



Logarithmic imputation techniques for temporal surveys: a memory-based approach explored through simulation and real-life applications

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Abstract

This research introduces memory-based logarithmic imputation techniques and the resulting estimators to address missing data within the temporal surveys. The mean square error of the resulting memory type estimators is reported to the first order approximation and the efficiency conditions are obtained by comparing the properties of the proposed and adapted imputation methods. The study contains a comprehensive simulation study to evaluate the performance of the resulting estimators under various conditions, providing insights into their applicability. Furthermore, the proposed methods are also illustrated through some real-life applications. The findings of simulation and real data application demonstrate the effectiveness of the memory type logarithmic imputation methods, providing insights into its application across different survey contexts and highlighting its potential to enhance data completeness and reliability in temporal survey analysis.

Keywords Exponentially weighted moving average · Memory type ratio and product imputation methods · Mean square error · Relative efficiency

Mathematics Subject Classification 62D10 · 62D05

1 Introduction

In sample surveys, missing data is a ubiquitous challenge experienced in several research areas, spanning from biomedical sciences to social sciences and beyond. It refers to the missing values in a dataset that are expected to be recorded or collected. The prevalence of missing data may compromise the validity and integrity of analyses, potentially leading to biased results and erroneous inferences. Therefore, understanding the nature of missing

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data, its causes, and its implications is important for the survey practitioners aiming to obtain accurate and reliable findings.

Missing data can manifest in several forms, including missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR). MCAR indicates that the probability of a data point being missing is unrelated to both observed and unobserved data. MAR implies that the probability of missingness may depend on observed data but not on unobserved data. MNAR indicates that the probability of missingness depends on unobserved data, leading to the potential biases in analyses. Researchers consider a range of strategies to address missing data including imputation methods. Imputation methods involve estimating missing values based on observed data with methods, namely, mean imputation, ratio imputation, power ratio imputation, regression imputation, exponential imputation, and compromised imputation commonly used. For more detailed study about various imputation methods, reader may refer to Rubin (1976), Heitjan and Basu (1996), Al-Omari and Bouza (2015), Bhushan and Pandey (2018), Bhattacharyya et al. (2021), Bhushan et al. (2023a, 2023b), Prasad and Yadav (2023), Shahzad et al. (2022), Bhushan and Kumar (2023, 2024), Pandey et al. (2021, 2024), Alomair and Shahzad (2023), and Yadav and Prasad (2024).

Consider a finite population $P = \{P_1, P_2, \dots, P_N\}$ of size N from which a random sample $s = \{p_1, p_2, \dots, p_n\}$ of size n is selected utilizing simple random sampling without replacement scheme to estimate the population mean $\bar{Y}$ of study variable y . Let y_i and x_i denote the study variable y and auxiliary variable x for the i^{th} unit of a finite population of size N . Let r represent the number of responding units among the sampled n units, and R and $\bar{R}$ represent the set of responding and non-responding units, respectively. The unit y_i is observed if $i \in R$, while unobserved/missing if $i \in \bar{R}$ and imputed values are computed. We suppose that the imputation is performed with the addition of an auxiliary variable x , such that the i^{th} unit of the variable x , i.e., x_i is known and positive for every $i \in s = R \cup \bar{R}$. In other words, the data on $x_s = \{x_i : i \in s\}$ are known. Let $\bar{X} = \sum_{i=1}^N x_i / N$ and $\bar{Y} = \sum_{i=1}^N y_i / N$ be the population means of the auxiliary variable x and the study variable y , respectively. Similarly, let $S_x^2 = \sum_{i=1}^N (x_i - \bar{X})^2 / (N - 1)$ and $S_y^2 = \sum_{i=1}^N (y_i - \bar{Y})^2 / (N - 1)$ be the population variances of the auxiliary variable x and the study variable y , respectively. Let $\bar{x}_n = \sum_{i=1}^n x_i / n$ be the sample mean of the auxiliary variable x based on sample size n . Also, let $\bar{x}_r = \sum_{i=1}^r x_i / r$ and $\bar{y}_r = \sum_{i=1}^r y_i / r$ be the response means of the auxiliary variable x and the study variable y , respectively. Further, let $C_x = S_x / \bar{X}$ and $C_y = S_y / \bar{Y}$ be the population coefficients of variation of the auxiliary variable x and the study variable y , respectively. Moreover, let ρ_{xy} be the population correlation coefficient between the variables x and y .

The auxiliary information plays an important role throughout various domains like experimental design, statistical modelling, and survey sampling. In essence, auxiliary data serves as a valuable tool to enhance the accuracy, efficiency, and interpretation of statistical estimates. This additional information may originate from diverse sources like expert opinions, population-based surveys, census data, and assumptions regarding demographic parameters. In the presence of auxiliary data, we categorize the imputation methods in the following schemes.

Scheme I If $\bar{X}$ is available and $\bar{x}_n$ is used.

Scheme II If $\bar{X}$ is available and $\bar{x}_r$ is used.

Scheme III If $\bar{X}$ is not available and $(\bar{x}_n, \bar{x}_r)$ are used.

When there is a positive correlation between the supplementary variable x and the main variable y , the ratio imputations provide accurate results. Utilizing the notations given in

Lee et al. (1994), the classical ratio imputation methods (CRIMs) and the resulting classical ratio estimators (CREs) under the above schemes are given in Table 1.

The logarithmic imputation methods (LIMs) and the resulting logarithmic estimators (LEs) under the schemes I–III are given in Table 2.

Searls (1964) demonstrated that multiplying a tuning constant in the estimator enhance its efficiency. With this approach, Bhushan et al. (2023a) suggested Searls type logarithmic imputation methods (SLIMs) and the resulting Searls type logarithmic estimators (SLEs) under the schemes I–III are given in Table 3.

The mean square error (MSE) of the resulting CREs, LEs, and SLEs is given in Appendix A for ready reference.

Table 1 CRIMs and CREs under schemes I–III

Schemes	CRIMs	Resulting CREs
I	$y_{j_{r_1}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[n\bar{y}_r \frac{\bar{x}}{\bar{x}_n} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{r_1} = \bar{y}_r \frac{\bar{x}}{\bar{x}_n}$
II	$y_{j_{r_2}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[n\bar{y}_r \frac{\bar{x}}{\bar{x}_r} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{r_2} = \bar{y}_r \frac{\bar{x}}{\bar{x}_r}$
III	$y_{j_{r_3}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[n\bar{y}_r \frac{\bar{x}_n}{\bar{x}_r} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{r_3} = \bar{y}_r \frac{\bar{x}_n}{\bar{x}_r}$

Table 2 LIMs and LEs under schemes I–III

Schemes	LIMs	Resulting LEs
I	$y_{j_{l_1}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[n\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_n}{\bar{x}} \right) \right\}^{\varphi_1} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{l_1} = \bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_n}{\bar{x}} \right) \right\}^{\varphi_1}$
II	$y_{j_{l_2}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[n\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_r}{\bar{x}} \right) \right\}^{\varphi_2} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{l_2} = \bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_r}{\bar{x}} \right) \right\}^{\varphi_2}$
III	$y_{j_{l_3}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[n\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_n}{\bar{x}_n} \right) \right\}^{\varphi_3} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{l_3} = \bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_n}{\bar{x}_n} \right) \right\}^{\varphi_3}$

φ_1 , φ_2 , and φ_3 are suitably chosen scalars which are given in Appendix A

Table 3 SLIMs and SLEs under schemes I–III

Schemes	SLIMs	Resulting SLEs
I	$y_{j_{sl_1}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[nA_1\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_n}{\bar{x}} \right) \right\}^{\varphi_1} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{sl_1} = A_1\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_n}{\bar{x}} \right) \right\}^{\varphi_1}$
II	$y_{j_{sl_2}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[nA_2\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_r}{\bar{x}} \right) \right\}^{\varphi_2} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{sl_2} = A_2\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_r}{\bar{x}} \right) \right\}^{\varphi_2}$
III	$y_{j_{sl_3}} = \begin{cases} y_j & \text{for } j \in R \\ \frac{1}{n-r} \left[nA_3\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_n}{\bar{x}_n} \right) \right\}^{\varphi_3} - r\bar{y}_r \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{sl_3} = A_3\bar{y}_r \left\{ 1 + \log \left(\frac{\bar{x}_n}{\bar{x}_n} \right) \right\}^{\varphi_3}$

A_1 , A_2 , and A_3 are suitably chosen scalars which are given in Appendix A

It is well-established that the efficient utilization of auxiliary information reduces the variance or mean square error of the estimators. It is common practice to obtain the study and auxiliary variables by utilizing just the current sample y_t at time t . However, the performance of the estimator may also be enhanced by utilizing data from past and more current samples, such as $\bar{y}_{t-1}$ from time $t - 1$, $\bar{y}_{t-2}$ from time $t - 2$, and so on. It is typically necessary when conducting surveys over a specific time period such as a quarterly, monthly, or yearly period. Very limited studies are available in the literature for the estimation of parameters utilizing EWMA statistics. Memory type ratio and product estimators for the population mean by employing EWMA statistics have been proposed under simple random sampling (SRS) by Noor-ul-Amin (2021), under stratified random sampling by Aslam et al. (2020), under ranked set sampling by Aslam et al. (2023). However, utilizing the EWMA statistic, Bhushan et al. (2023c) proposed memory type logarithmic estimators of population mean. Sharma et al. (2024) introduced an estimation procedures for population mean using EWMA for time scaled survey. Furthermore, Qureshi et al. (2024) used EWMA statistics and suggested memory type ratio and product estimators for population variance. Kumar et al. (2024) suggested general class of memory type estimators for population variance using EWMA statistics.

