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Original Article

Mean estimation using an efficient class of estimators based on simple random sampling: Simulation and applications

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ABSTRACT

In this article, we offer simple random sampling (SRS) based efficient class of estimators of population mean $\bar{Y}$ utilizing additional information. The expression of the mean square error of the proposed class of estimators is deduced up to first degree approximation. The efficiency conditions are established which are enhanced numerically utilizing a simulation study consummated over symmetrical and asymmetrical populations. Real data sets are also utilized to exemplify the suggested estimators. The numerical findings are appeared rather acceptable demonstrating better advancement over the ordinary estimators.

1. Introduction

In the survey research, over the last few decades, the utilization of supplementary informations associated with supplementary variable becomes a remarkable evolutions for improving the efficiency of the estimators. When such supplementary informations are correlated strongly with the main variable, the ratio methods of estimation is the most widely used method. However, the product methods of estimation are found to be most efficient when main variable is negatively correlated with the supplementary variable. Adapting these supplementary informations, various estimators had been suggested for evaluating the population mean/total. Ref. [1] suggested a power ratio estimator using auxiliary information. Ref. [2] build up an alternative ratio estimator utilizing the variation coefficient (C_x) of supplementary variable. Ref. [3] suggested ratio category of exponential estimator for computing the population mean. Ref. [4] analyzed two ratio kinds of estimators such that the skewness coefficient ($\beta_1(x)$) and kurtosis coefficient ($\beta_2(x)$) of the supplementary variable are in working order. Ref. [5] investigated a rectified ratio estimator by utilizing the available amount

of correlation coefficient. Ref. [6] advocated a new ratio estimators of population mean under SRS and ranked set sampling (RSS). Ref. [7] constructed an improved regression classes of population mean estimators. [8] utilized supplementary data and developed an improved Searls category of logarithmic estimators. Ref. [9] invoked a ratio cum exponential ratio estimator of population mean. Ref. [10] employed various available parameters of supplementary variable and constructed a class of ratio estimator for population mean. For mean estimation utilizing SRS, [11] designed an efficacious population mean based generalized class of estimators. Ref. [12] considered supplementary information and designed a modified class of logarithmic estimators, while, [13] developed an improved class of estimators. Ref. [14] suggested modified regression classes of exponential estimators. Ref. [15] developed a ratio type estimator utilizing supplementary information. Ref. [16] utilized some robust parameters to draw an optimized inference for population mean. This manuscript deals with an efficient class of estimators of population mean utilizing various supplementary information under SRS. The preeminent contribution of the current manuscript is pointed out below:

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- To investigate an efficient class of population mean estimators in SRS utilizing supplementary information,
- To mathematically liken the efficiency of the suggested estimators against the existing estimators utilizing the MSE/variance criteria,
- To empirically liken the efficacy of the investigated estimators with the ordinary estimators through the use of simulation study and real data application.

A few sections make up the remainder of the article. Section 2 delineates the notation, methodology, and existing works, while the proposed estimators are presented in Section 3. The efficiency conditions are generated in Section 4 that are numerically enhanced utilizing a simulation study in Section 5. An application of the proposed methods is demonstrated in Section 6. In Section 7, the manuscript is finally concluded.

2. Existing estimators

Let a survey population $Q = (Q_1, Q_2, \dots, Q_N)$ be based on N identifiable units from that a size n sample is selected utilizing simple random sampling without replacement (SRSWOR). Let (y_i, x_i) be the i^{th} unit of the main and supplementary variables. The population and sample means of the main variable y are $(\bar{Y} = \sum_{i=1}^N y_i/N, \bar{y} = \sum_{i=1}^n y_i/n)$, while the population and sample means of the variable x are $(\bar{X} = \sum_{i=1}^N x_i/N, \bar{x} = \sum_{i=1}^n x_i/n)$. The sample and population standard deviation of variable y are $s_y = \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2/(n-1)}$ and $S_y = \sqrt{\sum_{i=1}^N (y_i - \bar{Y})^2/(N-1)}$, respectively, while the sample and population standard deviation of variable x are $s_x = \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2/(n-1)}$ and $S_x = \sqrt{\sum_{i=1}^N (x_i - \bar{X})^2/(N-1)}$, respectively. The population variation coefficient for variables (y, x) are $(C_y = S_y/\bar{Y}, C_x = S_x/\bar{X})$. Let M_x be the median of variable x . The population correlation coefficient between variables (x, y) is given by ρ_{xy} .

