



Original article

A new sine-arisen probabilistic model and artificial neural network methods for statistical modeling of the music engineering and reliability data

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ABSTRACT

Probability-arisen models play a considerable role in preparing a crucial stage for decision-making concerning reliability, engineering, and more closely related scenarios. Bearing in mind the consequential roles of probability-arisen models, we introduce and implement a new probabilistic model that has arisen by using the sine function, namely, the sine very flexible Weibull (SVF-Weibull) distribution. The proposed SVF-Weibull distribution is a result of a combination of the very flexible Weibull distribution with the sine-based strategy. For the SVF-Weibull distribution, point estimates are obtained. The assessment of the point estimates of the SVF-Weibull distribution is done via a simulation study. Finally, the consequential role of the SVF-Weibull distribution, illustrated by considering reliability and music engineering data sets. Furthermore, we implement some machine learning tools for predicting the reliability and music engineering data sets. The performances of the machine learning tools are assessed across many hidden variables. Our findings suggest that the artificial neural network method is more optimal than other methods for predicting the reliability and music engineering data sets.

1. Introduction

Predominantly, probability distributions have significant applications [1,2], especially, the Weibull distribution (involving two parameters) has a deep and profound role in data modeling in all areas of application. On one side, the Weibull distribution is considered a special case of some available distributions, for instance, the exponentiated Weibull distribution of Mudholkar and Srivastava [3] and Kumaraswamy Weibull distribution of Cordeiro et al. [4]. On the other side, the Weibull distribution also generalizes some existing distributions, for instance, Rayleigh and exponential distributions. For a concise description; see [5].

Suppose Z ($Z \in \mathbb{R}^+$) has the Weibull distribution having the cumulative distribution function (CDF) $F(z; \Xi)$. Then, its $F(z; \Xi)$ is

$$F(z; \Xi) = 1 - e^{-\beta z^\theta}, \quad z \geq 0, \quad (1)$$

where $\theta > 0$, $\beta > 0$, and $\Xi = (\theta, \beta)$.

The Weibull distribution has been significantly extended and implemented due to its consequential and momentous role in applied science.

For some very recent modifications of $F(z; \Xi)$ in (1) with implications in different sectors, we refer Alshenawy et al. [6], Ghazal [7], and Shama et al. [8], among others.

Bebbington et al. [9] studied an alternative model to (1) called the flexible Weibull extension. They studied different reliability properties of the flexible Weibull extension. Its CDF $F(z; \kappa)$ is

$$F(z; \kappa) = 1 - e^{-A(z; \kappa)}, \quad z \geq 0, \quad (2)$$

where $A(z; \kappa) = e^{\left(\gamma z - \frac{\lambda}{z}\right)}$ and $\kappa = (\gamma, \lambda)$. The distribution with $F(z; \kappa)$ in (2) has been further generalized by several authors; see [10–14].

Taking motivation from (1) and (2), Ahmad [15] suggested a more flexible probabilistic model. This probabilistic model was called very flexible Weibull (VFW) distribution by the author (i.e., Ahmad [15]). The VFW distribution arose by using a mixture of the logarithm of cumulative hazard functions. The CDF $F(z; \Psi)$ of the VFW distribution is

$$F(z; \Psi) = 1 - e^{-B(z; \Psi)}, \quad z \geq 0. \quad (3)$$

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The probability density function $f(z; \boldsymbol{\psi})$ of the VFW distribution is

$$f(z; \boldsymbol{\psi}) = \left(\delta \eta z^{\delta-1} + \frac{1}{z^2} \right) B(z; \boldsymbol{\psi}) e^{-B(z; \boldsymbol{\psi})}, \quad z > 0, \quad (4)$$

where $B(z; \boldsymbol{\psi}) = e^{\left(\eta z^{\delta} - \frac{1}{z} \right)}$ and $\boldsymbol{\psi} = (\delta, \eta)$.

Considering an application, in [15], the author has shown that the VFW distribution outperforms a number of other flexibility distributions that are famous and frequently studied in the literature. Brito et al. [16] also supported the claim of the optimal fitting ability of the VFW distribution by studying some complete data sets. They considered some classical and Bayesian inferential methods using the VFW distribution under the presence of progressive type-II censoring. They indicated that the VFW distribution is a promising model but lacks further studies on its properties and practical implications under different phenomena or situations.

Inspired by the facts given about the VFW distribution in [16], we introduce its further modified version by taking into account a trigonometric function. Moreover, in order to address the gap in the implementation of the VFW distribution in practical domains, as highlighted by Brito et al. [16], we employ the newly updated version of the VFW distribution in the fields of reliability and music engineering. The new version of the VFW distribution is obtained by utilizing the sine-originated method of Alshanbari et al. [17]. Its CDF $G(z; \boldsymbol{\psi})$ is

$$G(z; \boldsymbol{\psi}) = 1 - \left(\frac{1 - \sin \left[\frac{\pi}{2} F(z; \boldsymbol{\psi}) \right]}{1 + \sin \left[\frac{\pi}{2} F(z; \boldsymbol{\psi}) \right]} \right)^2, \quad z \in \mathbb{R}. \quad (5)$$

In Section 2, we mix (3) with (5) to introduce a new version of the VFW distribution. We call it the sine very flexible Weibull (SVF-Weibull) distribution. Furthermore, Section 2 also showcases the visual representations of the SVF-Weibull distribution, in addition to its core functions. The SVF-Weibull distribution is a very flexible model, and its PDF has unimodal and bimodal behaviors. In addition to the unimodal and bimodal depictions of the PDF, the visual exploration of the HF of the SVF-Weibull distribution is also provided in Section 2. From the visual investigation of HF, we observe that the SVF-Weibull distribution can capture unimodal and modified unimodal forms of HF. The parameters estimation of the SVF-Weibull distribution and simulation study are provided in Section 3. The SVF-Weibull distribution is illustrated through two data sets in Section 4. For prediction purposes, certain machine learning tools are implemented in Section 5, where, Section 6 conclude the whole work and contribution of the paper.

2. The SVF-Weibull distribution

The main objective of this section is to familiarize readers with the essential distributional properties of the SVF-Weibull model. Suppose $Z (Z \in \mathbb{R}^+)$ follows the SVF-Weibull distribution. Via inserting (3) in (5), we get the CDF $G(z; \boldsymbol{\psi})$ of the SVF-Weibull distribution. Its form becomes

$$G(z; \boldsymbol{\psi}) = 1 - \left(\frac{1 - \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right]}{1 + \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right]} \right)^2. \quad (6)$$

Accompanying $G(z; \boldsymbol{\psi})$ in (6), the probability density function (PDF) $g(z; \boldsymbol{\psi})$ is

$$g(z; \boldsymbol{\psi}) = \frac{2\pi \left(\delta \eta z^{\delta-1} + \frac{1}{z^2} \right) B(z; \boldsymbol{\psi}) e^{-B(z; \boldsymbol{\psi})} \cos \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right]}{\left(1 + \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right] \right)^3} \times \left(1 - \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right] \right), \quad Z \in \mathbb{R}^+. \quad (7)$$

