

The embedding surface $z(r)$ can be obtained by simply integrating Eq. (26). The embedding diagram associated with the shape function $\varphi_S(r) = r - \frac{r^3}{r^2 + \frac{r_0^3}{B_1} - r_0^2} + B_1$ can be depicted in Fig. 9 (left panel) and Fig. 10 (left panel) for Model-I. Likewise, the embedding diagram related to the shape function $\varphi_S(r) = r - \frac{r^5}{r^4 + \frac{r_0^5}{B_2} - r_0^4}$ is illustrated in Fig. 9 (right panel) and Fig. 10 (right panel) corresponding to the Model-II.

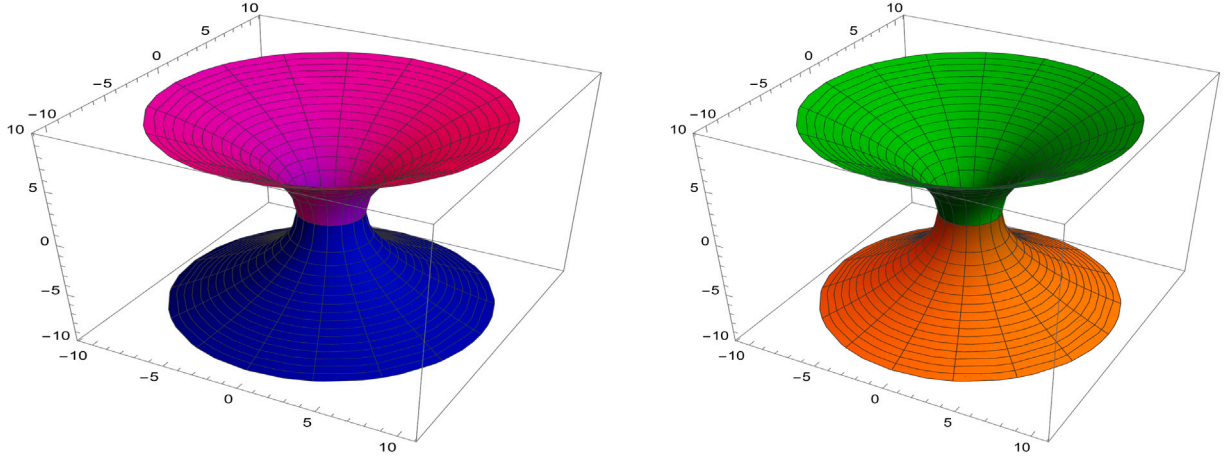


Fig. 10. 3-D WH embedding surface diagram for Model-I (left panel) and Model-II (right panel).

7. Stability evaluation of WH configurations

Now, in the present section, we provide the equilibrium criterion within $4\mathcal{D}$ EGB gravity, governed by the TOV equation [96,97], to establish the equilibrium configuration of the WH as follows.

$$\frac{dP_r}{dr} + \frac{\Lambda'(r)(P_r + \rho)}{2} + \frac{2(P_r - P_t)}{r} = 0, \quad (27)$$

where $\Lambda(r) = 2 \times \chi(r)$.

The equilibrium of the three forces acting on the WH is given by Eq. (27), which is written as $F_{hf} + F_{gf} + F_{af} = 0$, where

- $F_{gf} = \frac{-\Lambda'(r)(P_r + \rho)}{2}$, i.e. the gravitational attraction acting on a mass.
- $F_{af} = \frac{2(P_t - P_r)}{r}$, i.e. System anisotropy gives rise to the formation of a symmetric force.
- $F_{hf} = -\frac{dP_r}{dr}$, i.e. The hydrostatic force arises from a fluid in equilibrium.

The expression for $\mathcal{F}_{hf}$, $\mathcal{F}_{af}$ and $\mathcal{F}_{gf}$ of Model-I are given by

$$\begin{aligned} \mathcal{F}_{hf} = & \frac{2B_1^2(4\varpi + 1)Y \left(2rB_1 - 2r_0\varpi(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi+1} \right)}{\left(r^2B_1 + r_0^2(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^3} + \frac{4r^3 + r^2B_1(3 - 6\varpi) - 2rY - 2B_1(2\varpi + 1)Y}{r^6} \\ & + \frac{B_1 \left(r_0^2(2\varpi - 3)(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} - 5r^2B_1 \right) (r^3(2\varpi - 1) + 2r(2\varpi Y + Y) + 2B_1(4\varpi + 1)Y)}{r^4 \left(r^2B_1 + r_0^2(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^2} \\ & + \frac{B_1 (r^2(6\varpi - 3) + 4\varpi Y + 2Y)}{r^5B_1 + r^3r_0^2(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi}} + \frac{6(-r^4 + r^3B_1(2\varpi - 1) + r^2Y + 2rB_1(2\varpi + 1)Y + B_1^2(4\varpi + 1)Y)}{r^7}, \end{aligned} \quad (28)$$