The mean square error (MSE) of the existing and proposed estimators can be determined utilizing the notation: $\bar{y} = (1 + e_0)\bar{Y}$ and $\bar{x} = (1 + e_1)\bar{X}$. It is obvious that $E(e_0) = E(e_1) = 0$, $E(e_0^2) = fC_y^2$, $E(e_1^2) = fC_x^2$, and $E(e_0e_1) = f\rho_{xy}C_yC_x$, where $f = (N - n)/Nn$.

Further, we will consider the well-established existing estimators for the estimation of population mean utilizing SRS.

The basic mean estimator is given in (2.1) as

$$t_m = \bar{y} \quad (2.1)$$

The variance of t_m is given in (2.2) as

$$V(t_m) = f\bar{Y}^2C_y^2 \quad (2.2)$$

The basic ratio estimator is given in (2.3) as

$$t_r = \bar{y} \frac{\bar{X}}{\bar{x}} \quad (2.3)$$

The MSE of t_r is given in (2.4) as

$$MSE(t_r) = f\bar{Y}^2(C_y^2 + C_x^2 - 2\rho_{xy}C_xC_y) \quad (2.4)$$

The basic regression estimator is given in (2.5) as

$$t_{lr} = \bar{y} + b(\bar{X} - \bar{x}) \quad (2.5)$$

where the regression coefficient of y on x is b .

The MSE of t_{lr} is given in (2.6) as

$$MSE(t_{lr}) = f\bar{Y}^2(C_y^2 + b^2C_x^2 - 2b\rho_{xy}C_xC_y) \quad (2.6)$$

The minimum MSE of t_{lr} at optimizing scalar $b = \rho_{xy}S_y/S_x$ is given in (2.7) as

$$\min.MSE(t_{lr}) = f\bar{Y}^2C_y^2(1 - \rho_{xy}^2) \quad (2.7)$$

Ref. [2] utilized the supplementary information and developed the estimator given in (2.8) as

$$t_{sd} = \bar{y} \left(\frac{\bar{X} + C_x}{\bar{x} + C_x} \right) \quad (2.8)$$

The MSE of t_{sd} is given in (2.9) as

$$MSE(t_{sd}) = f\bar{Y}^2 \left(C_y^2 + \delta_1^2 C_x^2 - 2\delta_1 \rho_{xy} C_x C_y \right) \quad (2.9)$$

where $\delta_1 = \bar{X}/(\bar{X} + C_x)$.

The exponential ratio estimator shown in (2.10) was suggested by [3] as

$$t_{bt} = \bar{y} \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \quad (2.10)$$

The MSE of t_{bt} is given in (2.11) as

$$MSE(t_{bt}) = f\bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} - \rho_{xy} C_x C_y \right) \quad (2.11)$$

Ref. [4] considered different supplementary information and constructed the estimators given in (2.12) and (2.13) as

$$t_{us1} = \bar{y} \left(\frac{C_x \bar{X} + \beta_2(x)}{C_x \bar{x} + \beta_2(x)} \right) \quad (2.12)$$

$$t_{us2} = \bar{y} \left(\frac{\beta_2(x) \bar{X} + C_x}{\beta_2(x) \bar{x} + C_x} \right) \quad (2.13)$$