Connected with $G(z; \boldsymbol{\psi})$ of the SVF-Weibull distribution in (6), the survival function $S(z; \boldsymbol{\psi})$ is

$$S(z; \boldsymbol{\psi}) = \left(\frac{1 - \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right]}{1 + \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right]} \right)^2,$$

with hazard function $h(z; \boldsymbol{\psi})$ is

$$h(z; \boldsymbol{\psi}) = \frac{2\pi \left(\delta \eta z^{\delta-1} + \frac{1}{z^2} \right) B(z; \boldsymbol{\psi}) e^{-B(z; \boldsymbol{\psi})} \cos \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right]}{\left(1 + \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right] \right) \left(1 - \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right] \right)},$$

and cumulative hazard function $H(z; \boldsymbol{\psi})$ is

$$H(z; \boldsymbol{\psi}) = -2 \log \left(\frac{1 - \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right]}{1 + \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z; \boldsymbol{\psi})} \right]} \right),$$

Fig. 1 delivers definite plots of $G(z; \boldsymbol{\psi})$ and $S(z; \boldsymbol{\psi})$. The graphs of $G(z; \boldsymbol{\psi})$ and $S(z; \boldsymbol{\psi})$ are delivered using $\delta = (0.4, 1.4, 0.1)$ and $\eta = (0.9, 0.2, 0.1)$. The visual investigation conducted in Fig. 1 confirms that the SVF-Weibull distribution possesses a valid and concise CDF, as the curves of $G(z; \boldsymbol{\psi})$ are situated within the range of 0 and 1.

Fig. 2 conveys certain demonstrations of $g(z; \boldsymbol{\psi})$. The visual demonstrations of $g(z; \boldsymbol{\psi})$ are provided for $\delta = (1.8, 17.2)$ and $\eta = (1.2, 1.2)$. The exploration, in Fig. 2, shows the uni-modal and bi-modal behaviors of $g(z; \boldsymbol{\psi})$ of the SVF-Weibull distribution.

Fig. 3 put on show the plots of $h(z; \boldsymbol{\psi})$. The visual demonstrations of $h(z; \boldsymbol{\psi})$ are provided for $\delta = (1.2, 12.2)$ and $\eta = (0.1, 0.001)$. The exploration, in Fig. 3, shows the non-monotonic behaviors (especially, the uni-modal and modified uni-modal) of $h(z; \boldsymbol{\psi})$ of the SVF-Weibull distribution.

3. Estimation and simulation

The purpose of this section is twofold. The first aim is to obtain the point estimates $\hat{\delta}$ and $\hat{\eta}$ of the parameters δ and η , respectively. Once we have obtained the estimates $\hat{\delta}$ and $\hat{\eta}$, we will proceed to the second objective of evaluating the accuracy and reliability of these estimates. This will be done through (by means of) a simulation study.

3.1. Estimation

Assuming randomly observed samples $z_1, z_2, \dots, z_n$ from $g(z; \boldsymbol{\psi})$ which is expressed by (7). Relating with $g(z; \boldsymbol{\psi})$, the likelihood function $\tau(\boldsymbol{\psi})$ is

$$\tau(\boldsymbol{\psi}) = \prod_{i=1}^n g(z_i; \boldsymbol{\psi}). \quad (8)$$

Using (7) in (8), we arrive at

$$\tau(\boldsymbol{\psi}) = \prod_{i=1}^n \frac{\left(\delta \eta z_i^{\delta-1} + \frac{1}{z_i^2} \right) B(z_i; \boldsymbol{\psi}) e^{-B(z_i; \boldsymbol{\psi})} \cos \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right]}{\left(1 + \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right] \right)^3} \times \left(1 - \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right] \right). \quad (9)$$

Without taking into account the constant terms, (10) provides the log likelihood function, say $\log \tau(\boldsymbol{\psi})$, associated with $\tau(\boldsymbol{\psi})$, as expressed by

$$\log \tau(\boldsymbol{\psi}) = \sum_{i=1}^n \log \left(\delta \eta z_i^{\delta-1} + \frac{1}{z_i^2} \right) + \sum_{i=1}^n \log B(z_i; \boldsymbol{\psi}) - \sum_{i=1}^n B(z_i; \boldsymbol{\psi}) - 3 \sum_{i=1}^n \log \left(1 + \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right] \right) + \sum_{i=1}^n \log \left(1 - \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right] \right) \quad (10)$$

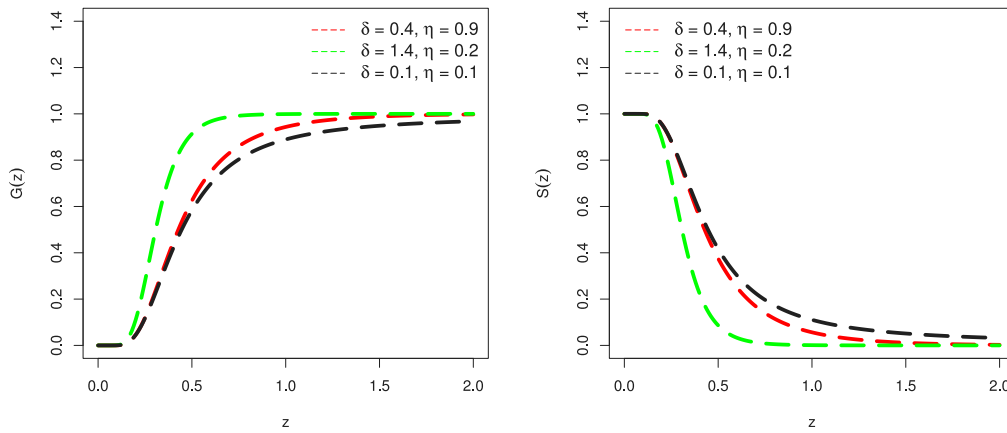


Fig. 1. The illustrations of $G(z; \boldsymbol{\psi})$ and $S(z; \boldsymbol{\psi})$ for $\delta = (0.4, 1.4, 0.1)$ and $\eta = (0.9, 0.2, 0.1)$.

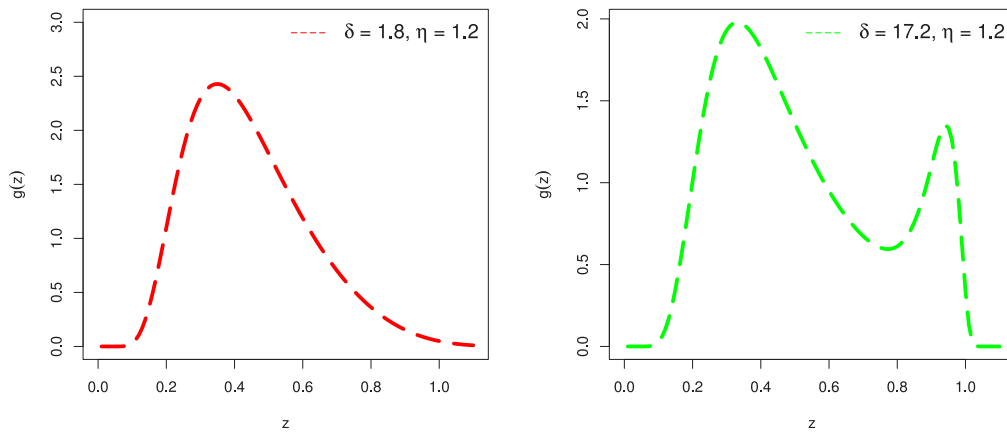


Fig. 2. The illustrations of $g(z; \boldsymbol{\psi})$ for $\delta = (1.8, 17.2)$ and $\eta = (1.2, 1.2)$.