$$\begin{aligned} \mathcal{F}_{af} = & \frac{1}{r^7} \left[2r^4 + 2r^3B_1\varpi^2 - 9r^3B_1\varpi + 3r^3B_1 - 4r^2Y + 4rB_1\varpi^2Y - 10rB_1\varpi Y - 10rB_1Y + 4B_1^2\varpi^2Y - 6B_1^2\varpi Y - 6B_1^2Y \right. \\ & \left. + \frac{4r^8B_1^3(\varpi^2 - 1)Y}{\left(r^2B_1 + r_0^2(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^3} + \frac{2r^3B_1(-2r^3(\varpi - 2)\varpi + 2r(1 - 2\varpi^2)Y + B_1(\varpi(7 - 2\varpi) + 3)Y)}{r^2B_1 + r_0^2(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi}} \right], \end{aligned} \quad (29)$$

$$\begin{aligned} \mathcal{F}_{gf} = & \frac{1}{2r^7} \left[\frac{2r^5B_1(r^3(\varpi + 1) + 2r(3\varpi + 2)Y + 2B_1(\varpi + 1)Y)}{\left(r^2B_1 + r_0^2(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^2} + \frac{-2r^3B_1(6\varpi + 1)Y - 2r^4(2\varpi + 1)(r^2 + 2Y)}{r^2B_1 + r_0^2(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi}} \right. \\ & \left. (2\varpi - 1)(r^3 + 2rY + 2B_1Y) - \frac{4r^8B_1^2(\varpi + 1)Y}{\left(r^2B_1 + r_0^2(r_0 - B_1) \left(\frac{r_0}{r} \right)^{2\varpi} \right)^3} \right]. \end{aligned} \quad (30)$$

The expression for $\mathcal{F}_{hf}$, $\mathcal{F}_{af}$ and $\mathcal{F}_{gf}$ of Model-II are given by

$$\begin{aligned} \mathcal{F}_{hf} = & \frac{4(-r^3(r + 1) + r_0^4 + r_0^3)(B_2 - 1)B_2Y(r^6(r + 1)^2B_2^2(2r + 3B_2 - 3)(B_2 - 1)^2(2r + 3B_2))}{r^2(r + 1)(r^3(r + 1)B_2 - (r_0^4(B_2 - 1)) - r_0^3(B_2 - 1))^3} \\ & + \frac{r_0^8(B_2 - 1)^2(2r + 3B_2) - r^3r_0^3B_2 - B_2(r(4r + 5) - r^3r_0^4(B_2 - 1)B_2(6(r + 1)B_2 + r(4r + 5)))}{r^2(r + 1)(r^3(r + 1)B_2 - (r_0^4(B_2 - 1)) - r_0^3(B_2 - 1))^3}, \end{aligned} \quad (31)$$

$$\mathcal{F}_{af} = \frac{4(-r^3(r + 1) + r_0^4 + r_0^3)(B_2 - 1)B_2Y(r^6(r + 1)^2B_2^2(2r + 3B_2 - 3)(B_2 - 1)^2(2r + 3B_2))}{r^2(r + 1)(r^3(r + 1)B_2 - (r_0^4(B_2 - 1)) - r_0^3(B_2 - 1))^3}$$