Temporal surveys play an important role in capturing dynamic trends and changes over time, providing valuable insights into evolving phenomena. Several studies have underscored the prevalence of missing data in temporal surveys, attributing it to various factors such as participant attrition, intermittent responses, and changes in survey structures over time. Traditional imputation methods, including mean imputation, regression imputation, and multiple imputation, have been widely employed to address missing data in surveys. While these methods offer practical solutions, their limitations in capturing temporal dependencies and nonlinear patterns in the data have spurred the exploration of more advanced techniques. The incorporation of memory mechanisms in imputation models has shown promise in addressing missingness in time-series data. Memory-based imputation methods consider the temporal sequence of observations, acknowledging that missing values may be influenced by past data points. However, the memory-based imputation methods in the context of temporal surveys still an underexplored area.

Despite the progress in imputation methods for classical surveys, there exists a notable research gap in the exploration of imputation techniques for the temporal surveys. This study is motivated by the need to bridge this gap, introducing the memory-based logarithmic imputation methods and the resulting estimators, which leverage the temporal patterns and relationships within the data. Unlike traditional imputation methods, these methods draw upon memory mechanisms to intelligently impute missing values, taking into account the past survey data.

Below is the design of the remaining content: Sect. 2 outlines the EWMA statistics with the notations. Section 3 provides the proposed memory type imputation techniques and resulting estimators, including their characteristics. In Sect. 4, these characteristics are compared with those of the traditional imputation techniques. Section 5 presents an extensive simulation study. Real data application of the suggested memory type imputation techniques is provided in Sect. 6. The discussion of results of the simulation and real data applications is given in Sect. 7. Lastly, Sect. 8 provides the conclusion.

2 EWMA statistics and some adapted memory type imputation methods

The EWMA is a statistical method utilized for characterizing or modelling time series data. In finance sector, it finds thorough application, especially in volatility modelling and technical analysis. The essence of EWMA lies in its moving average construction, where older data points are assigned less weights. The term “exponentially weighted” signifies that these weights reduce exponentially with each additional data point. The initial decision facing EWMA users is parameter selection as it determines the importance of the current observation in the computation. A higher parametric value results in greater precision in tracking the original time series utilizing EWMA.

Robert (1959) originally developed the idea of the EWMA statistic as a means to efficiently distinguish shifts in the mean of elementary processes within statistical quality control charts. By mixing past data with current data, the EWMA statistic enhances the accuracy of population parameter estimates. The EWMA statistics of variables (y, x) for $t > 1$ are given below:

$$Z_t = \kappa \bar{y} + (1 - \kappa)Z_{t-1}$$

$$\text{and } Q_t = \kappa \bar{x} + (1 - \kappa)Q_{t-1}.$$

The smoothing constant κ signifies the weight allocated to the data. With a higher κ , greater weight is assigned to the current observation, reducing the influence of past observations. Conversely, smaller values of κ amplify the importance of past observations, while reducing the impact of the current one. Typically, $0 \leq \kappa \leq 1$, denotes the degree of weight applied. When κ equals 1, all current data receive full weight, rendering the EWMA statistic equivalent to the standard sample mean. The index ‘ t ’ represents the time and Z_{t-1} denotes the previous information. Additionally, the initial values of Q_t and Z_t are considered based on the expected mean deduced from pilot surveys. It is observed that the statistics (Z_t, Q_t) are unbiased estimators for the population means ($\bar{Y}, \bar{X}$).

The corresponding imputed EWMA-based sample mean estimators for the missing data in the variables (y, x) for $t > 1$ are as follows:

$$Z_{tr} = \kappa \bar{y}_r + (1 - \kappa)Z_{tr-1}$$

$$\text{and } Q_{tr} = \kappa \bar{x}_r + (1 - \kappa)Q_{tr-1}.$$

Furthermore, the non-imputed sample means of variables (y, x) consisting of EWMA are:

$$Z_m = \kappa \bar{y}_n + (1 - \kappa)Z_{m-1}$$

$$\text{and } Q_m = \kappa \bar{x}_n + (1 - \kappa)Q_{m-1}.$$

Regarding EWMA statistics, the imputation methods are categorized into the following schemes.

Scheme I If $\bar{X}$ is available and Q_m is used.

Scheme II If $\bar{X}$ is available and Q_{tr} is used.

Scheme III If $\bar{X}$ is not available and (Q_m , and Q_{tr}) are used.

Under the above schemes, we prescribe some memory type ratio imputation methods (MTRIMs) and the corresponding resulting memory type ratio estimators (MTREs) in Table 4.

The superscript m used in the imputation methods and the resulting estimators refers the “memory type imputation methods and the resultant memory type estimators”.

Table 4 MTRIMs and MTREs under schemes I–III

Schemes	MTRIMs	Resulting MTRE
I	$y_{j_{r_1}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left(nZ_{ir} \frac{\bar{X}}{Q_m} - rZ_{ir} \right) & \text{for } j \in \bar{R} \end{cases}$	$t_{r_1}^m = Z_{ir} \frac{\bar{X}}{Q_m}$
II	$y_{j_{r_2}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left(nZ_{ir} \frac{\bar{X}}{Q_r} - rZ_{ir} \right) & \text{for } j \in \bar{R} \end{cases}$	$t_{r_2}^m = Z_{ir} \frac{\bar{X}}{Q_r}$
III	$y_{j_{r_3}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left(nZ_{ir} \frac{Q_m}{Q_r} - rZ_{ir} \right) & \text{for } j \in \bar{R} \end{cases}$	$t_{r_3}^m = Z_{ir} \frac{Q_m}{Q_r}$

Theorem 2.1 The MSEs of the resultant MTREs are given to the first order of approximation as

$$MSE(t_{r_1}^m) = \bar{Y}^2 \Phi (\lambda_r C_y^2 + \lambda_n C_x^2 - 2\lambda_n \rho_{xy} C_x C_y) \quad (2.1)$$

$$MSE(t_{r_2}^m) = \bar{Y}^2 \Phi \lambda_r (C_y^2 + C_x^2 - 2\rho_{xy} C_x C_y) \quad (2.2)$$

$$MSE(t_{r_3}^m) = \bar{Y}^2 \Phi (\lambda_r C_y^2 + \lambda_{rn} C_x^2 - 2\lambda_{rn} \rho_{xy} C_x C_y) \quad (2.3)$$

where $\Phi = \frac{\kappa}{(2-\kappa)}$, $\lambda_r = \left(\frac{1}{r} - \frac{1}{N} \right)$, $\lambda_n = \left(\frac{1}{n} - \frac{1}{N} \right)$, and $\lambda_{rn} = \left(\frac{1}{r} - \frac{1}{n} \right)$.

Proof To find out the mean square error of the resultant estimators, let us define some error terms as $e_0 = (Z_{ir} - \bar{Y})/\bar{Y}$, $e_1 = (Q_{ir} - \bar{X})/\bar{X}$, and $e_2 = (Q_m - \bar{X})/\bar{X}$ such that $E(e_0) = E(e_1) = E(e_2) = 0$, $E(e_0^2) = \lambda_r \Phi C_y^2$, $E(e_1^2) = \lambda_r \Phi C_x^2$, $E(e_2^2) = \lambda_n \Phi C_x^2$, $E(e_0 e_1) = \lambda_r \Phi \rho_{xy} C_x C_y$, $E(e_0 e_2) = \lambda_n \Phi \rho_{xy} C_x C_y$, $E(e_1 e_2) = \lambda_n \Phi C_x^2$.

Consider the MTRE under Scheme I as

$$t_{r_1}^m = Z_{ir} \frac{\bar{X}}{Q_m} \quad (2.4)$$

Employing the notation defined in Section 2, we write $t_{r_1}^m$ as

$$\begin{aligned} t_{r_1}^m &= \bar{Y}(1 + e_0) \frac{\bar{X}}{\bar{X}(1 + e_2)} \\ &= \bar{Y}(1 + e_0)(1 + e_2)^{-1} \end{aligned}$$

Utilizing Taylor's series expansion, we get

$$= \bar{Y}(1 + e_0)(1 - e_2 + e_2^2 - \dots)$$

Multiplying and neglecting the error terms of right-side having power more than 2, we get

$$t_{r_1}^m = \bar{Y}(1 + e_0 - e_2 - e_0 e_2 + e_2^2) \quad (2.5)$$

Subtract $\bar{Y}$ on both sides of (2.5), we get

$$t_{r_1}^m - \bar{Y} = \bar{Y}(e_0 - e_2 - e_0 e_2 + e_2^2) \quad (2.6)$$

Square and employ “Expectation” on both sides of (2.6), we get

$$MSE(t_{r_1}^m) = \bar{Y}^2 \Phi(\lambda_r C_y^2 + \lambda_n C_x^2 - 2\lambda_n \rho_{xy} C_x C_y)$$

As above, the MSE of the MTREs $t_{r_2}^m$ and $t_{r_3}^m$ can be obtained. $\square$

We prescribe some memory type logarithmic imputation methods (MTLIMs) and the resultant memory type logarithmic estimators (MTLEs) under the above schemes in Table 5:

Theorem 2.2 *The minimum MSEs of the resultant MTLEs $t_{l_i}^m$, $i = 1, 2, 3$ under schemes I–III are given to the first order of approximation as*

$$\begin{aligned} \min.MSE(t_{l_1}^m) &= \bar{Y}^2 \Phi C_y^2 (\lambda_r - \lambda_n \rho_{xy}^2), \\ \min.MSE(t_{l_2}^m) &= \bar{Y}^2 \Phi \lambda_r C_y^2 (1 - \rho_{xy}^2), \\ \text{and} \quad \min.MSE(t_{l_3}^m) &= \bar{Y}^2 \Phi \lambda_r C_y^2 + \bar{Y}^2 \Phi \lambda_{rn} C_y^2 (1 - \rho_{xy}^2). \end{aligned}$$

Proof Consider the MTLE $t_{l_1}^m$ under scheme I as

$$t_{l_1}^m = Z_{tr} \left\{ 1 + \log \left(\frac{Q_m}{\bar{X}} \right) \right\}^{\Pi_1}. \quad (2.7)$$