The MSEs of t_{us1} and t_{us2} are given in (2.14) and (2.15), respectively, as

$$MSE(t_{us1}) = f\bar{Y}^2 \left(C_y^2 + \delta_2^2 C_x^2 - 2\delta_2 \rho_{xy} C_x C_y \right) \quad (2.14)$$

$$MSE(t_{us2}) = f\bar{Y}^2 \left(C_y^2 + \delta_3^2 C_x^2 - 2\delta_3 \rho_{xy} C_x C_y \right) \quad (2.15)$$

where $\delta_2 = \bar{X}C_x/(C_x\bar{X} + \beta_2(x))$ and $\delta_3 = \bar{X}\beta_2(x)/(\beta_2(x)\bar{X} + C_x)$.

Ref. [5] considered the correlation coefficient between supplementary and main variables and designed the estimator given in (2.16) as

$$t_{st} = \bar{y} \left(\frac{\bar{X} + \rho_{xy}}{\bar{x} + \rho_{xy}} \right) \quad (2.16)$$

The MSE of t_{st} is given in (2.17) as

$$MSE(t_{st}) = f\bar{Y}^2 \left(C_y^2 + \delta_4^2 C_x^2 - 2\delta_4 \rho_{xy} C_x C_y \right) \quad (2.17)$$

where $\delta_4 = \bar{X}/(\bar{X} + \rho_{xy})$.

[17] adopted coefficient of kurtosis as supplementary information and suggested the estimator given in (2.18) as

$$t_s = \bar{y} \left(\frac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)} \right) \quad (2.18)$$

The MSE of t_s is given in (2.19) as

$$MSE(t_s) = f\bar{Y}^2 \left(C_y^2 + \delta_5^2 C_x^2 - 2\delta_5 \rho_{xy} C_x C_y \right) \quad (2.19)$$

where $\delta_5 = \bar{X}/(\bar{X} + \beta_2(x))$.

[18] suggested the generalized exponential ratio estimator given in (2.20) as

$$t_{sn} = \bar{y} \exp \left\{ \frac{(m\bar{X} + n) - (m\bar{x} + n)}{(m\bar{X} + n) + (m\bar{x} + n)} \right\} \quad (2.20)$$

where m and n are either available parameters of the supplementary variable or the real values.

The MSE of t_{sn} is given in (2.21) as

$$MSE(t_{sn}) = f\bar{Y}^2 \left(C_y^2 + \delta_6^2 C_x^2 - \delta_6 \rho_{xy} C_x C_y \right) \quad (2.21)$$

where $\delta_6 = m\bar{X}/2(m\bar{X} + n)$.

[9] examined the ratio cum exponential ratio estimator given in (2.22) as

$$t_k = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^\alpha \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \quad (2.22)$$

where α is an optimizing scalar. The minimum MSE of t_k at optimizing scalar $\alpha_{(opt)} = (2\rho_{xy}C_y - C_x)/2C_x$ is given in (2.23) as

$$MSE(t_k) = f\bar{Y}^2 C_y^2 (1 - \rho_{xy}^2) \quad (2.23)$$

[10] advocated the modified ratio estimators based on different supplementary information given from (2.24)–(2.32) as

$$t_{y1} = \bar{y} \left(\frac{\beta_2(x)M_x\bar{X} + \rho_{xy}}{\beta_2(x)M_x\bar{x} + \rho_{xy}} \right) \quad (2.24)$$

$$t_{y2} = \bar{y} \left(\frac{\beta_2(x)M_x\bar{X} + \rho_{xy}C_x}{\beta_2(x)M_x\bar{x} + \rho_{xy}C_x} \right) \quad (2.25)$$

$$t_{y3} = \bar{y} \left(\frac{\beta_1(x)M_x\bar{X} + \rho_{xy}}{\beta_1(x)M_x\bar{x} + \rho_{xy}} \right) \quad (2.26)$$