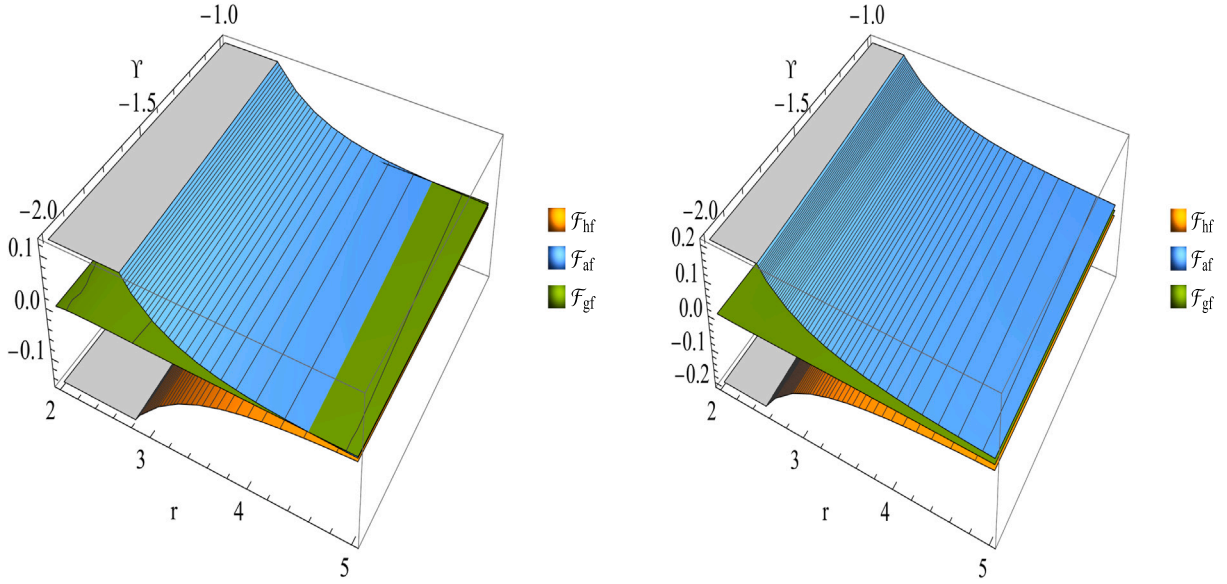


Fig. 11. Development of different forces ($\mathcal{F}_{hf}$, $\mathcal{F}_{af}$ and $\mathcal{F}_{gf}$) for Model-I (left) and Model-II (right) against the radial distance r .

$$+ \frac{4(r^4 + r^3 - r_0^3(r_0 + 1))(B_2 - 1)^2 B_2^2 Y((-r^3(r + 1) + r_0^4 + r_0^3)^2 B_2 - r_0^3(r_0 + 1)(r^3(3r + 2) + r_0^4 + r_0^3))}{2r^6(r + 1)(r^3(r + 1)B_2 - (r_0^4(B_2 - 1)) - r_0^3(B_2 - 1))^3}, \quad (32)$$

$$\mathcal{F}_{gf} = \frac{-(r_0^4(B_2 - 1)(r^3 + 2rY + 2B_2Y)) + r^3(r + 1)(B_2 - 1)B_2^2(r^3 + 2rY + 2(B_2 - 1)Y) - r_0^3(B_2 - 1)}{2r^7(r + 1)^2(r^3(r + 1)B_2 - (r_0^4(B_2 - 1)) - r_0^3(B_2 - 1))^3} \\ + \frac{-r^3r_0^4(r(2B_2 + 3) - 2B_2 + 4) - r^3(r + 1)r_0^3(r(2B_2 + 3) - 2B_2 + 4) + 2(r - 1)r_0^7 + (r - 1)r_0^8 + (r - 1)r_0^6}{2r^7(r + 1)^2(r^3(r + 1)B_2 - (r_0^4(B_2 - 1)) - r_0^3(B_2 - 1))^3}. \quad (33)$$

Since WH configuration with different forces namely $\mathcal{F}_{hf}$, $\mathcal{F}_{af}$ and $\mathcal{F}_{gf}$ are depicted in Fig. 11. The behavior of these forces depicted visually (in Model-I) illustrates an equilibrium state, as observed in Fig. 11. Also, Fig. 11 (left) represents the visual development of different forces, and the $\mathcal{F}_{hf}$ force is exhibited in the upper region. In contrast, the $\mathcal{F}_{af}$ force is presented in the lower region. Consequently, Fig. 11 shows the influence of $\mathcal{F}_{gf}$ force is approximately zero, as the hydrostatic force ($\mathcal{F}_{hf}$) exerts greater influence relative to the $\mathcal{F}_{af}$ force. As a result, $\mathcal{F}_{af} + \mathcal{F}_{hf} + \mathcal{F}_{gf} = 0$ can be concluded for Model-I. In a similar manner, the equilibrium condition is fulfilled for Model-II in Fig. 11 (right). Here, $\mathcal{F}_{gf}$ in Fig. 11 depicted the characteristics of gravitational force. The conditions for equilibrium are validated, since only the $\mathcal{F}_{hf}$ and $\mathcal{F}_{af}$ forces manifest this behavior.

8. Deflection angle

This section examines the deflection angle formed by WH geometry influenced by two different kinds of redshift functions in the background of Karmarkar conditions. It is very important to mention that the mass and energy create the curvature of space-time, which affects the velocity of light. The curvature of space-time can induce the bending of light in proximity to a massive object, such as a black hole or wormhole. The gravitational lensing, particularly its strong variant, has significantly intensified after the studies conducted by Virbhadrha and his corroborators [98,99]. Further, Bozza, in Ref. [100], presented an analytical method to calculate gravitational lensing in the strong field limit for any spherically symmetric spacetime. This methodology has been utilized in other following investigations, including Refs. [101,102]. This context prompts the application of this analytical technique in our ongoing research. We employ a numerical method to examine the deflection angle in proximity to the wormhole's throat to accomplish this objective. For a comprehensive derivation of the deflection angle, related to black hole can be found [103,104]. The formula for deflection angle Γ for the Morris–Thorne Wormhole geometry can be read as [105]

$$\Gamma = -\pi + 2 \int_{r_{cp}}^{\infty} \frac{e^{\chi(r)} dr}{r^2 \sqrt{(1 - \frac{\varphi_S(r)}{r})(\frac{1}{\beta^2} - \frac{e^{\chi(r)}}{r^2})}}, \quad (34)$$

where r_{cp} is the closest path of light near the throat and β is the impact parameter. For null geodesic, we have the relation between β and r_{cp} , defined by