Employing the earlier notation, the MTLE $t_{l_1}^m$ may be written as

Table 5 MTLIMs and MTLEs under schemes I–III

Schemes	MTLIMs	Resulting MTLEs
I	$y_{j_{l_1}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left[nZ_{tr} \left\{ 1 + \log \left(\frac{Q_m}{\bar{X}} \right) \right\}^{\Pi_1} - rZ_{tr} \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{l_1}^m = Z_{tr} \left\{ 1 + \log \left(\frac{Q_m}{\bar{X}} \right) \right\}^{\Pi_1}$
II	$y_{j_{l_2}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left[nZ_{tr} \left\{ 1 + \log \left(\frac{Q_x}{\bar{X}} \right) \right\}^{\Pi_2} - rZ_{tr} \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{l_2}^m = Z_{tr} \left\{ 1 + \log \left(\frac{Q_x}{\bar{X}} \right) \right\}^{\Pi_2}$
III	$y_{j_{l_3}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left[nZ_{tr} \left\{ 1 + \log \left(\frac{Q_{tr}}{Q_m} \right) \right\}^{\Pi_3} - rZ_{tr} \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{l_3}^m = Z_{tr} \left\{ 1 + \log \left(\frac{Q_{tr}}{Q_m} \right) \right\}^{\Pi_3}$

Π_1 , Π_2 , and Π_3 are suitably chosen scalars which are determined later in the proof of Theorem 2.2

$$\begin{aligned}
t_{l_1}^m &= \bar{Y}(1 + e_0) \left\{ 1 + \log \left(\frac{\bar{X}(1 + e_2)}{\bar{X}} \right) \right\}^{\Pi_1}, \\
&= \bar{Y}(1 + e_0) \{ 1 + \log(1 + e_2) \}^{\Pi_1}, \\
&= \bar{Y}(1 + e_0) \left\{ 1 + \left(e_2 - \frac{e_2^2}{2} + \dots \right) \right\}^{\Pi_1}.
\end{aligned}$$

Multiplying and neglecting the error terms of right-side having power more than 2, we get

$$t_{l_1}^m = \bar{Y} \left(1 + e_0 + \Pi_1 e_2 - \Pi_1 e_2^2 + \frac{\Pi_1^2}{2} e_2^2 + \Pi_1 e_0 e_2 \right). \quad (2.8)$$

Subtract $\bar{Y}$ on both sides of (2.8), we get

$$t_{l_1}^m - \bar{Y} = \bar{Y} \left(e_0 + \Pi_1 e_2 - \Pi_1 e_2^2 + \frac{\Pi_1^2}{2} e_2^2 + \Pi_1 e_0 e_2 \right). \quad (2.9)$$

Square and employ “Expectation” on both sides of (2.9), we get

$$MSE(t_{l_1}^m) = \bar{Y}^2 \Phi(\lambda_r C_y^2 + \Pi_1^2 \lambda_n C_x^2 - 2\Pi_1 \lambda_n \rho_{xy} C_x C_y). \quad (2.10)$$

Minimizing (2.10) with respect to (w.r.t.) Π_1 , we get

$$\Pi_{1(opt)} = -\rho_{xy} \frac{C_y}{C_x}.$$

Replacing Π_1 with $\Pi_{1(opt)}$ in (2.10), we get

$$\min.MSE(t_{l_1}^m) = \bar{Y}^2 \Phi C_y^2 (\lambda_r - \lambda_n \rho_{xy}^2). \quad (2.11)$$

As above, the minimum MSE of the MTLEs $t_{l_2}^m$ and $t_{l_3}^m$ may be obtained. $\square$

3 Proposed memory type imputation methods

Memory type imputation methods consist of techniques where missing values are replaced with values from neighbouring time points or observations. These methods usually use past or current data to estimate the missing values, thereby leveraging temporal or spatial relationships within the dataset. We propose the memory type Searls logarithmic imputation methods (MTSLIMs) and the resultant memory type Searls logarithmic estimators (MTSLEs) under the schemes I–III in Table 6.

Λ_i , $i = 1, 2, 3$ are suitably chosen scalars to optimize the MSE.

Remark 3.1 The computational complexity of the solution for memory-type Searls logarithmic imputation methods depends on the specific algorithm and implementation used. Typically, such methods involve iterative optimization or approximation techniques, depending on the sparsity of the data and the dimensionality. As for an algorithm, while there is not a standardized algorithm specifically termed for memory-type Searls logarithmic imputation,

Table 6 MTSLIMs and MTSLEs under schemes I–III

Schemes	MTSLIMs	Resulting MTSLEs
I	$y_{j_{sl_1}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left[nA_1 Z_{ir} \left\{ 1 + \log \left(\frac{Q_m}{\bar{X}} \right) \right\}^{\Pi_1} - rZ_{ir} \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{sl_1}^m = A_1 Z_{ir} \left\{ 1 + \log \left(\frac{Q_m}{\bar{X}} \right) \right\}^{\Pi_1}$
II	$y_{j_{sl_2}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left[nA_2 Z_{ir} \left\{ 1 + \log \left(\frac{Q_m}{\bar{X}} \right) \right\}^{\Pi_2} - rZ_{ir} \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{sl_2}^m = A_2 Z_{ir} \left\{ 1 + \log \left(\frac{Q_m}{\bar{X}} \right) \right\}^{\Pi_2}$
III	$y_{j_{sl_3}}^m = \begin{cases} Z_{ij} & \text{for } j \in R \\ \frac{1}{n-r} \left[nA_3 Z_{ir} \left\{ 1 + \log \left(\frac{Q_m}{\bar{Q}_m} \right) \right\}^{\Pi_3} - rZ_{ir} \right] & \text{for } j \in \bar{R} \end{cases}$	$t_{sl_3}^m = A_3 Z_{ir} \left\{ 1 + \log \left(\frac{Q_m}{\bar{Q}_m} \right) \right\}^{\Pi_3}$

these methods often utilize logarithmic transformations combined with advanced optimization approaches. Employing existing imputation frameworks to logarithmic transformations is a feasible strategy to achieve such implementations.

Theorem 3.1 *The minimum MSE of the resultant MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ under schemes I–III is given as*

$$\min.MSE(t_{sl_i}^m) = \bar{Y}^2 \left(1 - \frac{D_i^2}{C_i} \right)$$

where $C_1 = 1 + \Phi \lambda_r C_y^2 + 2\Pi_1(\Pi_1 - 1)\Phi \lambda_n C_x^2 + 4\Pi_1 \Phi \lambda_n \rho_{xy} C_x C_y$, $D_1 = 1 + \left(\frac{\Pi_1^2}{2} - \Pi_1 \right) \Phi \lambda_n C_x^2 + \Pi_1 \Phi \lambda_n \rho_{xy} C_x C_y$, $C_2 = 1 + \Phi \lambda_r C_y^2 + 2\Pi_2(\Pi_2 - 1)\Phi \lambda_r C_x^2 + 4\Pi_2 \Phi \lambda_r \rho_{xy} C_x C_y$,

$D_2 = 1 + \left(\frac{\Pi_2^2}{2} - \Pi_2 \right) \Phi \lambda_r C_x^2 + \Pi_2 \Phi \lambda_r \rho_{xy} C_x C_y$, $C_3 = 1 + \Phi \lambda_r C_y^2 + 2\Pi_3(\Pi_3 - 1)\Phi \lambda_{rn} C_x^2 + 4\Pi_3 \Phi \lambda_{rn} \rho_{xy} C_x C_y$, and $D_3 = 1 + \left(\frac{\Pi_3^2}{2} - \Pi_3 \right) \Phi \lambda_{rn} C_x^2 + \Pi_3 \Phi \lambda_{rn} \rho_{xy} C_x C_y$.

Proof Consider the MTSLE $t_{sl_1}^m$ under scheme I as

$$t_{sl_1}^m = A_1 Z_{ir} \left\{ 1 + \log \left(\frac{Q_m}{\bar{X}} \right) \right\}^{\Pi_1}. \quad (3.1)$$

Employing the earlier notation, the MTSLE $t_{sl_1}^m$ may be written as

$$\begin{aligned}
t_{sl_1}^m &= A_1 \bar{Y}(1 + e_0) \left\{ 1 + \log \left(\frac{\bar{X}(1 + e_2)}{\bar{X}} \right) \right\}^{\Pi_1}, \\
&= A_1 \bar{Y}(1 + e_0) \{ 1 + \log(1 + e_2) \}^{\Pi_1}, \\
&= A_1 \bar{Y}(1 + e_0) \left\{ 1 + \left(e_2 - \frac{e_2^2}{2} + \dots \right) \right\}^{\Pi_1}.
\end{aligned}$$

Multiplying and neglecting the error terms of right-side having power more than 2, we get

$$t_{sl_1}^m = A_1 \bar{Y} \left(1 + e_0 + \Pi_1 e_2 - \Pi_1 e_2^2 + \frac{\Pi_1^2}{2} e_2^2 + \Pi_1 e_0 e_2 \right). \quad (3.2)$$

Subtract $\bar{Y}$ on both sides of (3.2), we get

$$t_{sl_1}^m - \bar{Y} = \bar{Y} \left\{ A_1 \left(1 + e_0 + \Pi_1 e_2 - \Pi_1 e_2^2 + \frac{\Pi_1^2}{2} e_2^2 + \Pi_1 e_0 e_2 \right) - 1 \right\}. \quad (3.3)$$