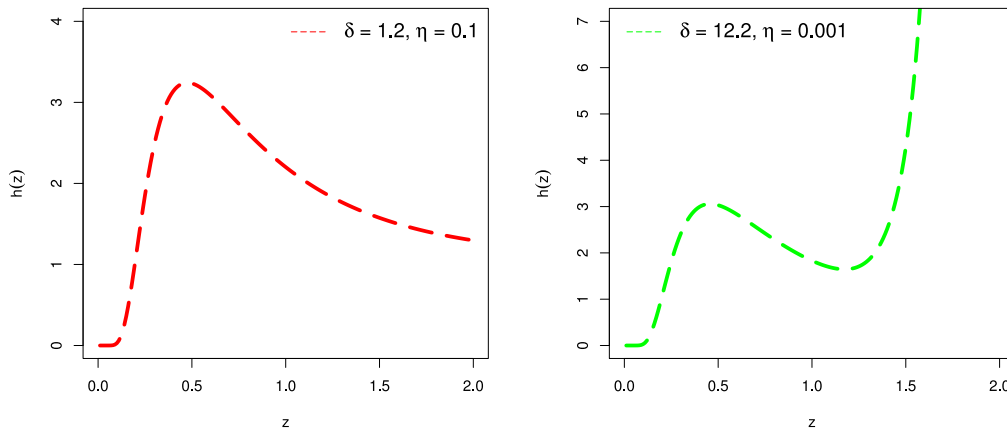


Fig. 3. The illustrations of $h(z; \boldsymbol{\psi})$ for $\delta = (1.2, 12.2)$ and $\eta = (0.1, 0.001)$.

$$+ \sum_{i=1}^n \log \left(\cos \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right] \right).$$

Corresponding to δ and η , the partial derivatives linking with $\log \tau(\boldsymbol{\psi})$ in (10), are given by

$$\begin{aligned} \frac{\partial}{\partial \delta} \log \tau(\boldsymbol{\psi}) &= \sum_{i=1}^n \frac{\eta \left[\delta (\log z_i) z_i^{\delta-1} + z_i^{\delta-1} \right]}{\left(\delta \eta z_i^{\delta-1} + \frac{1}{z_i^2} \right)} + \sum_{i=1}^n \frac{B'_1(z_i; \boldsymbol{\psi})}{B(z_i; \boldsymbol{\psi})} \\ &\quad - \sum_{i=1}^n B'_1(z_i; \boldsymbol{\psi}) \end{aligned}$$

$$\begin{aligned} &- \frac{3\pi}{2} \sum_{i=1}^n \frac{B'_1(z_i; \boldsymbol{\psi}) e^{-B(z_i; \boldsymbol{\psi})} \cos \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right]}{\left(1 + \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right] \right)} \\ &- \frac{\pi}{2} \sum_{i=1}^n \frac{B'_1(z_i; \boldsymbol{\psi}) e^{-B(z_i; \boldsymbol{\psi})} \cos \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right]}{\left(1 - \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right] \right)} \\ &- \frac{\pi}{2} \sum_{i=1}^n \frac{B'_1(z_i; \boldsymbol{\psi}) e^{-B(z_i; \boldsymbol{\psi})} \sin \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right]}{\left(\cos \left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})} \right] \right)}, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial}{\partial \eta} \log \tau(\boldsymbol{\psi}) &= \sum_{i=1}^n \frac{\delta z_i^{\delta-1}}{\left(\delta \eta z_i^{\delta-1} + \frac{1}{z_i^2}\right)} + \sum_{i=1}^n \frac{B'_2(z_i; \boldsymbol{\psi})}{B(z_i; \boldsymbol{\psi})} - \sum_{i=1}^n B'_2(z_i; \boldsymbol{\psi}) \\ &\quad - \frac{3\pi}{2} \sum_{i=1}^n \frac{B'_2(z_i; \boldsymbol{\psi}) e^{-B(z_i; \boldsymbol{\psi})} \cos\left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})}\right]}{\left(1 + \sin\left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})}\right]\right)} \\ &\quad - \frac{\pi}{2} \sum_{i=1}^n \frac{B'_2(z_i; \boldsymbol{\psi}) e^{-B(z_i; \boldsymbol{\psi})} \cos\left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})}\right]}{\left(1 - \sin\left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})}\right]\right)} \\ &\quad - \frac{\pi}{2} \sum_{i=1}^n \frac{B'_2(z_i; \boldsymbol{\psi}) e^{-B(z_i; \boldsymbol{\psi})} \sin\left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})}\right]}{\left(\cos\left[\frac{\pi}{2} - \frac{\pi}{2} e^{-B(z_i; \boldsymbol{\psi})}\right]\right)}, \end{aligned}$$

respectively.

In the expressions that are presented above, the terms $B'_1(z_i; \boldsymbol{\psi})$ and $B'_2(z_i; \boldsymbol{\psi})$ are given by

$$\frac{\partial}{\partial \delta} \log \tau(\boldsymbol{\psi}) = B'_1(z_i; \boldsymbol{\psi}),$$

and

$$\frac{\partial}{\partial \eta} \log \tau(\boldsymbol{\psi}) = B'_2(z_i; \boldsymbol{\psi}),$$

respectively.

Solving the expressions $\frac{\partial}{\partial \delta} \log \tau(\boldsymbol{\psi}) = 0$ and $\frac{\partial}{\partial \eta} \log \tau(\boldsymbol{\psi}) = 0$, respectively, delivers the point estimates $(\hat{\delta}, \hat{\eta})$ of (δ, η) .

3.2. Simulation

This second piece of Section 3 provides the performance assessment of $\hat{\delta}$ and $\hat{\eta}$ of the SVF-Weibull distribution. The performance assessment of $\hat{\delta}$ and $\hat{\eta}$ is shown by incorporating different sets of simulation studies to check how $\hat{\delta}$ and $\hat{\eta}$ behave with increase in n (i.e., sample size).

To achieve the intended objective, we obtain random numbers using the inverse form of (6), as explicitly stated in the given expression:

$$B(z; \boldsymbol{\psi}) = -\log \left[1 - \frac{2}{\pi} \sin^{-1} \left(\frac{1 - \sqrt{1-u}}{1 + \sqrt{1-u}} \right) \right] \quad (11)$$

where $u \in (0, 1)$.

The results of the simulation are obtained by using random numbers that are generated using (11). This simulation study is iterated 1000 times to ensure robustness and reliability of the results and we represent it with N (i.e., the iterations).