$$\beta = r_{cp} e^{-\phi(r_{cp})}. \quad (35)$$

In the previous section, we have calculated two different kinds of wormhole solutions by considering Karmarkar condition. In order to study the deflection angles around the wormhole throat, we shall use the newly constructed wormhole solutions. One can calculate deflection angle Γ numerically from Eq. (34) by using two different redshift functions, $\chi(r) = \varpi \log(\frac{r_0}{r})$ [94], $\chi(r) = \log(1 + \frac{1}{r})$ [94] and related shape functions. Fig. 12 illustrates that the deflection angle approaches zero as the distance r_{cp} approaches infinity. Consequently, as the light beam moves away from the wormhole throat, where the gravitational influence of the wormhole is minimal, it remains unaffected by its initial path. Conversely, as the distance r_{cp} approaches the radius of the wormhole throat, the deflection angle markedly increases and becomes positive. Furthermore, within the wormhole throat, where the gravitational field is exceedingly intense, the deflection of light approaches to infinity. For the different values of involved parameters, one can see the minor effect on deflection angles for both model-I and model-II, it can be confirmed from Fig. 12.

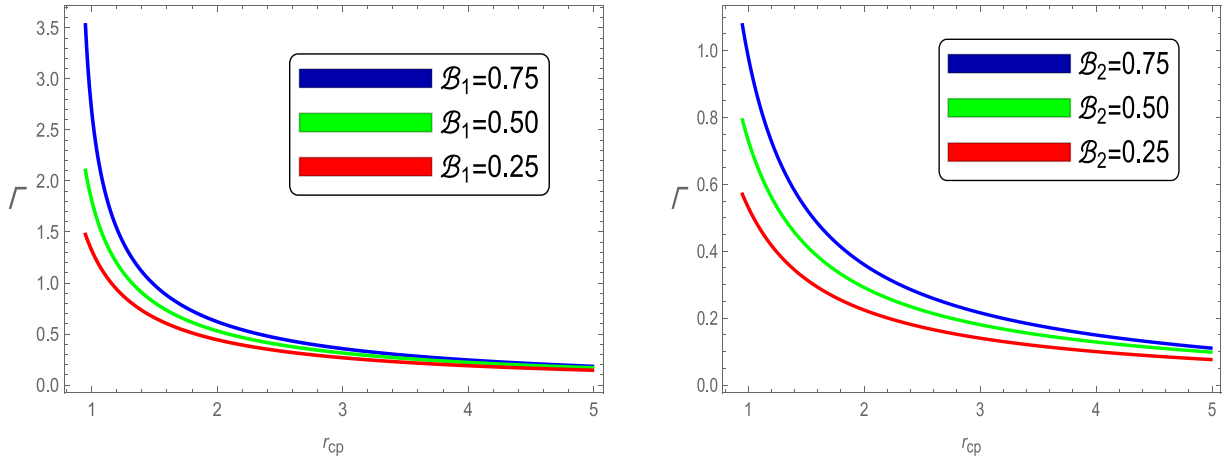


Fig. 12. Picture is showing the deflection angle for Model-I and Model-II.

9. Concluding remarks

WHs are an enigmatic, speculative formation in spacetime, the physical existence of which is still highly debated. Scientists and researchers have been delving thoroughly into the mathematically stated WH concept for a couple of decades. WH is known to depend on energy requirement violations, especially the NEC. The evidence that conventional matter validates all energy requirements is yet another assertion. Morris and Thorne [3] established the field of modern wormhole physics by proposing smooth traversable WHs made of exotic matter that contradicts the NEC [106]. Throughout our discussion, we explore a logarithmic redshift function, which enables a better comprehension of gravitational lensing and intriguing exact wormhole solutions. First and foremost, we have determined an accurate solution in 4D EGB-supported facilitated by an anisotropic material source with embedding class one technique.

For Model-I, with embedding class-1 approach, we determined the shape function $\varphi_S(r) = B_1 + r - \frac{r^3}{r^2 + (\frac{r_0^3}{B_1} - r_0^2)(\frac{r_0}{r})^{2\beta}}$ that validate all the prerequisites for the traversability of WH. Evidently, the derived shape function validates the flare-out constraint for the presumed WH throat radius $r_0 = 2$. The coupling strength between the geometry and matter is prescribed by the GB coupling constant Y . Fig. 3 gives a visual depiction conveys that the radial NEC is not validated. To achieve WH solutions for Model-I, the GB coupling parameter Y turns out to be a supplier of exotic matter and causes the invalidation of the WEC and NEC. Similarly for Model II, we get the expressions for matter components. In the present investigation, we derive the shape function which is given by $\xi_S(r) = r - \frac{r^5}{r^4 + \frac{r_0^5}{B_2} - r_0^4} + B_2$. Fig. 7 shows the violation of NEC ($\rho + P_r$). The WEC and NEC violations were caused by the GB coupling parameter Y , which provides the exotic matter needed in the vicinity of the WH. The WH solutions are in hydrostatic equilibrium for Model I and II (Fig. 8) under the mutual impact of the hydrostatic force (F_{hf}), anisotropic force (F_{af}) and gravitational force (F_{gf}).