Square and employ “Expectation” on both sides of (3.3), we get

$$\begin{aligned}
MSE(t_{sl_1}^m) &= \bar{Y}^2 \left[1 + A_1^2 \left\{ 1 + \Phi \lambda_r C_y^2 + 2\Pi_1(\Pi_1 - 1)\Phi \lambda_n C_x^2 + 4\Pi_1 \Phi \lambda_n \rho_{xy} C_x C_y \right\} \right. \\
&\quad \left. - 2A_1 \left\{ 1 + \left(\frac{\Pi_1^2}{2} - \Pi_1 \right) \Phi \lambda_n C_x^2 + \Pi_1 \Phi \lambda_n \rho_{xy} C_x C_y \right\} \right], \\
&= \bar{Y}^2 (1 + A_1^2 C_1 - 2A_1 D_1).
\end{aligned} \quad (3.4)$$

where $C_1 = 1 + \Phi \lambda_r C_y^2 + 2\Pi_1(\Pi_1 - 1)\Phi \lambda_n C_x^2 + 4\Pi_1 \Phi \lambda_n \rho_{xy} C_x C_y$ and

$$D_1 = 1 + \left(\frac{\Pi_1^2}{2} - \Pi_1 \right) \Phi \lambda_n C_x^2 + \Pi_1 \Phi \lambda_n \rho_{xy} C_x C_y.$$

Minimizing (3.4) w.r.t. A_1 , we get

$$A_{1(opt)} = \frac{D_1}{C_1}.$$

Replacing A_1 with $A_{1(opt)}$ in (3.4), we get

$$\min.MSE(t_{sl_1}^m) = \bar{Y}^2 \left(1 - \frac{D_1^2}{C_1} \right). \quad (3.5)$$

As above, the minimum MSE of the MTSLEs $t_{sl_2}^m$ and $t_{sl_3}^m$ may be obtained. In general, the minimum MSE of MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ is given below for ready reference.

$$\min.MSE(t_{sl_i}^m) = \bar{Y}^2 \left(1 - \frac{D_i^2}{C_i} \right). \quad (3.6)$$

□

Remark 3.2 It is to be noted that the simultaneous optimization of A_i , $i = 1, 2, 3$ and Π_i is not possible. Therefore, the optimum values of Π_i may be obtained by putting

$\Lambda_i = 1$, $i = 1, 2, 3$ in the respective MTSLEs and minimizing their MSEs wrt Π_i . The optimum value of Π_i is given below for ready reference.

$$\Pi_{i(opt)} = -\rho_{xy} \frac{C_y}{C_x}.$$

4 Efficiency criteria

In this part, the MTSLEs are compared with the corresponding CRIMs, LIMs, SLIMs, MTRIMs, and MTLIMs under schemes I–III.

• Comparison of MTSLEs with CRIMs

(i). The MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ outperform the CREs t_{r_i} under schemes I–III, if

$$MSE(t_{r_i}) > MSE(t_{sl_i}^m) \implies \frac{D_i^2}{C_i} > 1 - \lambda_r C_y^2 - \lambda_n C_x^2 + 2\lambda_n \rho_{xy} C_x C_y.$$

• Comparison of MTSLEs with LIMs

(i). The MTSLE $t_{sl_1}^m$ outperforms the LE t_{l_1} under scheme I, if

$$MSE(t_{l_1}) > MSE(t_{sl_1}^m) \implies \frac{D_1^2}{C_1} > 1 - C_y^2 (\lambda_r - \lambda_n \rho_{xy}^2).$$

(ii). The MTSLE $t_{sl_2}^m$ outperforms the LE t_{l_2} under scheme II, if

$$MSE(t_{l_2}) > MSE(t_{sl_2}^m) \implies \frac{D_2^2}{C_2} > 1 - C_y^2 \lambda_r (1 - \rho_{xy}^2).$$

(iii). The MTSLE $t_{sl_3}^m$ outperforms the LE t_{l_3} under scheme III, if

$$MSE(t_{l_3}) > MSE(t_{sl_3}^m) \implies \frac{D_3^2}{C_3} > 1 - \lambda_r C_y^2 + \lambda_m C_y^2 (1 - \rho_{xy}^2).$$

• Comparison of MTSLEs with SLIMs

(i). The MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ outperform the SLEs t_{sl_i} under schemes I–III, if

$$MSE(t_{sl_i}) > MSE(t_{sl_i}^m) \implies \frac{D_i^2}{C_i} > \frac{B_i^2}{A_i}$$

• Comparison of MTSLEs with MTRIMs

(i). The MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ outperform the MTREs $t_{r_i}^m$ under schemes I–III, if

$$MSE(t_{r_i}^m) > MSE(t_{sl_i}^m) \implies \frac{D_i^2}{C_i} > 1 - \Phi C_y^2 (\lambda_r - \lambda_n \rho_{xy}^2).$$

• Comparison of MTSLEs with MTLIMs

(i). The MTSLE $t_{sl_1}^m$ outperforms the MTLE $t_{l_1}^m$ under scheme I, if

$$MSE(t_{l_1}^m) > MSE(t_{sl_1}^m) \implies \frac{D_1^2}{C_1} > 1 - \Phi C_y^2(\lambda_r - \lambda_n \rho_{xy}^2).$$

(ii). The MTSLE $t_{sl_2}^m$ outperforms the MTLE $t_{l_2}^m$ under scheme II, if

$$MSE(t_{l_2}^m) > MSE(t_{sl_2}^m) \implies \frac{D_2^2}{C_2} > 1 - \Phi C_y^2 \lambda_r (1 - \rho_{xy}^2).$$

(iii). The MTSLE $t_{sl_3}^m$ outperforms the MTLE $t_{l_3}^m$ under scheme III, if

$$MSE(t_{l_3}^m) > MSE(t_{sl_3}^m) \implies \frac{D_3^2}{C_3} > 1 - \Phi \lambda_r C_y^2 - \Phi \lambda_{rn} C_y^2 (1 - \rho_{xy}^2).$$

In these conditions, the proposed MTSLIMs outperform to their conventional counterparts under schemes I–III.

5 Simulation study

A simulation study is critical for evaluating the performance of statistical approaches in controlled environments. It allows researchers to assess robustness, efficiency, and accuracy across different scenarios. Simulation study gives insights into shortcomings and strengths of the techniques, driving future advancements and applications. Additionally, it helps validate theoretical findings and ensure reliability before deployment in practical contexts.

In this section, the proposed MTSLIMs are compared to the current traditional and memory type imputation methods using a simulation study. In the process of simulation study, employ *R* software to produce a population of size $N=1000$ from bivariate normal distribution using parameters given as $\bar{X} = 15$, $\bar{Y} = 10$, $\sigma_x = 7$, $\sigma_y = 5$, and several amounts of $\rho_{xy}=0.1, 0.5$, and 0.9 . Additionally, consider several values of smoothing constant $\kappa = 0.1, 0.5$, and 0.9 . A total of 15,000 samples are generated with sizes $n=10$ having responding units (8, 9), $n=100$ having responding units (80, 90), and $n=300$ having responding units (280, 290). The total 15,000 values are computed for each estimator taken into consideration in this research using the samples that were previously selected. Each estimator's MSE and RE are calculated using (5.1) and (5.2), respectively.

Consisting of simulation with 15,000 iterations, the MSE and RE are calculated, respectively by employing the under mentioned equations:

$$MSE(t_k) = \frac{1}{15,000} \sum_{k=1}^{15,000} (t_k - \bar{Y})^2 \quad (5.1)$$

$$\text{and } RE(t_k) = \frac{MSE(t_{r_i})}{MSE(t_k)}, \quad (5.2)$$

where $t_k = t_{r_i}$ $i = 1, 2, 3, t_{l_i}, t_{sl_i}, t_{r_i}^m, t_{l_i}^m$, and $t_{sl_i}^m$.

The simulation findings are given in Tables 7 and 8, respectively.

Table 7 MSE of memory type estimators for chosen values of correlation coefficient ρ_{xy} , sample size n , responding unit r , and smoothing constant κ