The behaviors of $\hat{\delta}$ and $\hat{\eta}$ are assessed by using the

- Bias

$$Bias(\boldsymbol{\psi}) = \frac{1}{N} \sum_{i=1}^N (\boldsymbol{\psi}_i - \boldsymbol{\psi}),$$

and

- Mean square error (MSE)

$$MSE(\boldsymbol{\psi}) = \frac{1}{N} \sum_{i=1}^N (\boldsymbol{\psi}_i - \boldsymbol{\psi})^2.$$

The process of acquiring the values of the MSE, bias, and MLEs $(\hat{\delta}, \hat{\eta})$, is carried out for:

(i) $\delta = 0.9, \eta = 1.2$, (ii) $\delta = 1.5, \eta = 1.0$ and (iii) $\delta = 1.4, \eta = 0.9$. The simulation results are obtained by incorporating the R-script with library(rootSolve), and method = L-BFGS-B. Tables 1–3 present the simulation results for $n = 50, 100, 200, \dots, 1000$.

For $\delta = 0.9, \eta = 1.2$, $\delta = 1.5, \eta = 1.0$, and $\delta = 1.4, \eta = 0.9$, the numerical presentation of results are illustrated in Tables 1, 2, and 3, respectively.

The numerical exploration of the simulation results in Tables 1–3, reveal that as we observe increments in n , we observe:

Table 1

The numerical presentation of the simulation results of the SVF-Weibull distribution for $\delta = 0.9$ and $\eta = 1.2$.

n	Parameters	MLE	MSE	Bias
50	δ	0.9707857	0.0057234099	0.025909238
	η	1.2876409	0.0348760923	0.032069439
100	δ	0.9291597	0.0049806877	0.018159662
	η	1.2385910	0.0235314543	0.027590649
200	δ	0.9185640	0.0020823908	0.007564029
	η	1.2176180	0.0096569986	0.006617933
300	δ	0.9162742	0.0010892894	0.005274190
	η	1.2205560	0.0056856482	0.009555555
400	δ	0.9162085	0.0008797781	0.005208492
	η	1.2149750	0.0035647665	0.003975207
500	δ	0.9145770	0.0005992584	0.003576999
	η	1.2129220	0.0026875986	0.001921692
600	δ	0.9144889	0.0004681682	0.003488912
	η	1.2144370	0.0020396660	0.003437467
700	δ	0.9142255	0.0003985600	0.003225512
	η	1.2123270	0.0014362377	0.001327309
800	δ	0.9146159	0.0003196487	0.003615857
	η	1.2127990	0.0014061005	0.001798717
900	δ	0.9128133	0.0002224341	0.001813276
	η	1.2128550	0.0011273400	0.001854732
1000	δ	0.9130480	0.0001745326	0.002047963
	η	1.2132830	0.0008512831	0.002282957

Table 2

The numerical presentation of the simulation results of the SVF-Weibull distribution for $\delta = 1.5$ and $\eta = 1$.

n	Parameters	MLE	MSE	Bias
50	δ	1.586539	0.018987650	0.021765093
	η	1.070985	0.020875393	0.024098240
100	δ	1.527106	0.013344505	0.016106359
	η	1.030925	0.017333765	0.019925175
200	δ	1.525510	0.007351119	0.014510146
	η	1.020180	0.007764831	0.009180205
300	δ	1.519412	0.004555874	0.008411696
	η	1.020509	0.005431596	0.009509195
400	δ	1.515986	0.003268140	0.004986429
	η	1.018391	0.004049169	0.007390807
500	δ	1.516716	0.002469497	0.005716492
	η	1.015717	0.003020755	0.004717276
600	δ	1.512809	0.002112210	0.001808978
	η	1.015432	0.002437949	0.004431559
700	δ	1.513703	0.001985515	0.002702754
	η	1.013279	0.001964134	0.002279423
800	δ	1.512777	0.001645280	0.001777479
	η	1.012900	0.001882446	0.001899633
900	δ	1.513535	0.001373773	0.002535254
	η	1.014544	0.001620137	0.003544336
1000	δ	1.513055	0.001355501	0.002055385
	η	1.012539	0.001510801	0.001539481

- stability of $\hat{\delta}$ and $\hat{\eta}$, i.e., tend to stable.
- decrementation in the MSEs of $\hat{\delta}$ and $\hat{\eta}$.
- biases of $\hat{\delta}$ and $\hat{\eta}$ tend to zero.

4. Implementations of the SVF-Weibull model

In this area of the article, we make arrangements for presenting two data examples in order to reveal the importance of the SVF-Weibull distribution. The SVF-Weibull distribution is first exemplified using the data on failure times. In addition to the reliability data example,

Table 3

The numerical presentation of the simulation results of the SVF-Weibull distribution for $\delta = 1.4$ and $\eta = 0.9$.

n	Parameters	MLE	MSE	Bias
50	δ	1.4898820	0.017653091	0.025234903
	η	0.9409834	0.018543067	0.017896243
100	δ	1.4217480	0.012246013	0.020747526
	η	0.9152300	0.013219490	0.014230000
200	δ	1.4114060	0.006235260	0.010405798
	η	0.9091437	0.005640561	0.008143688
300	δ	1.4063350	0.003571102	0.005334695
	η	0.9057016	0.003727469	0.004701586
400	δ	1.4058130	0.002862664	0.004813004
	η	0.9043351	0.002833274	0.003335081
500	δ	1.4041230	0.002154063	0.003123044
	η	0.9021094	0.002238644	0.001109443
600	δ	1.4048870	0.001870609	0.003886682
	η	0.9037810	0.002030669	0.002780956
700	δ	1.4041710	0.001451041	0.003171254
	η	0.9026554	0.001620184	0.001655428
800	δ	1.4029000	0.001237211	0.001899831
	η	0.9019898	0.001376179	0.000989831
900	δ	1.4030930	0.001217126	0.002092559
	η	0.9039578	0.001264708	0.002957827
1000	δ	1.4034550	0.001062924	0.002454515
	η	0.8991219	0.001151372	-0.001878134

we consider another example from music engineering to establish the second exemplification of the SVF-Weibull distribution.

4.1. Summary and graphical descriptions of the data set

The very first example (be regarded as Data 1) consist of fifty observations. It shows the failure time of electronic stuffs/equipment (noted per 1000 h); see [18]. The summary of Data 1 (about three decimal points) are: 0.036 (minimum observation), 15.080 (maximum observation), 3.343 (mean point), 1.414 (media point), 0.207 (1st quartile point), 4.498 (3rd quartile point), 1.416 (skewness of Data 1), 4.084 (kurtosis of Data 1), 17.484 (variance), and 15.044 (range of Data 1).

The audio engineer is known to use a range of music equipment in order to prepare recordings. When looking at potential musical equipments, the durability of headphones is an important factor to consider (be regarded as Data 2); see [19]. The summary of Data 2 (about three decimal points) are: 1.321 (minimum observation), 5.064 (maximum observation), 2.711 (mean point), 2.635 (media point), 2.104 (1st quartile point), 3.146 (3rd quartile point), 0.632 (skewness of Data 2), 3.286 (kurtosis of Data 2), 0.555 (variance), and 3.742 (range of Data 2).