$$t_{y4} = \bar{y} \left(\frac{\beta_1(x)M_x\bar{X} + \rho_{xy}C_x}{\beta_1(x)M_x\bar{x} + \rho_{xy}C_x} \right) \quad (2.27)$$

$$t_{y5} = \bar{y} \left(\frac{n\bar{X} + \rho_{xy}}{n\bar{x} + \rho_{xy}} \right) \quad (2.28)$$

$$t_{y6} = \bar{y} \left(\frac{n\bar{X} + C_x}{n\bar{x} + C_x} \right) \quad (2.29)$$

$$t_{y7} = \bar{y} \left(\frac{n\bar{X} + \rho_{xy}C_x}{n\bar{x} + \rho_{xy}C_x} \right) \quad (2.30)$$

$$t_{y8} = \bar{y} \left(\frac{n\rho_{xy}\bar{X} + C_x}{n\rho_{xy}\bar{x} + C_x} \right) \quad (2.31)$$

$$t_{y9} = \bar{y} \left(\frac{nC_x\bar{X} + \rho_{xy}}{nC_x\bar{x} + \rho_{xy}} \right) \quad (2.32)$$

The MSE of t_{yi} , $i = 1, 2, \dots, 9$ is given in (2.33) as

$$MSE(t_{yi}) = f\bar{Y}^2 (C_y^2 + \Delta_i^2 C_x^2 - 2\Delta_i \rho_{xy} C_x C_y) \quad (2.33)$$

where $\Delta_1 = \beta_2(x)M_x\bar{X}/(\beta_2(x)M_x\bar{x} + \rho_{xy})$, $\Delta_2 = \beta_2(x)M_x\bar{X}/(\beta_2(x)M_x\bar{x} + \rho_{xy}C_x)$, $\Delta_3 = \beta_1(x)M_x\bar{X}/(\beta_1(x)M_x\bar{x} + \rho_{xy})$, $\Delta_4 = \beta_1(x)M_x\bar{X}/(\beta_1(x)M_x\bar{x} + \rho_{xy}C_x)$, $\Delta_5 = n\bar{X}/(n\bar{x} + \rho_{xy})$, $\Delta_6 = n\bar{X}/(n\bar{x} + C_x)$, $\Delta_7 = n\bar{X}/(n\bar{x} + \rho_{xy}C_x)$, $\Delta_8 = n\rho_{xy}\bar{X}/(n\rho_{xy}\bar{x} + C_x)$, and $\Delta_9 = nC_x\bar{X}/(nC_x\bar{x} + \rho_{xy})$.

[19] suggested the estimator given in (2.34) as

$$t_{by} = \gamma\bar{y} + (1 - \gamma)\bar{y} \left(\frac{ab\bar{X} + cd}{ab\bar{x} + cd} \right) \quad (2.34)$$

where γ is an optimizing scalar. The minimum MSE of t_{by} at optimizing scalar $\gamma_{(opt)} = 1 - \rho_{xy}(C_y/\theta C_x)$, $\theta = ab\bar{X}/(ab\bar{x} + cd)$ is given in (2.35) as

$$\min.MSE(t_{by}) = f\bar{Y}^2 C_y^2 (1 - \rho_{xy}^2) \quad (2.35)$$

3. Suggested class of estimators

To find out the best possible method of estimation over the various entrants, we propose an efficient class of estimators for the population mean utilizing SRS and studied its characteristics. In (3.1), the suggested estimators is shown as

$$t_a = \left\{ w_1\bar{y} + w_2\bar{y} \left(\frac{a\bar{X} + b}{a\bar{x} + b} \right)^g \right\} \exp \left\{ \frac{(a\bar{X} + b) - (a\bar{x} + b)}{(a\bar{X} + b) + (a\bar{x} + b)} \right\}, \quad (3.1)$$

where w_1 and w_2 are scalars, whereas the constant g assumes real values to generate various estimators. Also, (a, b) are either real values or function of available parameters of supplementary variable x , namely, variation coefficient (C_x), skewness coefficient ($\beta_1(x)$), kurtosis coefficient ($\beta_2(x)$), mean ($\bar{X}$), standard deviation (S_x), median (M_x), etc.