We have encountered several related studies on WHs within various modified gravity theories with our current research. In recent times, Kavya et al. [107] showcase the development of energy conditions within $F(R, \mathcal{L}_m)$ by generating feasible WH solutions using specific shape functions and energy density. NEC was found to be violated for WH Model A in their inspection, still, it is valid for Model B. Furthermore, WH solutions in $F(R, \mathcal{L}_m)$ are contributed by Solanki and his associates [108] utilizing different equations of state and a specific shape function. Mishra and his collaborators [109] explored the traversable WH geometry by utilizing a constant redshift and different shape functions. However, our inspection deviates from the prescribed studies since we opt for a generic form of redshift function relying on the radial distance. In continuation, we also explore some more recent works including [86,110,111], the authors employing the embedding class one approach by taking the radial dependent form of $\phi = \frac{-2C}{r}$ where C is arbitrary, to obtain the form functions. In contrast to the present work, we choose a logarithm form of redshift function ($\beta \log(\frac{r_0}{r}) \& \log(1 + \frac{1}{r})$) which secures prominent importance because of its competency to offer an enhanced comprehension of the gravitational effects in the vicinity of WHs throat. Gravitational lensing [65] is another significant aspect related to WH dynamics. It modifies the observed traveling redshift light via WH by inducing it to bend. A logarithm redshift function is a more appropriate representation of these distortions and gives researchers a more thorough grasp of the gravitational lensing phenomena surrounding the WH. The intense gravitational field within the WH throat causes light deflection to nearly reach infinity. Fig. 12 indicates that variations in parameters have a minimal effect on the deflection angles in both Model-I and Model-II. Furthermore, because WHs are often associated with strong gravitational fields that produce time dilation effects, a logarithmic redshift function fits these temporal distortions better. In addition to improving prediction for WH observational signatures [112], this choice aids in distinguishing WHs from other astronomical events. Employing a logarithm form of redshift function is crucial for enhancing the understanding and observation strength of these intriguing cosmic creations.

CRediT authorship contribution statement

S.K. Maurya: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Jitendra Kumar:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Sourav Chaudhary:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Investigation, Formal analysis, Data curation, Conceptualization. **A. Errehymy:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources,

Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **G. Mustafa:** Writing – review & editing, Writing – original draft, Visualization, Validation, Resources, Investigation, Formal analysis, Data curation, Conceptualization. **K. Myrzakulov:** Writing – review & editing, Writing – original draft, Visualization, Validation, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no competing interests that could influence the results or interpretation of this research. They have no financial, personal, or professional affiliations that might be perceived as a bias in the conduct or reporting of the study. This includes any financial relationships with organizations or entities that have a direct interest in the subject matter discussed in this manuscript. All authors have read and approved the final version of the manuscript and agree to be accountable for all aspects of the work.

Acknowledgments

The author SKM acknowledges that this research work is also supported by the TRC Project (Grant No. BFP/RGP/CBS/24/203). Also, this research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP23487178). S. K. Maurya appreciates the administration of the University of Nizwa in the Sultanate of Oman for their unwavering support and encouragement. Jitendra Kumar acknowledges support from the authority of the Central University of Haryana (CUH). AE thanks the National Research Foundation of South Africa for the award of a postdoctoral fellowship.

Data availability

This manuscript has no associated data or the data will not be deposited (There is no observational data related to this article. The necessary calculations and graphic discussion are already available in the manuscript.)