For Data 1, the plots of the normal quantile, histogram, violin plot, and box plot are described in Fig. 4. These plots show that Data 1 is skewed to right. For Data 2, these plots are presented through Fig. 5. These plots show that Data 2 is uni-modal and skewed to right. In addition to summary plots, the observations of these data sets are illustrated visually through Fig. 6.

4.2. Rival models along with selection standards

Here, we discuss some competing models against the SVF-Weibull distribution as best possible alternatives for Data 1 and Data 2. The competing models are (i) Weibull distribution (regarded as Distribution-1 for data analyses) and F-Weibull distribution (regarded as Distribution-2 for data analyses). The SFs of Distribution-1 and Distribution-2 are given by

$$S(z; \Xi) = e^{-\beta z^\theta}, \quad z > 0,$$

Table 4

The values of $\hat{\delta}, \hat{\eta}, \hat{\theta}, \hat{\beta}, \hat{\lambda}$, and $\hat{\gamma}$ of the SVF-Weibull, Distribution-1, and Distribution-2 for Data 1.

Dist.	$\hat{\delta}$	$\hat{\eta}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\gamma}$
SVF-Weibull	0.0963	1.0770	–	–	–	–
Distribution-1	–	–	0.6606	0.5416	–	–
Distribution-2	–	–	–	–	0.1803	0.0990

and

$$S(z; \kappa) = e^{-A(z; \kappa)}, \quad z > 0,$$

respectively.

For Data 1 and Data 2, the identification of the optimal model among the SVF-Weibull distribution, Distribution-1, and Distribution-2 is made using certain selection standards (decisive tools). From the term standard tools, we meant to say the information criteria (IC).

The selection standards or IC are given by:

- Akaike IC (we use and display by κ_1^*) calculated as

$$2k - 2\ell(\cdot),$$

- Consistent akaike IC (it is referred and display it with κ_2^{**}) calculated as

$$\frac{2nk}{n-k-1} - 2\ell(\cdot),$$

- Bayesian IC (we use and display it with κ_3^{***}) calculated as

$$k \log(n) - \ell(\cdot),$$

and

- Hannan-Quinn IC (we use and display through κ_4^{****}) calculated as

$$2k \log[\log(n)] - 2\ell(\cdot).$$

4.3. Failure times data

Associated with Data 1, the values of $\hat{\delta}, \hat{\eta}, \hat{\theta}, \hat{\beta}, \hat{\lambda}$, and $\hat{\gamma}$ of the SVF-Weibull, Distribution-1, and Distribution-2 are given in Table 4.

Furthermore, the values of $\kappa_1^*, \kappa_2^{**}, \kappa_3^{***}$, and κ_4^{****} of the SVF-Weibull, Distribution-1, and Distribution-2, for Data 1 are displayed by Table 5.

The SVF-Weibull distribution results corresponding to Data 1 are: $\kappa_1^* = 192.981, \kappa_2^{**} = 193.236, \kappa_3^{***} = 196.805$, and $\kappa_4^{****} = 195.437$. For Distribution-1, we have $\kappa_1^* = 208.728, \kappa_2^{**} = 208.984, \kappa_3^{***} = 212.552$, and $\kappa_4^{****} = 210.184$. Furthermore, for Distribution-2, we have $\kappa_1^* = 195.851, \kappa_2^{**} = 196.106, \kappa_3^{***} = 199.675$, and $\kappa_4^{****} = 197.307$. The given and stated facts favors the optimal fitting ability of the SVF-Weibull distribution for Data 1.

Associated with Data 1, besides the given results favoring to the SVF-Weibull distribution, we also implement another approach to again confirms its optimality using Data 1. The second given approach is based via implementing the visual illustrations. For instance, we use the empirical PDF approach (left-panel plot), empirical CDF method (middle-panel plot), and Kaplan–Meier SF approach (right-panel plot). Thus, associated to failure times data, the fitted plots of the SVF-Weibull distribution is displayed by Fig. 7. The empirical exploration in Fig. 7 shows the SVF-Weibull distribution provides the optimal (or close) fit to Data 1.

4.4. Music engineering data

The reported values for $\hat{\delta}, \hat{\eta}, \hat{\theta}, \hat{\beta}, \hat{\lambda}$, and $\hat{\gamma}$ of the SVF-Weibull, Distribution-1, and Distribution-2 associated with Data 2 can be found in Table 6.

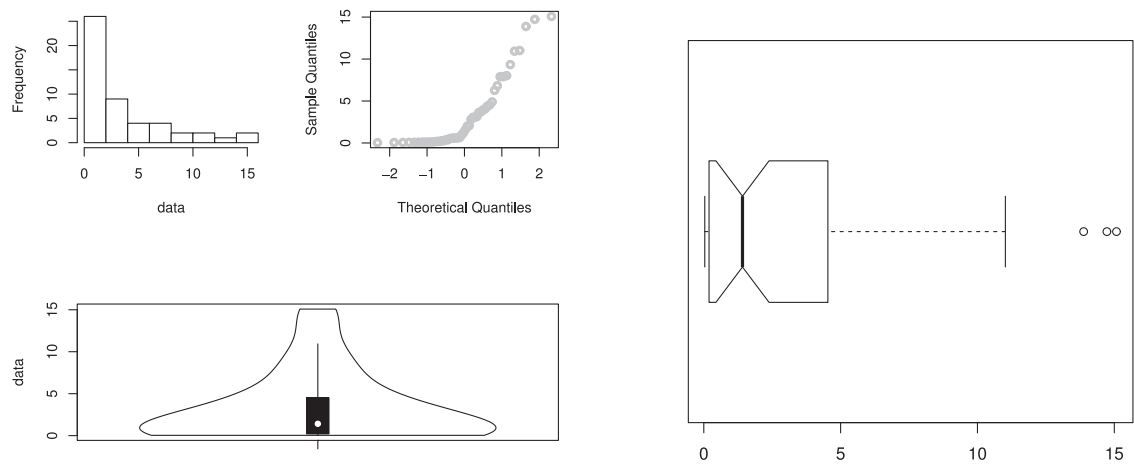


Fig. 4. Summary plots of Data 1.

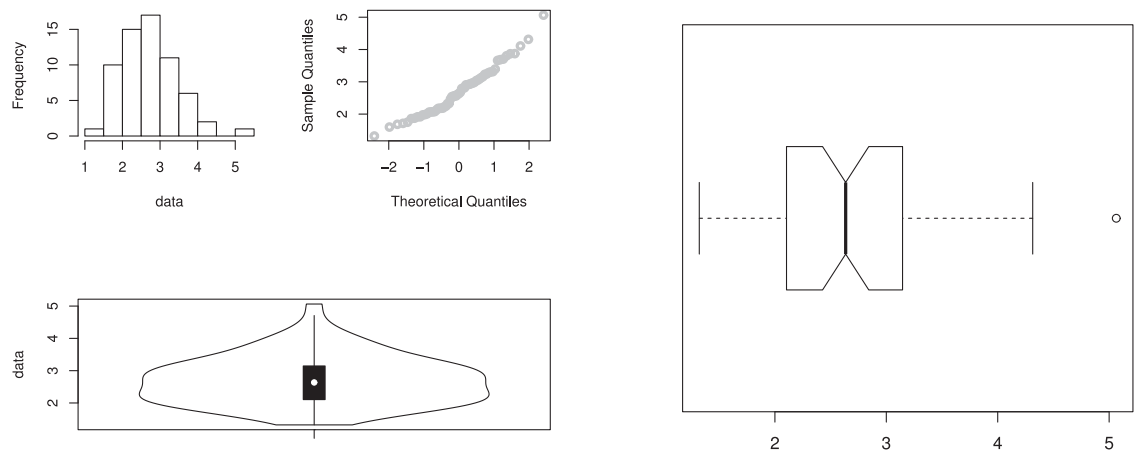


Fig. 5. Summary plots of Data 2.