It is possible to describe the suggested estimator t_a as a function of e' s as

$$t_a - \bar{Y} = \bar{Y} \left[w_1 \left\{ 1 + e_0 - \frac{\nu}{2}e_1 - \frac{\nu}{2}e_0e_1 + \frac{3}{8}\nu^2e_1^2 \right\} + w_2 \left\{ 1 + e_0 - \nu ge_1 - \frac{\nu}{2}e_1 + \frac{g(g+1)}{2}\nu^2e_1^2 \right\} - 1 \right], \quad (3.2)$$

where $\nu = a\bar{X}/(a\bar{X} + b)^{-1}$.

The suggested estimator's MSE up to first degree approximation may be obtained by squaring both sides of (3.2).

$$\begin{aligned} MSE(t_a) &= \\ &= \bar{Y}^2 \left[1 + w_1^2 \left\{ 1 + fC_y^2 + \frac{5}{4}\nu^2fC_x^2 - 2\nu f\rho_{xy}C_yC_x \right\} + w_2^2 \left\{ 1 + fC_y^2 + (2\nu^2g^2 + \nu^2g + \nu^2)fC_x^2 - (2\nu + 4\nu g)f\rho_{xy}C_yC_x \right\} \right. \\ &\quad + 2w_1w_2 \left\{ 1 + fC_y^2 + \left\{ \frac{g(g+1)}{2}\nu^2 + \nu^2 \right\} fC_x^2 - 2\nu(1 + g)f\rho_{xy}C_yC_x \right\} \\ &\quad - 2w_1 \left(1 + \frac{3}{8}\nu^2fC_x^2 - \frac{\nu}{2}f\rho_{xy}C_yC_x \right) \\ &\quad \left. - 2w_2 \left\{ 1 + \left\{ \frac{g(g+1)}{2}\nu^2 + \frac{3}{8}\nu^2 - \frac{1}{2}g\nu^2 \right\} fC_x^2 - \left(\frac{\nu}{2} + \nu g \right) f\rho_{xy}C_yC_x \right\} \right] \\ &= \bar{Y}^2 [1 + w_1^2 L_1 + w_2^2 L_2 + 2w_1w_2 L_3 - 2w_1 L_4 - 2w_2 L_5], \end{aligned} \quad (3.3)$$

where

$$L_1 = 1 + fC_y^2 + \frac{5}{4}\nu^2fC_x^2 - 2\nu f\rho_{xy}C_yC_x,$$

$$L_2 = 1 + fC_y^2 + (2\nu^2g^2 + \nu^2g + \nu^2)fC_x^2 - (2\nu + 4\nu g)f\rho_{xy}C_yC_x,$$

$$L_3 = 1 + fC_y^2 + \left\{ \frac{g(g+1)}{2}\nu^2 + \nu^2 \right\} fC_x^2 - 2\nu(1 + g)f\rho_{xy}C_yC_x,$$

$$L_4 = 1 + \frac{3}{8}\nu^2fC_x^2 - \frac{\nu}{2}f\rho_{xy}C_yC_x,$$

$$L_5 = 1 + \left\{ \frac{g(g+1)}{2}\nu^2 + \frac{3}{8}\nu^2 - \frac{1}{2}g\nu^2 \right\} fC_x^2 - \left(\frac{\nu}{2} + \nu g \right) f\rho_{xy}C_yC_x.$$