References

- [1] L. Flamm, Beitrage zur Einsteinschen Gravitationstheorie, *Phys. Z.* 17 (1916) 448.
- [2] A. Einstein, N. Rosen, *Phys. Rev.* 48 (1935) 73–77.
- [3] M.S. Morris, K.S. Thorne, *Am. J. Phys.* 56 (1988) 395–412.
- [4] T. Harko, F.S.N. Lobo, M.K. Mak, S.V. Sushkov, *Phys. Rev. D* 87 (6) (2013) 067504.
- [5] M.K. Zangeneh, F.S.N. Lobo, N. Riazi, *Phys. Rev. D* 90 (2) (2014) 024072.
- [6] A. Banerjee, M.K. Jasim, S.G. Ghosh, *Ann. Phys.* 433 (2021) 168575.
- [7] R. Shaikh, S. Kar, *Phys. Rev. D* 94 (2) (2016) 024011.
- [8] K.A. Bronnikov, A.M. Galiakhmetov, *Gravit. Cosmol.* 21 (4) (2015) 283–288.
- [9] J.L. Rosa, J.P.S. Lemos, F.S.N. Lobo, *Phys. Rev. D* 98 (6) (2018) 064054.
- [10] J.L. Rosa, P.M. Kull, *Eur. Phys. J. C* 82 (12) (2022) 1154.
- [11] J.L. Rosa, *Phys. Rev. D* 104 (6) (2021) 064002.
- [12] S. Capozziello, T. Harko, T.S. Koivisto, F.S.N. Lobo, G.J. Olmo, *Phys. Rev. D* 86 (2012) 127504.
- [13] V. De Falco, E. Battista, S. Capozziello, M. De Laurentis, *Phys. Rev. D* 101 (10) (2020) 104037.
- [14] V. De Falco, E. Battista, S. Capozziello, M. De Laurentis, *Phys. Rev. D* 103 (4) (2021) 044007.
- [15] V. De Falco, *Phys. Rev. D* 108 (2) (2023) 024051.
- [16] V. De Falco, E. Battista, S. Capozziello, M. De Laurentis, *Eur. Phys. J. C* 81 (2) (2021) 157.
- [17] R. Deshpande, O. Lunin, *Nuclear Phys. B* 996 (2023) 116355.
- [18] B. Bhawal, S. Kar, *Phys. Rev. D* 46 (1992) 2464–2468.
- [19] M.R. Mehdizadeh, *Eur. Phys. J. C* 80 (4) (2020) 310.
- [20] M. Ilyas, K. Bamba, J. Cosmol. Astropart. Phys. 10 (2023) 038.
- [21] M. Zubair, M. Farooq, *Eur. Phys. J. C* 83 (6) (2023) 507.
- [22] G. Dotti, J. Oliva, R. Troncoso, *Phys. Rev. D* 76 (2007) 064038.
- [23] H. Maeda, M. Nozawa, *Phys. Rev. D* 78 (2008) 024005.
- [24] D. Glavan, C. Lin, *Phys. Rev. Lett.* 124 (8) (2020) 081301.
- [25] S. Hansraj, C. Hansraj, N. Mkhize, A. Errehymy, S.K. Maurya, *Phys. Dark Univ.* 47 (2025) 101757.
- [26] S. Hansraj, S. Remember Mngadi, A. Errehymy, *Nuclear Phys. B* 1010 (2025) 116753.
- [27] S. Hansraj, S.R. Mngadi, A. Errehymy, *Eur. Phys. J. C* 84 (11) (2024) 1146.
- [28] A. Errehymy, S.K. Maurya, G. Mustafa, S. Hansraj, H.I. Alrebd, A.H. Abdel-Aty, *Fortsch. Phys.* 71 (10–11) (2023) 2300052.
- [29] S.K. Maurya, A. Errehymy, K.N. Singh, N. Al-Harbi, K.S. Nisar, A.H. Abdel-Aty, *Eur. Phys. J. C* 83 (10) (2023) 968.
- [30] Y. Myrzakulov, K. Myrzakulov, S. Upadhyay, D.V. Singh, *Int. J. Geom. Meth. Mod. Phys.* 20 (07) (2023) 2350121.
- [31] A. Kumar, D.V. Singh, Y. Myrzakulov, G. Yergaliyeva, S. Upadhyay, *Eur. Phys. J. Plus* 138 (12) (2023) 1071.
- [32] D.G. Boulware, S. Deser, *Phys. Rev. Lett.* 55 (1985) 2656, <http://dx.doi.org/10.1103/PhysRevLett.55.2656>.
- [33] R.C. Myers, M.J. Perry, *Ann. Phys.* 172 (1986) 304, [http://dx.doi.org/10.1016/0003-4916\(86\)90186-7](http://dx.doi.org/10.1016/0003-4916(86)90186-7).
- [34] D. Glavan, C. Lin, *Phys. Rev. Lett.* 124 (8) (2020) 081301, <http://dx.doi.org/10.1103/PhysRevLett.124.081301>, arXiv:1905.03601, [gr-qc].
- [35] A. de la Cruz-Dombriz, D. Saez-Gomez, *Entropy* 14 (2012) 1717–1770, <http://dx.doi.org/10.3390/e14091717>, arXiv:1207.2663, [gr-qc].
- [36] S.G. Ghosh, D.V. Singh, S.D. Maharaj, *Phys. Rev. D* 97 (10) (2018) 104050, <http://dx.doi.org/10.1103/PhysRevD.97.104050>.
- [37] K. Aoki, M.A. Gorji, S. Mizuno, S. Mukohyama, *JCAP* 01 (2021) 054, <http://dx.doi.org/10.1088/1475-7516/2021/01/054>, arXiv:2010.03973, [gr-qc].
- [38] M. Visser, S. Kar, N. Dadhich, *Phys. Rev. Lett.* 90 (2003) 201102.
- [39] M. Visser, *Phys. Rev. D* 39 (1989) 3182.
- [40] E. Poisson, M. Visser, *Phys. Rev. D* 52 (1995) 7318.
- [41] F.S.N. Lobo, *Cl. Quantum Grav.* 21 (2004) 4811.
- [42] J.P.S. Lemos, F.S.N. Lobo, *Phys. Rev. D* 69 (2004) 104007.
- [43] F. Rahaman, M. Kalam, S. Chakraborty, *Gen. Relativity Gravitation* 38 (2006) 1687.
- [44] C. Bejarano, E.F. Eiroa, C. Simeone, *Phys. Rev. D* 75 (2007) 027501.
- [45] E. Gravanis, S. Willison, *Phys. Rev. D* 75 (2007) 084025.
- [46] S.D. Forghani, S.H. Mazharimousavi, M. Halilsoy, *Phys. Lett. B* 804 (2020) 135374.
- [47] S. Chaudhary, S.K. Maurya, J. Kumar, S. Ray, *Astropart. Phys.* 162 (2024) 103002.
- [48] R. Korolev, F.S.N. Lobo, S.V. Sushkov, *Phys. Rev. D* 101 (2020) 124057.
- [49] A. Restuccia, F. Tello-Ortiz, *Eur. Phys. J. C* 80 (2020) 580.