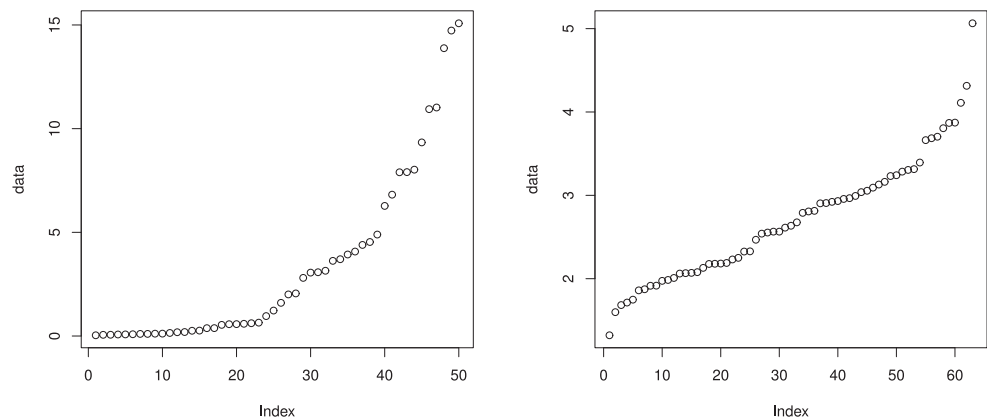


Fig. 6. Illustrations of both data sets.

Table 5
The values of κ_1^* , κ_2^{**} , κ_3^{***} , and κ_4^{****} of the SVF-Weibull, Distribution-1, and Distribution-2 for Data 1.

Dist.	κ_1^*	κ_2^{**}	κ_3^{***}	κ_4^{****}
SVF-Weibull	192.981	193.236	196.805	195.437
Distribution-1	208.728	208.984	212.552	210.184
Distribution-2	195.851	196.106	199.675	197.307

Additionally, when considering Data 2, the values of κ_1^* , κ_2^{**} , κ_3^{***} , and κ_4^{****} for the SVF-Weibull, Distribution-1, and Distribution-2 can be found in Table 7.

Affiliated with Data 2, the SVF-Weibull distribution showcases the following: $\kappa_1^* = 140.911$, $\kappa_2^{**} = 141.111$, $\kappa_3^{***} = 145.197$, and $\kappa_4^{****} = 142.597$. For Distribution-1, we have $\kappa_1^* = 146.993$, $\kappa_2^{**} = 147.193$, $\kappa_3^{***} = 151.280$, and $\kappa_4^{****} = 148.679$. Furthermore, for Distribution-2, we have $\kappa_1^* = 145.625$, $\kappa_2^{**} = 145.825$, $\kappa_3^{***} = 149.911$, and $\kappa_4^{****} = 147.311$. The obtained and displayed facts favors the superior fitting capability of the SVF-Weibull model for Data 2 as well.

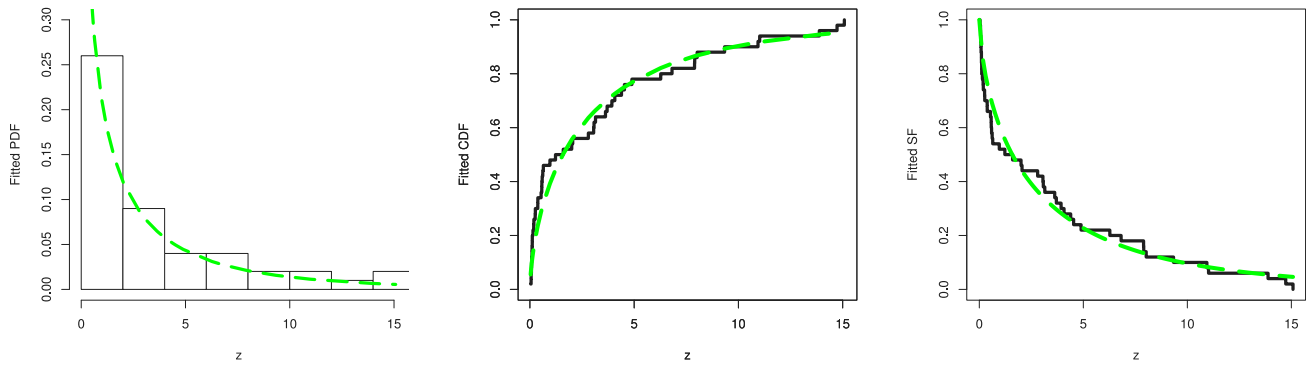


Fig. 7. The display of the visual performances of the SVF-Weibull distribution for Data 1.

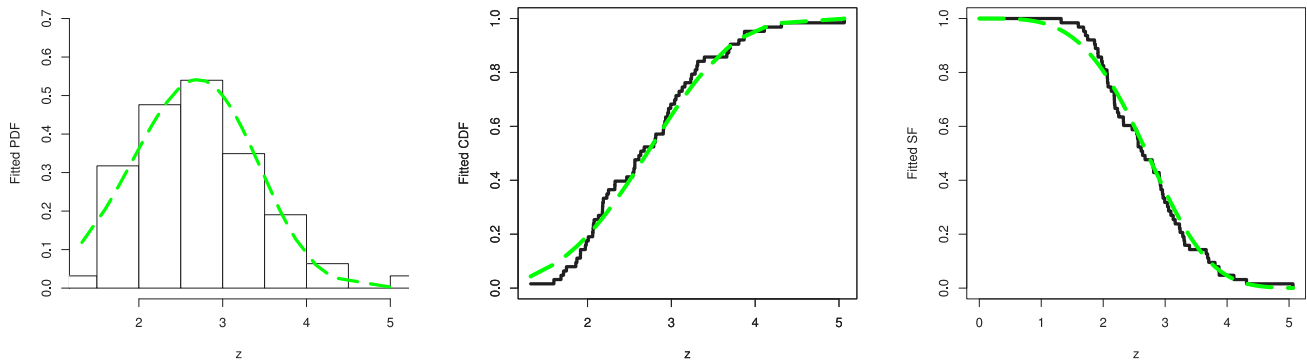


Fig. 8. The display of the visual performances of the SVF-Weibull distribution for Data 2.

Table 6

The point estimates for $\hat{\delta}, \hat{\eta}, \hat{\theta}, \hat{\beta}, \hat{\lambda}$, and $\hat{\gamma}$ of the SVF-Weibull, Distribution-1, and Distribution-2 using Data 2.