$\kappa = 0.1$																
Scheme I		ρ_{xy}	n	r	t_{r_1}	t_{l_1}	$t_{s_{l_1}}$	$\kappa = 0.5$			$\kappa = 0.9$					
								$t_{r_1}^m$	$t_{l_1}^m$	$t_{s_{l_1}}^m$	$t_{r_1}^m$	$t_{l_1}^m$	$t_{s_{l_1}}^m$			
	0.1	10	8	5.3366	2.7495	2.6471	0.2809	0.1447	0.1444	1.7789	0.9165	0.9047	4.3663	2.2496	2.1804	
			9	4.9129	2.4276	2.3483	0.2586	0.1278	0.1275	1.6376	0.8092	0.8001	4.0197	1.9862	1.9327	
		100	80	0.4615	0.2844	0.2835	0.0243	0.0150	0.0150	0.1538	0.0948	0.0947	0.3776	0.2327	0.2321	
			90	0.4263	0.2496	0.2489	0.0224	0.0131	0.0131	0.1421	0.0832	0.0831	0.3488	0.2042	0.2038	
		300	280	0.1090	0.0640	0.0640	0.0057	0.0034	0.0034	0.0363	0.0213	0.0213	0.0892	0.0524	0.0523	
			290	0.1059	0.0609	0.0609	0.0056	0.0032	0.0032	0.0353	0.0203	0.0203	0.0867	0.0499	0.0498	
	0.5	10	8	3.1898	2.1876	2.0882	0.1679	0.1151	0.1148	1.0633	0.7292	0.7177	2.6099	1.7898	1.7227	
			9	2.8165	1.8734	1.7961	0.1482	0.0986	0.0984	0.9388	0.6245	0.6156	2.3044	1.5328	1.4806	
		100	80	0.2812	0.2262	0.2254	0.0148	0.0119	0.0119	0.0937	0.0754	0.0753	0.2301	0.1851	0.1845	
			90	0.2473	0.1927	0.1920	0.0130	0.0101	0.0101	0.0824	0.0642	0.0641	0.2023	0.1576	0.1572	
		300	280	0.0628	0.0492	0.0491	0.0033	0.0026	0.0026	0.0209	0.0164	0.0164	0.0514	0.0402	0.0402	
			290	0.0598	0.0462	0.0461	0.0031	0.0024	0.0024	0.0199	0.0154	0.0154	0.0489	0.0378	0.0378	
	0.9	10	8	1.1125	1.0146	0.9637	0.0586	0.0534	0.0533	0.3708	0.3382	0.3324	0.9102	0.8301	0.7958	
			9	0.7700	0.6825	0.6474	0.0405	0.0359	0.0358	0.2567	0.2275	0.2235	0.6300	0.5584	0.5347	
		100	80	0.1048	0.1034	0.1030	0.0055	0.0054	0.0054	0.0349	0.0345	0.0344	0.0857	0.0846	0.0843	
			90	0.0709	0.0695	0.0692	0.0037	0.0037	0.0037	0.0236	0.0232	0.0231	0.0580	0.0569	0.0567	
		300	280	0.0171	0.0169	0.0169	0.0009	0.0008	0.0008	0.0057	0.0056	0.0056	0.0140	0.0138	0.0138	
			290	0.0141	0.0139	0.0138	0.0007	0.0007	0.0007	0.0047	0.0046	0.0046	0.0115	0.0113	0.0113	
	$\kappa = 0.1$															
	Scheme II		ρ_{xy}	n	r	t_{r_2}	t_{l_2}	$t_{s_{l_2}}$	$\kappa = 0.5$			$\kappa = 0.9$				
									$t_{r_2}^m$	$t_{l_2}^m$	$t_{s_{l_2}}^m$	$t_{r_2}^m$	$t_{l_2}^m$	$t_{s_{l_2}}^m$		
		0.1	10	8	5.8961	2.6557	2.554	0.3103	0.1398	0.1395	1.9654	0.8852	0.8735	4.8241	2.1728	2.1041
				9	5.1547	2.3904	2.3113	0.2713	0.1258	0.1256	1.7182	0.7968	0.7877	4.2175	1.9558	1.9024
			100	80	0.5098	0.2834	0.2825	0.0268	0.0149	0.0149	0.1699	0.0945	0.0944	0.4171	0.2319	0.2313
90		0.4477		0.2492	0.2485	0.0236	0.0131	0.0131	0.1492	0.0831	0.0830	0.3663	0.2039	0.2035		
300		280	0.0171	0.0169	0.0169	0.0009	0.0008	0.0008	0.0057	0.0056	0.0056	0.0140	0.0138	0.0138		
		290	0.0141	0.0139	0.0138	0.0007	0.0007	0.0007	0.0047	0.0046	0.0046	0.0115	0.0113	0.0113		

Table 7 (continued)

$\kappa = 0.1$										$\kappa = 0.5$					$\kappa = 0.9$				
Scheme III	ρ_{xy}	n	r	t_{r_3}	t_{l_5}	t_{sl_3}	$t_{r_3}^m$	$t_{l_5}^m$	$t_{sl_3}^m$	$t_{r_3}^m$	$t_{l_5}^m$	$t_{sl_3}^m$	$t_{r_3}^m$	$t_{l_5}^m$	$t_{sl_3}^m$	$t_{r_3}^m$	$t_{l_5}^m$	$t_{sl_3}^m$	$t_{r_3}^m$
100		90	9	2.6858	2.5800	2.4949	0.1414	0.1358	0.1355	0.8953	0.8600	0.8503	2.1975	2.1109	2.0535				
			80	0.2790	0.2637	0.2629	0.0147	0.0139	0.0139	0.0930	0.0879	0.0878	0.2283	0.2158	0.2152				
			90	0.2447	0.2380	0.2374	0.0129	0.0125	0.0125	0.0816	0.0793	0.0793	0.2002	0.1947	0.1943				
			280	0.0623	0.0609	0.0609	0.0033	0.0032	0.0032	0.0208	0.0203	0.0203	0.051	0.0498	0.0498				
			290	0.0592	0.0586	0.0585	0.0031	0.0031	0.0031	0.0197	0.0195	0.0195	0.0485	0.0479	0.0479				
300	0.9	10	8	2.5401	2.5154	2.4195	0.1337	0.1324	0.1321	0.8467	0.8385	0.8274	2.0782	2.0580	1.9933				
			9	2.4776	2.4678	2.3867	0.1304	0.1299	0.1296	0.8259	0.8226	0.8133	2.0271	2.0191	1.9644				
			80	0.2317	0.2313	0.2306	0.0122	0.0122	0.0122	0.0772	0.0771	0.0770	0.1896	0.1892	0.1888				
			90	0.2249	0.2247	0.2241	0.0118	0.0118	0.0118	0.0750	0.0749	0.0748	0.1840	0.1838	0.1835				
			280	0.0581	0.0581	0.0580	0.0030	0.0030	0.0030	0.0193	0.0193	0.0193	0.0475	0.0475	0.0475				
		290		0.0574	0.0574	0.0574	0.0030	0.0030	0.0030	0.0191	0.0191	0.0191	0.0470	0.0470	0.0470				

Table 8 RE of memory type estimators for chosen values of correlation coefficient ρ_{xy} , sample size n , responding unit r , and smoothing constant κ

$\kappa = 0.1$															
$\kappa = 0.5$															
Scheme I	ρ_{xy}	n	r	t_{r_1}	t_{l_1}	t_{d_1}	$t_{r_1}^m$	$t_{l_1}^m$	$t_{d_1}^m$	$\kappa = 0.9$					
0.1	0.1	10	8	0.5849	1.1352	1.1791	11.1123	21.5682	21.6129	3.4500	0.7148	1.3874	1.4315		
			9	0.5615	1.1364	1.1747	10.6685	21.5910	21.6298	3.4091	1.6845	0.6863	1.3889	1.4274	
		100	80	0.6238	1.0123	1.0154	11.8516	19.2338	19.2369	3.0369	1.8713	0.7624	1.2373	1.2403	
			90	0.5932	1.0129	1.0156	11.2701	19.2454	19.2482	3.0388	1.7795	0.725	1.2380	1.2407	
		300	280	0.5905	1.0057	1.0064	11.2204	19.1092	19.1099	3.0172	1.7716	0.7218	1.2292	1.2299	
	290		0.5788	1.0060	1.0066	10.9967	19.1135	19.1141	3.0179	1.7363	0.7074	1.2295	1.2302		
	0.5	10	8	0.9416	1.3731	1.4384	17.8909	26.0880	26.1538	4.1192	4.1849	1.1509	1.6782	1.7436	
			9	0.9477	1.4249	1.4862	18.0068	27.0722	27.1338	4.2746	4.3361	1.1583	1.7415	1.8028	
		100	80	0.9891	1.2294	1.2341	18.7921	23.3577	23.3624	2.9672	3.6881	1.2088	1.5025	1.5073	
			90	0.9882	1.2684	1.2729	18.7761	24.0993	24.1038	2.9647	3.8051	1.2078	1.5502	1.5548	
300		280	0.9911	1.2660	1.2672	18.8302	24.0538	24.0550	2.9732	3.7980	1.2113	1.5473	1.5485		
0.9	0.1	10	8	2.7168	2.9788	3.1362	51.6186	56.5973	56.7543	8.1503	8.9364	9.0936	3.7981		
			9	3.4981	3.9465	4.1606	66.4637	74.9834	75.1953	10.4943	11.8395	12.0521	4.8235	5.0372	
		100	80	2.6765	2.7132	2.7237	50.8536	51.5504	51.5609	8.0295	8.1395	8.1500	3.2713	3.3161	
			90	3.4798	3.5474	3.5616	66.1156	67.4012	67.4153	10.4393	10.6423	10.6564	4.2530	4.3499	
		300	280	3.6625	3.7133	3.7172	69.5877	70.5532	70.5570	10.9875	11.1400	11.1438	4.4764	4.5385	
	290		4.2247	4.2959	4.3006	80.2701	81.6226	81.6273	12.6742	12.8878	12.8924	5.1636	5.2552		
	$\kappa = 0.1$														
	$\kappa = 0.5$														
	Scheme II	ρ_{xy}	n	r	t_{r_2}	t_{l_2}	t_{d_2}	$t_{r_2}^m$	$t_{l_2}^m$	$t_{d_2}^m$	$\kappa = 0.9$				
	0.1	0.1	10	8	0.5294	1.1753	1.2221	10.0579	22.3304	22.3779	1.5881	3.5259	3.5732	0.647	1.4365
9				0.5352	1.1540	1.1935	10.1681	21.9265	21.9665	1.6055	3.4621	3.5019	0.6541	1.4105	1.4501
100			80	0.5647	1.0158	1.0189	10.7302	19.2998	19.3029	1.6942	3.0473	3.0505	0.6902	1.2415	1.2447
			90	0.5648	1.0145	1.0173	10.7312	19.2762	19.2789	1.6944	3.0436	3.0464	0.6903	1.2400	1.2428
0.5			10	8	0.5294	1.1753	1.2221	10.0579	22.3304	22.3779	1.5881	3.5259	3.5732	0.647	1.4365
		9		0.5352	1.1540	1.1935	10.1681	21.9265	21.9665	1.6055	3.4621	3.5019	0.6541	1.4105	1.4501
		100	80	0.5647	1.0158	1.0189	10.7302	19.2998	19.3029	1.6942	3.0473	3.0505	0.6902	1.2415	1.2447
			90	0.5648	1.0145	1.0173	10.7312	19.2762	19.2789	1.6944	3.0436	3.0464	0.6903	1.2400	1.2428
		300	280	0.5648	1.0145	1.0173	10.7312	19.2762	19.2789	1.6944	3.0436	3.0464	0.6903	1.2400	1.2428

Table 8 (continued)