One may minimize (3.3) to get the optimal values of w_1 and w_2 as

$$w_{1(opt)} = \frac{(L_2 L_4 - L_3 L_5)}{(L_1 L_2 - L_3^2)},$$

$$w_{2(opt)} = \frac{(L_1 L_5 - L_3 L_4)}{(L_1 L_2 - L_3^2)}.$$

The minimum MSE may be obtained by putting optimizing scalars $w_{1(opt)}$ and $w_{2(opt)}$ in (3.3) as

$$\min.MSE(t_a) = \bar{Y}^2 \left[1 - \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)} \right]. \quad (3.4)$$

3.1. Few individuals of the estimators t_a

It is important to note that the class of estimators are generalized form of class of estimators for several values of a and b which are either actual numbers or the function of the available values of variable x . Utilizing these values, a wide range of estimators can be rendered. Few estimators of them are enlisted in Table 1.

4. Efficiency conditions

The efficiency conditions are determined by likening the MSE of the suggested estimators with the MSE of the existing estimators.

$$MSE(t_a) < V(t_m) \implies \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)} > 1 - fC_y^2$$

$$MSE(t_a) < MSE(t_r) \implies \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f(C_y^2 + C_x^2 - 2\rho_{xy}C_x C_y)$$

Table 1Few individuals of the estimators t_a .

Members of t_a	a	b
$t_{a_1} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{\bar{x}}{\bar{x}} \right)^g \right] \exp \left(\frac{\bar{x} - \bar{x}}{\bar{x} + \bar{x}} \right)$	1	0
$t_{a_2} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{\bar{x} + \beta_2(x)}{\bar{x} + \beta_2(x)} \right)^g \right] \exp \left(\frac{\bar{x} - \bar{x}}{\bar{x} + \bar{x} + 2\beta_2(x)} \right)$	1	$\beta_2(x)$
$t_{a_3} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{\bar{x} + C_x}{\bar{x} + C_x} \right)^g \right] \exp \left(\frac{\bar{x} - \bar{x}}{\bar{x} + \bar{x} + 2C_x} \right)$	1	C_x
$t_{a_4} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{C_x \bar{x} + \beta_2(x)}{C_x \bar{x} + \beta_2(x)} \right)^g \right] \exp \left\{ \frac{C_x(\bar{x} - \bar{x})}{C_x(\bar{x} + \bar{x}) + 2\beta_2(x)} \right\}$	C_x	$\beta_2(x)$
$t_{a_5} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{\beta_2(x) \bar{x} + C_x}{\beta_2(x) \bar{x} + C_x} \right)^g \right] \exp \left\{ \frac{\beta_2(x)(\bar{x} - \bar{x})}{\beta_2(x)(\bar{x} + \bar{x}) + 2C_x} \right\}$	$\beta_2(x)$	C_x
$t_{a_6} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{\bar{x} + C_x}{\bar{x} + C_x} \right)^g \right] \exp \left(\frac{\bar{x} - \bar{x}}{\bar{x} + \bar{x} + 2\rho_{xy}} \right)$	1	ρ_{xy}
$t_{a_7} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{C_x \bar{x} + \rho_{xy}}{C_x \bar{x} + \rho_{xy}} \right)^g \right] \exp \left\{ \frac{C_x(\bar{x} - \bar{x})}{C_x(\bar{x} + \bar{x}) + 2\rho_{xy}} \right\}$	C_x	ρ_{xy}
$t_{a_8} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{\rho_{xy} \bar{x} + C_x}{\rho_{xy} \bar{x} + C_x} \right)^g \right] \exp \left\{ \frac{\rho_{xy}(\bar{x} - \bar{x})}{\rho_{xy}(\bar{x} + \bar{x}) + 2C_x} \right\}$	ρ_{xy}	C_x
$t_{a_9} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{\beta_2(x) \bar{x} + \rho_{xy}}{\beta_2(x) \bar{x} + \rho_{xy}} \right)^g \right] \exp \left\{ \frac{\beta_2(x)(\bar{x} - \bar{x})}{\beta_2(x)(\bar{x} + \bar{x}) + 2\rho_{xy}} \right\}$	$\beta_2(x)$	ρ_{xy}
$t_{a_{10}} = \left[w_1 \bar{y} + w_2 \bar{y} \left(\frac{\rho_{xy} \bar{x} + \beta_2(x)}{\rho_{xy} \bar{x} + \beta_2(x)} \right)^g \right] \exp \left\{ \frac{\rho_{xy}(\bar{x} - \bar{x})}{\rho_{xy}(\bar{x} + \bar{x}) + 2\beta_2(x)} \right\}$	ρ_{xy}	$\beta_2(x)$