Dist.	$\hat{\delta}$	$\hat{\eta}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\gamma}$
SVF-Weibull	3.5831	0.0025	–	–	–	–
Distribution-1	–	–	3.8262	0.0148	–	–
Distribution-2	–	–	–	–	5.5895	0.6188

Table 7

The values of $\kappa_1^*, \kappa_2^{**}, \kappa_3^{***}$, and κ_4^{****} of the SVF-Weibull, Distribution-1, and Distribution-2 using Data 2.

Dist.	κ_1^*	κ_2^{**}	κ_3^{***}	κ_4^{****}
SVF-Weibull	140.911	141.111	145.197	142.597
Distribution-1	146.993	147.193	151.280	148.679
Distribution-2	145.625	145.825	149.911	147.311

Linking with Data 2, in addition to the given results favoring to the SVF-Weibull distribution, we again implement the above graphical approaches to confirms it optimality for Data 2. So, associated to music engineering data, Fig. 8 displays the empirical PDF (left-panel plot), empirical CDF (middle-panel plot), and Kaplan–Meier SF (right-panel plot). These plots substantiate the excellent fitting capacity of the SVF-Weibull distribution to Data 2.

5. Modeling with artificial neural network approach

Adopting the modeling with the artificial neural network (ANN) approach involves leveraging artificial neural networks to comprehend intricate data patterns. Through training the network, this method enables precise predictions and sophisticated modeling of complex relationships, enhancing our understanding of complex datasets.

5.1. Description of the ANN approach

The ANN is a versatile computational algorithm capable of analyzing a wide variety of non-linear scenarios. One notable advantage of ANN algorithms compared to other nonlinear tools is their ability to accurately approximate a wide range of functions. Furthermore, the development of artificial neural networks (ANNs) does not necessitate any prior assumptions, thereby providing an additional advantage. The network, on the other hand, is determined by the attributes of the data, as discussed in Zhang’s work [20]. The single hidden layer feed-forward network is widely recognized as the most prevalent algorithmic approach for predictive modeling, given its inclusion in a comprehensive array of networks. The algorithm is characterized by the connection of three levels: input layers, hidden layers, and output layers, as described by Peng et al. [21] and Ahmad et al. [11].

The mathematical expression representing the relationship between input layers $(\pi_1, \pi_2, \dots, \pi_{t-u})$ and output layer π_t is as follows:

$$\pi_t = v_0 + \sum_{g=1}^h \vartheta_g \mathbb{H} \left(\gamma_g + \sum_{n=1}^s \gamma_{gn} \pi_{t-n} \right) + \varepsilon_t. \quad (12)$$

The unknown parameters, denoted by $\vartheta_g (g = 1, 2, 3 \dots, h)$, $\gamma_{gn} (g = 1, 2, 3 \dots, h, n = 1, 2, \dots, s)$ are typically linked to the connection weights in Eq. (12). Furthermore, h and s , respectively, indicate the quantity of input layers and hidden layers.

5.2. The step indicator saturation approach

The technique of step indicator saturation (SIS) is extensively employed in outlier detection due to its improved performance compared to existing outlier selection methods. It is conventionally accepted to assume that the error conforms to a normal distribution. Nevertheless, it is crucial to acknowledge that this assumption is frequently violated in real-world data analysis situations [22]. From a mathematical

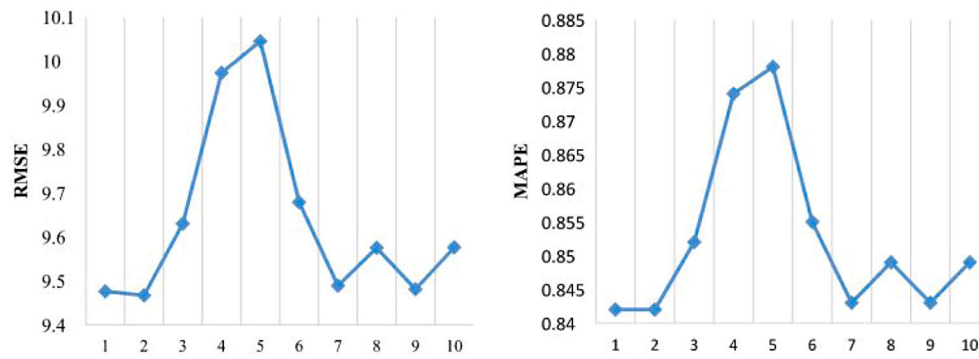


Fig. 9. The impact of the hidden variables on model performance.

perspective, it may be represented as

$$G_t = \omega + \sum_{d=1}^P K_d I + \xi_t, \quad (13)$$

where ω denotes the intercept term, K_d is basically the set of unknown parameters, capture the impact of dummies, and the error term ξ_t , within a model represents the random or stochastic component that captures unobserved factors affecting the dependent variable.

The SIS indicator set can be depicted in the following manner:

$$I_{it} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \vdots & \vdots & \vdots & 1 \end{bmatrix}.$$

In Eq. (13), the intercept term is included to account for the constant term in the linear relationship between the response variable G and the predictor variables. Additionally, the identity matrix I is included to ensure that each relevant observation is given equal weight in the estimation process. This matrix helps to account for any potential heteroscedasticity or unequal variances in the data. However, it is crucial to note that the least squares method, which is commonly used to estimate the coefficients in linear regression models, is not suitable when the number of variables is greater than the number of observations. In the field of econometrics, (13) mentioned before corresponds to a comprehensive unrestricted model. Autometrics is a methodological technique that falls under the category of alternative methodologies. The underlying methodology of this study adopts a general-to-specific approach. This approach involves the simultaneous execution of two tasks: model estimation and outlier detection. As part of the general-to-specific procedure, a dummy variable is assigned to each observation.

5.3. Modeling of the reliability data

This subsection seeks to examine how well the method works using reliability data, by implementing the same tools i.e., ANN and SIS. These tools basically assist us in understanding how reliable the method is in real-world scenarios and improve its performance.

A look at the Fig. 9 shows a noteworthy trend: both the root mean square error (RMSE) and mean absolute percentage error (MAPE) show an increased trajectory as the number of hidden variables increases from 1 to 5. However, after the fifth hidden variable, there is a clear trend reversal that causes both RMSE and MAPE to drop. This observation highlights the significance of establishing an ideal balance in figuring out the right number of hidden variables by pointing to a non-linear relationship between the number of hidden variables and the predictive performance of the model. In conclusion, it seems that using dual hidden variables results in the best model.

Table 8

Comparative analysis of models using the reliability data.