$\kappa = 0.1$															
$\kappa = 0.5$															
$\kappa = 0.9$															
Scheme II	ρ_{xy}	n	r	t_{r_2}	t_{l_2}	t_{s_2}	$t_{r_2}^m$	$t_{l_2}^m$	$t_{s_2}^m$						
	0.5	300	280	0.5669	1.0063	1.0070	10.7704	19.1205	19.1212	1.7006	3.019	3.0197	0.6928	1.2300	1.2307
			290	0.5670	1.0063	1.0069	10.7732	19.1191	19.1198	1.7010	3.0188	3.0195	0.6930	1.2299	1.2305
			8	0.9280	1.5159	1.5943	17.6311	28.8012	28.8802	2.7839	4.5476	4.6264	1.1342	1.8527	1.9313
		100	9	0.9422	1.4962	1.5635	17.9018	28.4277	28.4953	2.8266	4.4886	4.5561	1.1516	1.8287	1.8960
			80	0.9861	1.3130	1.3187	18.7352	24.9470	24.9527	2.9582	3.9390	3.9447	1.2052	1.6048	1.6105
			90	0.9868	1.3118	1.3168	18.7489	24.9251	24.9301	2.9603	3.9355	3.9405	1.2061	1.6034	1.6084
	0.9	300	280	0.9902	1.3013	1.3026	18.8131	24.7249	24.7262	2.9705	3.9039	3.9052	1.2102	1.5905	1.5917
			290	0.9896	1.3006	1.3017	18.8021	24.7106	24.7118	2.9688	3.9017	3.9029	1.2095	1.5896	1.5908
			8	4.7959	5.954	6.4613	91.1230	113.1262	113.6184	14.3878	17.862	18.3585	5.8617	7.2771	7.7814
		100	9	4.8610	5.8964	6.3263	92.3581	112.0317	112.4505	14.5829	17.6892	18.1112	5.9412	7.2067	7.6344
			80	5.0093	5.1767	5.2073	95.1771	98.3567	98.3872	15.0280	15.5300	15.5605	6.1225	6.3270	6.3576
			90	5.0151	5.1749	5.2018	95.2871	98.3238	98.3506	15.0453	15.5248	15.5516	6.1296	6.3249	6.3517
300	280	5.0287	5.1351	5.1417	95.546	97.5664	97.5731	15.0862	15.4052	15.4119	6.1462	6.2762	6.2829		
	290	5.0226	5.1286	5.1349	95.4294	97.4436	97.4500	15.0678	15.3858	15.3922	6.1387	6.2683	6.2746		
$\kappa = 0.1$															
$\kappa = 0.5$															
$\kappa = 0.9$															
Scheme III	ρ_{xy}	n	r	t_{r_3}	t_{l_3}	t_{s_3}	$t_{r_3}^m$	$t_{l_3}^m$	$t_{s_3}^m$						
0.1	10	8	0.8480	1.0310	1.0718	16.1120	19.5890	19.6305	2.5440	3.0930	3.1342	1.0364	1.2601	1.3010	
		9	0.9194	1.0137	1.0486	17.4689	19.2594	19.2948	2.7582	3.0410	3.0762	1.1237	1.2389	1.274	
		80	0.8565	1.0034	1.0063	16.2735	19.0644	19.0673	2.5695	3.0102	3.0131	1.0468	1.2264	1.2293	
	300	90	0.9219	1.0016	1.0041	17.5168	19.0300	19.0325	2.7658	3.0047	3.0073	1.1268	1.2241	1.2267	
		280	0.9339	1.0006	1.0012	17.7446	19.0111	19.0117	2.8018	3.0018	3.0024	1.1415	1.2229	1.2236	
		290	0.9654	1.0003	1.0009	18.3424	19.0056	19.0062	2.8962	3.0009	3.0015	1.1799	1.2226	1.2232	
0.5	10	8	0.9846	1.0737	1.1170	18.7072	20.3996	20.4435	2.9538	3.2210	3.2647	1.2034	1.3123	1.3557	

Table 8 (continued)

$\kappa = 0.1$										$\kappa = 0.5$					$\kappa = 0.9$				
Scheme III	ρ_{xy}	n	r	t_{r_3}	t_{l_3}	t_{sd_3}	$t_{r_3}^m$	$t_{l_3}^m$	$t_{sd_3}^m$	$t_{r_3}^m$	$t_{l_3}^m$	$t_{sd_3}^m$	$t_{r_3}^m$	$t_{l_3}^m$	$t_{sd_3}^m$				
100		300	9	0.9938	1.0346	1.0699	18.8831	19.6578	19.6934	2.9815	3.1039	3.1394	1.2147	1.2645	1.2999				
			80	0.9969	1.0547	1.0578	18.9418	20.0385	20.0416	2.9908	3.1640	3.1671	1.2185	1.2890	1.2922				
			90	0.9985	1.0268	1.0294	18.9721	19.5096	19.5122	2.9956	3.0805	3.0830	1.2204	1.2550	1.2576				
			280	0.9991	1.0219	1.0225	18.9825	19.4163	19.4169	2.9972	3.0657	3.0664	1.2211	1.2490	1.2496				
			290	0.9995	1.0110	1.0116	18.9906	19.2084	19.209	2.9985	3.0329	3.0335	1.2216	1.2356	1.2362				
100	0.9	10	8	1.1899	1.2016	1.2491	22.6076	22.8297	22.8778	3.5696	3.6047	3.6527	1.4543	1.4686	1.5163				
		9	1.0871	1.0915	1.1285	20.6555	20.7377	20.7751	3.2614	3.2744	3.3117	1.3287	1.3340	1.3711					
		80	1.2106	1.2127	1.2161	23.0023	23.0414	23.0448	3.6319	3.6381	3.6415	1.4797	1.4822	1.4856					
		90	1.0965	1.0973	1.1000	20.8328	20.8483	20.851	3.2894	3.2918	3.2945	1.3401	1.3411	1.3438					
		280	1.0801	1.0806	1.0812	20.5223	20.5308	20.5315	3.2404	3.2417	3.2424	1.3202	1.3207	1.3214					
100		300	290	1.0391	1.0393	1.0399	19.7423	19.7463	19.7469	3.1172	3.1178	3.1184	1.2700	1.2702	1.2708				

6 Real data illustrations

To support the practical execution of the proposed imputation methods and resultant estimations, we have conducted an empirical illustration. This is based on analyzing two carefully selected populations. The assessment and final findings depend on MSE and RE. Descriptions of both populations are described below:

• Population 1.

The population referenced is taken from Singh (2003), spanning pages 1116–1118. It consists of data on fish caught by marine recreational fishermen categorized by species group and year along with the Atlantic and Gulf coasts during 1992 to 1995. In this population, the amount of fish caught in 1995 denotes as the study variable, whereas the amount caught in 1994 denotes as an auxiliary variable. The characteristics of this population are as follows: $N = 69$, $n = 25$, $r = 20$, $\kappa = 0.2$, $\bar{X} = 4954.435$, $\bar{Y} = 4514.899$, $S_x^2 = 49829270$, $S_y^2 = 37199578$, $\rho_{xy} = 0.9601$.

• Population 2.

The population is sourced from Singh (2003), page 1113. It relies on a hypothetical scenario of a small village with only 30 old persons (aged over 50 years). In this context, the duration of sleep (in minutes) is taken as the study variable, whereas the age (in years) of the persons is taken as an auxiliary variable. The characteristics of this population are as follows: $N = 69$, $n = 10$, $r = 8$, $\kappa = 0.3$, $\bar{X} = 67.267$, $\bar{Y} = 384.2$, $S_x^2 = 85.237$, $S_y^2 = 3582.58$, $\rho_{xy} = -0.8552$.

In survey sampling, density and box plots are important for analyzing the distribution of study and auxiliary variables. Density plots reveal probability distributions, central tendencies, and anomalies, while box plots summarize key statistics like the median and outliers. These tools help identify variability, trends, and relationships, supporting better sampling design, modelling, and imputation. Therefore, to have better understanding about the data sets, the density and boxplots of the study and auxiliary variables of both populations are presented from Figs. 1, 2, 3 and 4, respectively.

The findings of the estimators for these populations are given in Table 9 in terms of MSE and RE.

7 Discussion of results

The important findings of the simulation study and real data illustrations are described below.

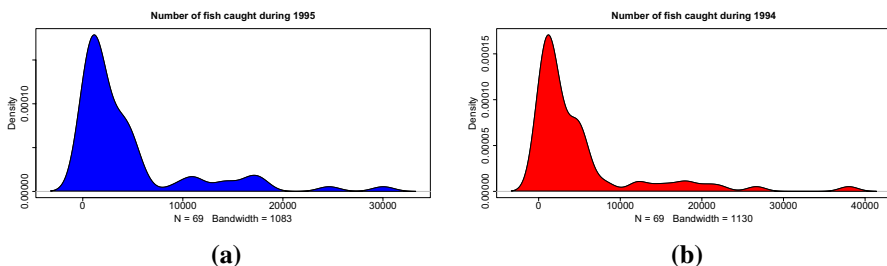


Fig. 1 The density plots for the **a** study variable and **b** supplementary variable of population 1

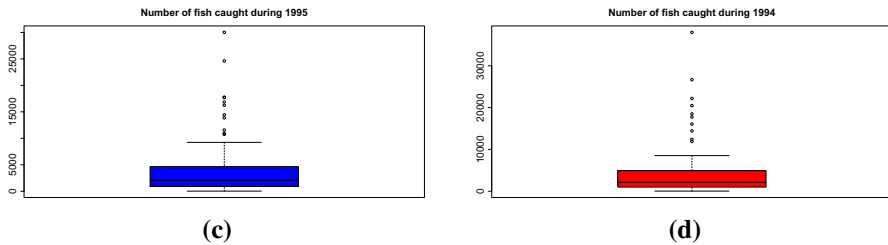


Fig. 2 The boxplots for the **c** study variable and **d** supplementary variable of population 1