$$MSE(t_a) < MSE(t_{lr}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f C_y^2 (1 - \rho_{xy}^2)$$

$$MSE(t_a) < MSE(t_{sd}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f(C_y^2 + \delta_1^2 C_x^2 - 2\delta_1 \rho_{xy} C_x C_y)$$

$$MSE(t_a) < MSE(t_{bt}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f \left(C_y^2 + \frac{1}{4} C_x^2 - \rho_{xy} C_x C_y \right)$$

$$MSE(t_a) < MSE(t_{us1}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f(C_y^2 + \delta_2^2 C_x^2 - 2\delta_2 \rho_{xy} C_x C_y)$$

$$MSE(t_a) < MSE(t_{us2}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f(C_y^2 + \delta_3^2 C_x^2 - 2\delta_3 \rho_{xy} C_x C_y)$$

$$MSE(t_a) < MSE(t_{st}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f(C_y^2 + \delta_4^2 C_x^2 - 2\delta_4 \rho_{xy} C_x C_y)$$

$$MSE(t_a) < MSE(t_s) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f(C_y^2 + \delta_5^2 C_x^2 - 2\delta_5 \rho_{xy} C_x C_y)$$

$$MSE(t_a) < MSE(t_{sn}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f(C_y^2 + \delta_6^2 C_x^2 - 2\delta_6 \rho_{xy} C_x C_y)$$

$$MSE(t_a) < MSE(t_k) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f C_y^2 (1 - \rho_{xy}^2)$$

$$MSE(t_a) < MSE(t_{y_i}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f(C_y^2 + \Delta_i^2 C_x^2 - 2\Delta_i \rho_{xy} C_x C_y)$$

$$MSE(t_a) < MSE(t_{by}) \Rightarrow \frac{(L_1 L_5^2 + L_2 L_4^2 - 2L_3 L_4 L_5)}{(L_1 L_2 - L_3^2)}$$

$$> 1 - f C_y^2 (1 - \rho_{xy}^2)$$

The suggested estimators will be efficient likened to the ordinary estimators under these conditions.

5. Simulation study

To enhance the mathematical results, a simulation study is performed over the artificially generated symmetric and asymmetric populations. Following [20], we considered the models given in (5.1) and (5.2) as:

$$y = 3.3 + \sqrt{(1 - \rho_{xy}^2)} y^* + \rho_{xy} \left(\frac{S_y}{S_x} \right) x^*, \quad (5.1)$$

$$x = 3.1 + x^*, \quad (5.2)$$

where, for the appropriate distributions, x^* and y^* are independent variables. Adapting these models, we have generated the below mentioned populations:

- (1). Assuming $x^* \sim N(10, 85)$ and $y^* \sim N(15, 74)$, a normal population of $N = 1000$ units is generated with a fluctuating correlation coefficient $\rho_{xy} = 0.3, 0.5, 0.7, 0.9$.
- (2). Assuming $x^* \sim G(0.04, 0.005)$ and $y^* \sim G(0.09, 0.007)$, a gamma population of $N = 1000$ units is generated with a fluctuating correlation coefficient $\rho_{xy} = 0.3, 0.5, 0.7, 0.9$.