Selection tools	ANN(H = 2)	SIS
RMSE	9.467	10.282
MAPE	0.842	0.900

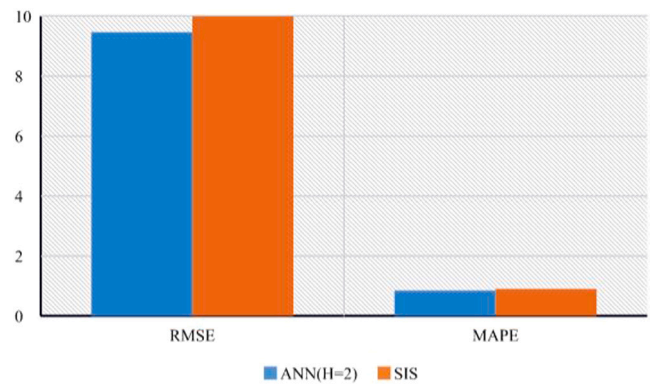


Fig. 10. Models evaluation using the bar chart for the reliability data.

In Table 8, the assessment metrics for the two models, specifically the ANN with two hidden variables ($H = 2$) and the SIS technique, provide useful insights into their respective performances. The ANN model, consisting of only two hidden variables, produces an RMSE value of 9.467. This value quantifies the average magnitude of prediction error. A lower RMSE is generally considered superior, as it suggests that the ANN model effectively reduces the differences between predicted and actual values. The ANN model exhibits a MAPE of 0.842, which represents the average percentage deviation between the predicted values and the actual values. A lower MAPE signifies that the model's forecasts are, on average, highly accurate in relation to the real values. Fig. 10 corroborates the results presented in Table 8, providing additional evidence of the enhanced predictive capability of the ANN model. The collective results confirm the high predictive capability of the ANN model in estimating the reliability data, offering valuable insights for making well-informed judgments and strategic planning.

5.4. Modeling of the music engineering data

This subsection also compares the performance of the SIS technique and ANN. The hidden variables play a crucial role in improving model performance inside the ANN framework. To ascertain the effect on model performance, the study begins by simulating the ANN with a variety of hidden variables. To be more precise, the hidden variables are systematically changed in steps one to ten, with the accompanying errors being recorded at each step. This method makes it possible

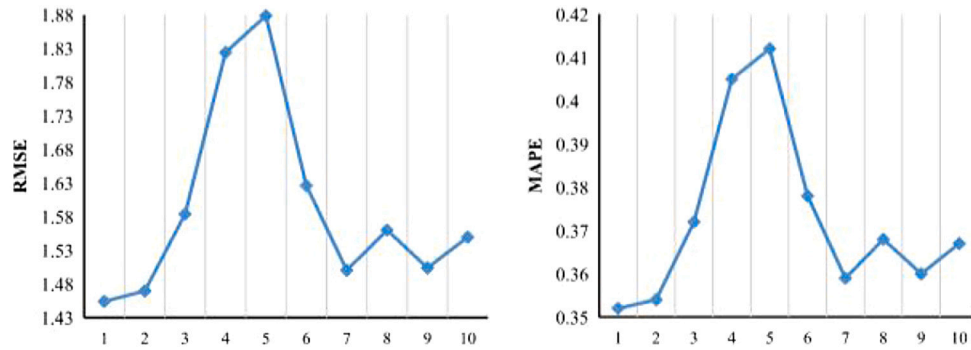


Fig. 11. The impact of the hidden variables on model performance.

Table 9

Comparative analysis of models using music engineering data.

Selection tools	ANN(H = 1)	SIS
RMSE	1.454	1.877
MAPE	0.352	0.916

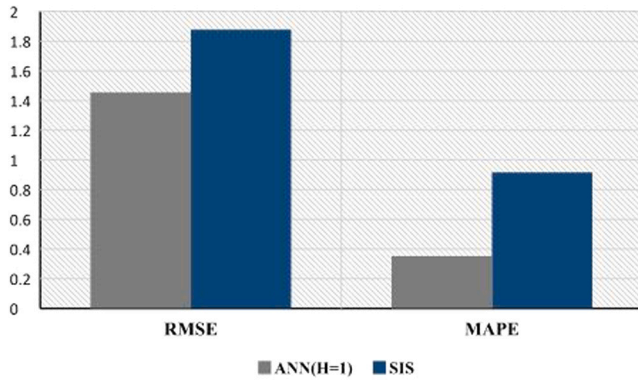


Fig. 12. Models evaluation using the bar chart for the music engineering data.

to investigate in-depth how hidden factors affect the accuracy of the model.

Examining Fig. 11, reveals a notable trend: as the number of hidden variables increases from 1 to 5, both RMSE and MAPE demonstrate an upward trajectory. However, beyond the fifth hidden variable, a reversal in the trend is evident, leading to a subsequent decrease in both RMSE and MAPE. This observation suggests a non-linear relationship between the number of hidden variables and the model's predictive performance, emphasizing the importance of finding an optimal balance in determining the appropriate number of hidden variables. In conclusion, the optimal model appears to be achieved when employing a single hidden variable.

The evaluation measures for the two models, namely the ANN with one hidden variable ($H = 1$) and the SIS technique, offer valuable insights into their individual performances, as shown in Table 9. The ANN model with a single hidden variable yields an RMSE value of 1.454, which represents the average magnitude of prediction errors. A smaller RMSE is typically seen as superior, indicating that the ANN model effectively minimizes the disparities between anticipated and actual values. The ANN model has a MAPE of 0.352, indicating the average percentage difference between the predicted values and the actual values. A reduced MAPE indicates that the model's predictions are, on average, quite close to the actual values. The plots in Fig. 12 support the findings in Table 9, further demonstrating the ANN model's improved predictive ability. The combined findings affirm the ANN model's strong predictability in forecasting music data devices; lifetimes, providing significant perspectives for making informed decisions

and strategic planning. This section also seeks to examine how well the method works using reliability data, by implementing the same tools i.e., ANN and SIS. These tools basically assist us in understanding how reliable the method is in real-world scenarios and improve its performance.

6. Conclusion

This paper studied a new probabilistic model called the SVF-Weibull distribution. For the VF-Weibull distribution, the point estimates and different simulation studies based on these point estimates were also provided. The SVF-Weibull distribution was also illustrated empirically using two data sets which were taken from the reliability and music engineering. Certain IC measures, expressed with κ_1^* , κ_2^{**} , κ_3^{***} , and κ_4^{****} , were used to point out the optimal model between the SVF-Weibull distribution and competing distribution which were expressed by Distribution-1 and Distribution-2. Furthermore, we employ the well-established ANN technique, known for its exceptional efficacy when applied to intricate datasets. An exhaustive evaluation of ANN's performance across a variety of hidden variables yields a comprehensive understanding of its adaptability. This all-encompassing evaluation, in conjunction with a comparative analysis of the SIS method, offers an immense amount of information regarding the relative effectiveness of such a technique. The study's findings clearly demonstrate that the ANN technique outperforms the SIS method in precisely modeling the reliability as well as the musical data.

Due to the promising results (in terms of optimal fitting power) of the SVF-Weibull model in music engineering and reliability sector, it can also be applied to other fields such as healthcare, financial sector, and hydrological field, among others. Moreover, one should also contemplate the possibility of utilizing a bivariate extension of the SVF-Weibull distribution to address bivariate data sets.

CRedit authorship contribution statement

Junqiao Zhu: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Marwa M. Mohie El-Din:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Jin-Taek Seong:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Yusra A. Tashkandy:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **M.E. Bakr:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Anoop Kumar:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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