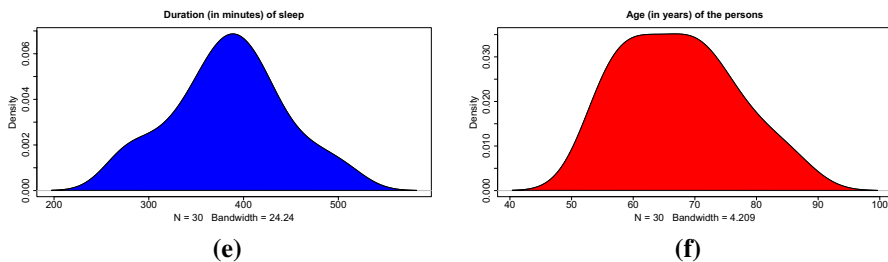


Fig. 3 The density plots for the **e** study variable and **f** supplementary variable of population 2

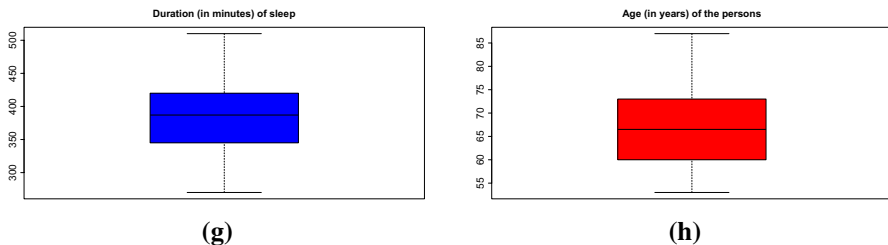


Fig. 4 The boxplots for the **g** study variable and **h** supplementary variable of population 2

1. From the simulation results reported in Table 7, it is seen that:

- The MSEs of the proposed MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ are least in comparison to the CREs t_{r_i} , LEs t_{l_i} , SLEs t_{sl_i} , MTREs $t_{r_i}^m$, and MTLEs $t_{l_i}^m$ under schemes I, II, and III. For example, under scheme I, at $\rho_{xy} = 0.1$, $n = 10$, $r = 8$, and $\kappa = 0.1$, MSE of the CRE t_{r_1} is 5.3366, MSE of the LE t_{l_1} is 2.7495, MSE of the SLE t_{sl_1} is 2.6471, MSE of the MTRE $t_{r_1}^m$ is 0.2809, MSE of the MTLE $t_{l_1}^m$ is 0.1447, MSE of the MTSLE $t_{sl_1}^m$ is 0.1444. This shows that the MTSLE $t_{sl_1}^m$ is more efficient than the existing estimators. Also, the proposed MTSLE $t_{sl_1}^m$ outperforms for other values of smoothing constant $\kappa = 0.5, 0.9$. Moreover, the similar tendency in the results can also be observed from the schemes II and III.
- The MSEs of the proposed MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ decrease as the correlation coefficient ρ_{xy} values vary from 0.1 to 0.9. Consequently, the supplemental information increases the estimators' efficiency. For example, under scheme I, at $n = 300$,

Table 9 MSE and RE of the conventional and memory type estimators utilizing real populations

Schemes	Estimators	Population 1		Population 2	
		MSE	RE	MSE	RE
Scheme I	$\bar{y}$	1320855	1.00	328.80	1.00
	t_{r_1}	454694.80	2.90	873.55	0.37
	t_{l_1}	446204.10	2.96	153.91	2.13
	t_{sl_1}	403222.60	3.27	153.82	2.14
	$t_{r_1}^m$	50521.65	26.14	154.15	2.13
	$t_{l_1}^m$	49578.24	26.64	27.16	12.10
	$t_{sl_1}^m$	49013.33	26.94	27.15	12.11
Scheme II	t_{r_2}	115120.80	11.47	1077.84	0.30
	t_{l_2}	103301.40	12.78	88.32	3.72
	t_{sl_2}	71754.92	18.40	87.79	3.74
	$t_{r_2}^m$	12791.20	103.26	190.20	1.72
	$t_{l_2}^m$	11477.93	115.07	15.58	21.09
	$t_{sl_2}^m$	11062.19	119.40	15.57	21.11
Scheme III	t_{r_3}	981280.60	1.34	533.08	0.61
	t_{l_3}	977951.80	1.35	263.21	1.24
	t_{sl_3}	913654.2	1.44	263.03	1.25
	$t_{r_3}^m$	150364.00	8.78	109.89	2.99
	$t_{l_3}^m$	149994.20	8.80	62.27	5.27
	$t_{sl_3}^m$	107819.80	12.25	46.44	7.07

$r = 280$, and $\kappa = 0.1$, the MSE values of the proposed MTSLE $t_{sl_i}^m$ are 0.0032 and 0.0008 for $\rho_{xy} = 0.1, 0.9$, respectively. The similar tendency in the MSE values can be viewed from schemes II and III.

- For each constant value of the correlation coefficient ρ_{xy} , the MSEs of the suggested MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ decrease as sample size n increases from 10 to 300. This demonstrates that the suggested MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ are superior than the CREs t_{r_i} , LEs t_{l_i} , SLEs t_{sl_i} , MTREs $t_{r_i}^m$, and MTLEs $t_{l_i}^m$ under each scheme, even when utilizing varied sample sizes.

2. From the simulation results reported in Table 8, it is seen that:

- The REs of the proposed MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ are maximum in comparison to the CREs t_{r_i} , LEs t_{l_i} , SLEs t_{sl_i} , MTREs $t_{r_i}^m$, and MTLEs $t_{l_i}^m$ under schemes I, II, and III. For example, under scheme I, at $\rho_{xy} = 0.1$, $n = 10$, $r = 8$, and $\kappa = 0.1$, RE of the CRE t_{r_1} is 0.5849, RE of the LE t_{l_1} is 1.1352, RE of the SLE t_{sl_1} is 1.1791, RE of the MTRE $t_{r_1}^m$ is 11.1123, RE of the MTLE $t_{l_1}^m$ is 21.5682, and RE of the MTSLE $t_{sl_1}^m$ is 21.6129. This shows that the MTSLE $t_{sl_1}^m$ is more efficient than the existing estimators. The proposed MTSLE $t_{sl_1}^m$ outperforms for other values of smoothing constant $\kappa = 0.5, 0.9$. Moreover, the similar tendency in the results can also be observed from the schemes II and III.
- The REs of the proposed MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ increase as the correlation coefficient ρ_{xy} values vary from 0.1 to 0.9 under schemes I, II, and III. As a result, the auxiliary information increases the estimators' efficiency. For example, under

- scheme I, at $n = 300$, $r = 280$, and $\kappa = 0.1$, the RE values of the proposed MTSLE $t_{sl_1}^m$ are 19.1099 and 70.5570 for $\rho_{xy} = 0.1, 0.9$, respectively. The similar tendency in the RE values can be viewed from schemes II and III.
- For higher value of the correlation coefficient ρ_{xy} , the REs of the suggested MTSLEs increase as sample size n increases from 10 to 300. This demonstrates that the suggested MTSLEs are superior than the CREs, LEs, SLEs, MTREs, and MTLEs under each scheme, even when utilizing varied sample sizes.
3. From the simulation results of Tables 7 and 8, it is also observed that:
- The results of the MTSLEs are improved in terms of minimum MSE and maximum RE as the amount of the smoothing constant κ increases, which is employed to give weight to the previous and current observations.
 - Furthermore, when κ rises, the information that is being processed now is given greater weight than information that is previously known, which raises the MSE and reduces the RE of memory-based estimators.
4. From the results of real data illustration reported in Tables 9, it is observed that:
- In population 1, the proposed MTSLIMs and the resultant MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ under all schemes outperform the CRIMs, LIMs, SLIMs, MTRIMs, MTLIMs and the corresponding resultant CREs t_{r_i} , $i = 1, 2, 3$, LEs t_{l_i} , SLEs t_{sl_i} , MTREs $t_{r_i}^m$, and MTLEs $t_{l_i}^m$, respectively. For example, in scheme I, the $MSE(t_{sl_1}^m) = 49013.33$ which is less than the $MSE(t_{r_1}) = 1320855$, $MSE(t_{l_1}) = 454694.80$, $MSE(t_{sl_1}) = 403222.60$, $MSE(t_{r_1}^m) = 50521.65$, and $MSE(t_{l_1}^m) = 49578.24$, while the $RE(t_{sl_1}^m) = 26.94$ which is greater than the $RE(t_{r_1}) = 2.90$, $RE(t_{l_1}) = 2.96$, $RE(t_{sl_1}) = 3.27$, $RE(t_{r_1}^m) = 26.14$, and $RE(t_{l_1}^m) = 26.64$.
 - In population 2, the proposed MTSLIMs and the resultant MTSLEs $t_{sl_i}^m$, $i = 1, 2, 3$ under all schemes outperform the CRIMs, LIMs, SLIMs, MTRIMs, MTLIMs and the corresponding resultant CREs t_{r_i} , $i = 1, 2, 3$, LEs t_{l_i} , SLEs t_{sl_i} , MTREs $t_{r_i}^m$, and MTLEs $t_{l_i}^m$. For example, in scheme I, the $MSE(t_{sl_1}^m) = 27.15$ which is less than the $MSE(t_{r_1}) = 873.55$, $MSE(t_{l_1}) = 153.91$, $MSE(t_{sl_1}) = 153.82$, $MSE(t_{r_1}^m) = 154.15$, and $MSE(t_{l_1}^m) = 27.16$, while the $RE(t_{sl_1}^m) = 12.11$ which is greater than the $RE(t_{r_1}) = 0.37$, $RE(t_{l_1}) = 2.13$, $RE(t_{sl_1}) = 2.14$, $RE(t_{r_1}^m) = 2.13$, and $RE(t_{l_1}^m) = 12.10$.
 - The performance of the proposed MTSLIMs and the resultant MTSLEs is best in scheme II as compare to scheme I and scheme III in both the populations.