- [50] A. Errehymy, Y. Khedif, O. Donmez, M. Daoud, K. Myrzakulov, S. Bekov, *Eur. Phys. J. C* 84 (9) (2024) 908.
- [51] A. Errehymy, *Phys. Dark Univ.* 44 (2024) 101438.
- [52] A. Errehymy, S.K. Maurya, S. Hansraj, M. Daoud, H.I. Alrebdy, A. Abdel-Aty, *Ann. Phys.* 535 (8) (2023) 2300178.
- [53] E. Elizalde, M. Khurshudyan, *Internat. J. Modern Phys. D* 28 (2019) 1950172.
- [54] P. Sahoo, S. Mandal, P. Sahoo, *New Astron.* 80 (2020) 101421.
- [55] A. Errehymy, S.K. Maurya, G.E. Vilcu, M.A. Khan, M. Daoud, *Astropart. Phys.* 160 (2024) 102972.
- [56] K.N. Singh, A. Banerjee, F. Rahaman, M.K. Jasim, *Phys. Rev. D* 101 (2020) 084012.
- [57] F. Rahaman, S. Islam, P.K.F. Kuhfittig, S. Ray, *Phys. Rev. D* 86 (2012) 106010.
- [58] P. Bhar, F. Rahaman, *Eur. Phys. J. C* 74 (2014) 3213.
- [59] E. Battista, S. Capozziello, A. Errehymy, *Eur. Phys. J. C* 84 (12) (2024) 1314.
- [60] A. Errehymy, A. Banerjee, O. Donmez, M. Daoud, K.S. Nisar, A.H. Abdel-Aty, *Gen. Relativity Gravitation* 56 (6) (2024) 76.
- [61] A. Errehymy, A. Banerjee, S. Hansraj, O. Donmez, K.S. Nisar, A.H. Abdel-Aty, *Eur. Phys. J. C* 84 (6) (2024) 573.
- [62] F. Rahaman, P.K.F. Kuhfittig, S. Ray, N. Islam, *Eur. Phys. J. C* 74 (2014) 2750.
- [63] F. Rahaman, P. Salucci, P.K.F. Kuhfittig, S. Ray, M. Rahaman, *Ann. Phys.* 350 (2014) 561.
- [64] P.K.F. Kuhfittig, *Eur. Phys. J. C* 74 (99) (2014) 2818.
- [65] K.K. Nandi, Y.Z. Zhang, A.V. Zakharov, *Phys. Rev. D* 74 (2006) 024020.
- [66] M. Yu. Piotrovich, et al., *MNRAS* 498 (2020) 3684.
- [67] D. Glavan, C. Lin, *Phys. Rev. Lett.* 124 (2020) 081301.
- [68] S. Nojiri, S.D. Odintsov, *Phys. Lett. B* 631 (2005) 1.
- [69] K.R. Karmarkar, *Proc. Ind. Acad. Sci. Sect. A* 27 (1948) 56.
- [70] G. Abbas, S. Qaisar, W. Javed, M.A. Meraj, *Iran. J. Sci. Technol. A* 42 (2018) 1659.
- [71] P. Fuloria, N. Pant, *Eur. Phys. J. A* 53 (2017) 227.
- [72] P.K.F. Kuhfittig, *Ann. Phys.* 392 (2018) 63.
- [73] S. Gedela, R.K. Bisht, N. Pant, *Eur. Phys. J. A* 54 (2018) 207.
- [74] P.K.F. Kuhfittig, *Pramana* 92 (2019) 75.
- [75] L.P. Eisenhart, *Riemannian Geometry*, Princeton University Press, Princeton, United States of America, 1966.
- [76] J. Eiesland, *Trans. Amer. Math. Soc.* 27 (1925) 213.
- [77] M.R. Mehdizadeh, A.H. Ziaie, *Phys. Rev. D* 95 (2017) 064049.
- [78] P.H.F. de Oliveira, G. Alencar, I. Carneiro Jardim, R.R. Landim, *Symmetry* 15 (2023) 383.
- [79] J.L. Rosa, P.M. Kull, *Eur. Phys. J. C* 82 (2022) 1154.
- [80] A.S. Nezhad, M.R. Mehdizadeh, H. Golchin, *arXiv:2312.02714*, [gr-qc].
- [81] N. Godani, G.C. Samanta, *New Astron.* 80 (2020) 101399.
- [82] N. Godani, G.C. Samanta, *Eur. Phys. J. C* 80 (2020) 30.
- [83] M. Sharif, I. Nawazish, *Ann. Phys.* 389 (2018) 283.
- [84] U.K. Sharma, Shweta, A.K. Mishra, *Int. J. Geom. Meth. Mod. Phys.* 19 (2022) 2250019.
- [85] A. Malik, A. Ashraf, F. Mofarreh, A. Ali, M. Shoaib, *Int. J. Geom. Meth. Mod. Phys.* 20 (2023) 2350145.
- [86] A. Malik, F. Mofarreh, A. Zia, A. Ali, *Chin. Phys. C* 46 (2022) 095104.
- [87] A. Ditta, I. Hussain, G. Mustafa, A. Errehymy, M. Daoud, *Eur. Phys. J. C* 81 (10) (2021) 880, <http://dx.doi.org/10.1140/epjc/s10052-021-09668-7>.
- [88] N.U. Molla, H. Chaudhary, U. Debnath, G. Mustafa, S.K. Maurya, *Eur. Phys. J. C* 85 (1) (2025) 15, <http://dx.doi.org/10.1140/epjc/s10052-024-13720-7>, *arXiv:2406.09492*, [gr-qc].
- [89] G. Mustafa, A. Errehymy, F. Javed, S. Hansraj, S.K. Maurya, S. Sadiq, *Embedding Class One Galactic Halo Wormhole Geometries within Torsion and Matter Coupling Gravity Formalism*. Available at SSRN 4496249.
- [90] M. Kord Zangeneh, F.S.N. Lobo, M.H. Dehghani, *Phys. Rev. D* 92 (2015) 124049.
- [91] M. Cataldo, P. Labrana, S. del Campo, J. Crisostomo, P. Salgado, *Phys. Rev. D* 78 (2008) 104006.
- [92] M. Cataldo, P. Meza, P. Minning, *Phys. Rev. D* 83 (2011) 044050.
- [93] M. Jamil, et al., *Eur. Phys. J. C* 67 (2010) 513.
- [94] F. Parsaei, S. Rastgoo, P.K. Sahoo, *Eur. Phys. J. Plus* 137 (2022) 1083.
- [95] S.V. Sushkov, S.M. Kozyrev, *Rev. D* 84 (2011) 124026.
- [96] R.C. Tolman, *Phys. Rev.* 55 (1939) 364.
- [97] J.R. Oppenheimer, G.M. Volkoff, *Phys. Rev.* 55 (1939) 374.
- [98] K.S. Virbhadra, G.F.R. Ellis, *Phys. Rev. D* 62 (2000) 084003.
- [99] V. Bozza, et al., *Gen. Relativity Gravitation* 33 (2001) 1535.
- [100] C.M. Claudel, K.S. Virbhadra, G.F.R. Ellis, *J. Math. Phys.* 42 (2001) 818.
- [101] J.P.S. Lemos, F.S.N. Lobo, S. Quinet de Oliveira, *Phys. Rev. D* 68 (2003) 064004.
- [102] J.M. Tejeiro, E.A. Larranaga, *Romanian J. Phys.* 57 (2012) 736.
- [103] R. Shaikh, P. Banerjee, S. Paul, T. Sarkar, *JCAP* 07 (2019) 028.
- [104] N. Godani, G.C. Samanta, *Ann. Phys.* 429 (2021) 168460.
- [105] V. Bozza, *Phys. Rev. D* 66 (2002) 103001.
- [106] Hochberg, M. Visser, *Null energy condition in dynamic wormholes*, *Phys. Rev. Lett.* 81 (1998) 746, <http://dx.doi.org/10.1103/PhysRevLett.81.746>.
- [107] N.S. Kavya, V. Venkatesha, G. Mustafa, P.K. Sahoo, S.D. Rashmi, *Chinese J. Phys.* 84 (2023) 1.
- [108] R. Solanki, Z. Hassan, P.K. Sahoo, *Chinese J. Phys.* 85 (2023) 74.
- [109] B. Mishra, A.S. Agrawal, S.K. Tripathy, S. Ray, *Internat. J. Modern Phys. A* 37 (2022) 2250010.
- [110] M. Sharif, A. Fatima, *Eur. Phys. J. Plus* 138 (2023) 196, *arXiv:2303.01737*, [gr-qc].
- [111] G. Mustafa, Z. Hassan, P.H.R.S. Moraes, P.K. Sahoo, *Phys. Lett. B* 821 (2021) 136612.
- [112] J. Peng, M. Guo, X.H. Feng, *Phys. Rev. D* 104 (12) (2021) 124010.