8 Conclusion

In temporal surveys based on missing data, maximizing the effectiveness of resultant estimators necessitates the simultaneous consideration of past and current sample data. To address this challenge, the author employs the EWMA statistic and proposed MTSLIMs alongside their resultant MTSLEs for estimating population means in the presence of missing data under SRS. The MSE of the resultant MTSLEs is approximated to the first order.

A comprehensive comparison is provided between the performance of the proposed MTSLIMs and the conventional imputation methods. This comparison is further substantiated

by an extensive simulation study performed in *R* software. The simulation results demonstrated that the proposed MTSLEs outperform the CREs, LEs, SLEs, MTREs, and MTLEs in terms of lesser MSE and higher RE for suitably chosen values of correlation coefficient ρ_{xy} , sample size n , responding unit r , and smoothing constant κ under schemes I, II, and III. Moreover, the efficiency of the proposed methods is illustrated through the applications to real populations. The findings of the real data illustration underscore the supremacy of the proposed MTSLEs over their conventional counterparts in terms of minimum MSE and maximum RE under schemes I, II, and III.

Therefore, the proposed MTSLEs and the corresponding resultant MTSLEs emerge as viable options for estimating the population mean in the presence of missing data in temporal surveys. Future research endeavours may explore MTSLEs and the corresponding MTSLEs beyond SRS, including stratified random sampling, ranked set sampling, stratified ranked set sampling, etc.

Appendix A

This section provides the *MSE* and optimum values of scalars of the resultant estimators discussed in Sect. 1.

$$\begin{aligned}MSE(t_{r_1}) &= \bar{Y}^2(\lambda_r C_y^2 + \lambda_n C_x^2 - 2\lambda_n \rho_{xy} C_x C_y) \\MSE(t_{r_2}) &= \bar{Y}^2 \lambda_r (C_y^2 + C_x^2 - 2\rho_{xy} C_x C_y) \\MSE(t_{r_3}) &= \bar{Y}^2(\lambda_r C_y^2 + \lambda_{rn} C_x^2 - 2\lambda_{rn} \rho_{xy} C_x C_y) \\min.MSE(t_{l_1}) &= \bar{Y}^2 C_y^2 (\lambda_r - \lambda_n \rho_{xy}^2) \\min.MSE(t_{l_2}) &= \bar{Y}^2 C_y^2 \lambda_r (1 - \rho_{xy}^2) \\min.MSE(t_{l_3}) &= \lambda_n \bar{Y}^2 C_y^2 + \lambda_{rn} \bar{Y}^2 C_x^2 (1 - \rho_{xy}^2) \\min.MSE(t_{sl_i}) &= \bar{Y}^2 \left(1 - \frac{B_i^2}{A_i} \right), \quad i = 1, 2, 3\end{aligned}$$

The optimum values of scalars φ_i , $i = 1, 2, 3$ and A_i are given below.

$$\varphi_{i(opt)} = \rho_{xy} \frac{C_y}{C_x} \quad \text{and} \quad A_{i(opt)} = \frac{B_i}{A_i}$$

where

$$\begin{aligned}A_1 &= 1 + \lambda_r C_y^2 + 2\varphi_1(\varphi_1 - 1)\lambda_n C_x^2 + 4\varphi_1 \lambda_n \rho_{xy} C_x C_y, \\B_1 &= 1 + \frac{\varphi_1(\varphi_1 - 2)}{2} \lambda_n C_x^2 + \varphi_1 \lambda_n \rho_{xy} C_x C_y, \\A_2 &= 1 + \lambda_r C_y^2 + 2\varphi_2(\varphi_2 - 1)\lambda_r C_x^2 + 4\varphi_2 \lambda_r \rho_{xy} C_x C_y, \\B_2 &= 1 + \frac{\varphi_2(\varphi_2 - 2)}{2} \lambda_r C_x^2 + \varphi_2 \lambda_r \rho_{xy} C_x C_y,\end{aligned}$$

$$A_3 = 1 + \lambda_r C_y^2 + 2\varphi_3(\varphi_3 - 1)\lambda_{rn} C_x^2 + 4\varphi_3 \lambda_{rn} \rho_{xy} C_x C_y,$$

$$B_3 = 1 + \frac{\varphi_3(\varphi_3 - 2)}{2} \lambda_{rn} C_x^2 + \varphi_3 \lambda_{rn} \rho_{xy} C_x C_y.$$

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Data availability All data used for the empirical study is included in the manuscript.

Declarations

Conflict of interest The authors declare that they have no Conflict of interest.

Human and/or animals rights The authors declare that the research did not involve Human Participants and/or Animals.

References

- Alomair, M.A., Shahzad, U.: Compromised-imputation and EWMA-based memory-type mean estimators using quantile regression. *Symmetry* **15**(10), 1888 (2023)
- Al-Omari, A.I., Bouza, C.: Ratio estimators of the population mean with missing values using ranked set sampling. *Environmetrics* **26**(2), 67–76 (2015)
- Aslam, I., Noor-ul-Amin, M., Yasmeen, U., Hanif, M.: Memory type ratio and product estimators in stratified sampling. *J. Reliab. Stat. Stud.* **13**(1), 1–20 (2020)
- Aslam, I., Noor-ul-Amin, M., Hanif, M., Sharma, P.: Memory type ratio and product estimators under ranked-based sampling schemes. *Commun. Stat. Theory Methods* **52**(4), 1155–1177 (2023)
- Bhattacharyya, D., Singh, G.N., Jawa, T.M., Sayed-Ahmed, N., Pandey, A.K.: An exponential-cum-sine-type hybrid imputation technique for missing data. *Comput. Intell. Neurosci.* **2021**(1), 4845569 (2021)
- Bhushan, S., Pandey, A.P.: Optimality of ratio type estimation methods for population mean in presence of missing data. *Commun. Stat. Theory Methods* **47**(11), 2576–2589 (2018)
- Bhushan, S., Kumar, A., Pandey, A.P., Singh, S.: Estimation of population mean in presence of missing data under simple random sampling. *Commun. Stat. Simul. Comput.* **52**(12), 6048–6069 (2023)
- Bhushan, S., Kumar, A., Shukla, S., Bakr, M.E., Tashkandy, Y.A., Hossain, M.M.: New logarithmic type imputation techniques in presence of measurement errors. *Alex. Eng. J.* **71**, 707–730 (2023)
- Bhushan, S., Kumar, A., Al-Omari, A.I., Alomani, G.A.: Mean estimation for time-based surveys using memory-type logarithmic estimators. *Mathematics* **11**(9), 2125 (2023)
- Bhushan, S., Kumar, A.: Imputation of missing data using multi auxiliary information under ranked set sampling. *Commun. Stat. Simul. Comput.* (2023). <https://doi.org/10.1080/03610918.2023.2288796>
- Bhushan, S., Kumar, A.: Novel logarithmic imputation methods under ranked set sampling. *Meas. Interdiscip. Res. Perspect.* **22**(3), 235–257 (2024)
- Heitjan, D.F., Basu, S.: Distinguishing ‘missing at random’ and ‘missing completely at random’. *Am. Stat.* **50**, 207–213 (1996)
- Kumar, A., Anshika, Emam, W., Tashkandy, Y.: Memory type general class of estimators for population variance under simple random sampling. *Heliyon*, e36090 (2024). <https://doi.org/10.1016/j.heliyon.2024.e36090>
- Lee, H., Rancourt, E., Sarndal, C.E.: Experiments with variance estimation from survey data with imputed values. *J. Off. Stat.* **10**, 231–243 (1994)
- Noor-ul-Amin, M.: Memory type estimators of population mean using exponentially weighted moving averages for time scaled surveys. *Commun. Stat. Theory Methods* **50**(12), 2747–2758 (2021)
- Pandey, A.K., Singh, G.N., Bhattacharyya, D., Ali, A.Q., Al-Thubaiti, S., Yakout, H.A.: Some classes of logarithmic-type imputation techniques for handling missing data. *Comput. Intell. Neurosci.* **2021**(1), 8593261 (2021)
- Pandey, A.K., Singh, G.N., Bhattacharyya, D., Singh, P.K.: Efficient and alternative approaches for imputing missing data to estimate population mean. *Qual. Quant.* **58**, 5883–5897 (2024)

- Prasad, S., Yadav, V.K.: Imputation of missing data through product type exponential methods in sampling theory. *Rev. Colomb. Estad.* **46**(1), 111–127 (2023)
- Qureshi, M.N., Tariq, M.U., Hanif, M.: Memory-type ratio and product estimators for population variance using exponentially weighted moving averages for time-scaled surveys. *Commun. Stat. Simul. Comput.* **53**(3), 1484–1493 (2024)
- Roberts, S.: Control chart tests based on geometric moving averages. *Technometrics* **1**(3), 239–250 (1959)
- Rubin, R.B.: Inference and missing data. *Biometrika* **63**(3), 581–592 (1976)
- Sharma, Prayas, Singh, Poonam, Kumari, Mamta, Singh, Rajesh: Estimation procedures for population mean using EWMA for time scaled survey. *Sankhya B* (2024). <https://doi.org/10.1007/s13571-024-00347-7>
- Searls, D.T.: The utilization of a known coefficient of variation in the estimation procedure. *J. Am. Stat. Assoc.* **59**(308), 1225–1226 (1964)
- Shahzad, U., Al-Noor, N.H., Hanif, M., Sajjad, I., Anas, M.M.: Imputation based mean estimators in case of missing data utilizing robust regression and variance-covariance matrices. *Commun. Stat. Simul. Comput.* **51**(8), 4276–4295 (2022)
- Singh, S.: *Advanced Sampling Theory with Applications*. Springer Netherlands, Dordrecht (2003). <https://doi.org/10.1007/978-94-007-0789-4>
- Yadav, V.K., Prasad, S.: Generalized class of factor type exponential imputation techniques for population mean using simulation approach. *J. Stat. Comput. Simul.* **94**(9), 1997–2039 (2024). <https://doi.org/10.1080/00949655.2024.2310699>

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