We have drawn $n = 100$ units utilizing SRSWOR from both of these populations. With 10,000 iterations, MSE and PRE have been calculated utilizing the expressions given in (5.3) and (5.4), respectively:

$$MSE(T_i) = \frac{1}{10,000} \sum_{i=1}^{10,000} (T_i - \bar{Y})^2, \quad (5.3)$$

$$PRE = \frac{V(t_m)}{MSE(T_i)} \times 100, \quad (5.4)$$

where T_i represents both the suggested and current estimators. The outcomes of the simulated normal and gamma populations are reported, respectively, in Table 2 and Table 3.

5.1. Key results of simulation study

The key results are interpreted in the following points:

- Table 2 contains the results obtained from simulated normal population. These results show that the members t_{a_i} , $i = 1, 2, \dots, 10$ of the suggested estimators t_a are more efficient than the basic mean estimator t_m , basic ratio estimator t_r , basic regression estimator t_{lr} , and the estimators t_{sd} , t_{bt} , t_{up1} , t_{up2} , t_s , t_{sn} , t_k , t_{y_i} , $i = 1, 2, \dots, 9$, and t_{by} suggested by the references [2], [3], [4], [5], [17], [18], [9], [10], and [19], respectively, with least MSE and highest PRE.
- Additionally, according to the findings of Table 2, when ρ_{xy} grows, the MSE and PRE of the individuals t_{a_i} , $i = 1, 2, \dots, 10$ of the suggested estimators t_a decrease and increase.
- Table 3 contains the results calculated from simulated gamma population. These results show that the members t_{a_i} , $i = 1, 2, \dots, 10$ of the suggested estimators t_a are more efficient than the basic mean estimator t_m , basic ratio estimator t_r , basic regression estimator t_{lr} , and the estimators t_{sd} , t_{bt} , t_{up1} , t_{up2} , t_s , t_{sn} , t_k , t_{y_i} , $i = 1, 2, \dots, 9$, and t_{by} suggested by the references [2], [3], [4], [5], [17], [18], [9], [10], and [19], respectively, with least MSE and highest PRE.
- Additionally, according to the results of Table 3, when ρ_{xy} grows, the MSE and PRE of the individuals t_{a_i} , $i = 1, 2, \dots, 10$ of the suggested estimators t_a decrease and increase.

Table 2
MSE and PRE of estimators for simulated normal population.

ρ_{xy}	0.3		0.5		0.7		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	0.0079	100.00	0.0079	100.00	0.0079	100.00	0.0079	100.00
t_r	0.0077	102.19	0.0060	130.77	0.0043	182.60	0.0021	364.38
t_{lr}	0.0073	108.02	0.0060	130.78	0.0040	194.31	0.0014	537.55
t_{sd}	0.0075	104.73	0.0060	130.40	0.0045	174.03	0.0027	293.00
t_{bt}	0.0073	107.97	0.0065	121.77	0.0056	141.34	0.0043	183.23
t_{us_1}	0.0074	106.41	0.0069	114.39	0.0064	123.62	0.0059	134.18
t_{us_2}	0.0074	106.44	0.0069	114.39	0.0063	124.92	0.0055	144.22
t_{st}	0.0075	104.71	0.0060	130.40	0.0045	174.46	0.0026	304.85
t_s	0.0075	104.71	0.0060	130.40	0.0045	174.46	0.0026	304.85
t_{sn}	0.0073	107.97	0.0065	121.77	0.0056	141.34	0.0043	183.23
t_k	0.0073	108.02	0.0060	130.78	0.0040	194.31	0.0014	537.55
t_{y_1}	0.0077	102.20	0.0060	130.77	0.0043	182.57	0.0021	364.07
t_{y_2}	0.0077	102.21	0.0060	130.77	0.0043	182.51	0.0021	363.23
t_{y_3}	0.0077	101.99	0.0060	130.74	0.0043	184.08	0.0021	377.65
t_{y_4}	0.0078	101.52	0.0060	130.60	0.0043	187.26	0.0018	419.08
t_{y_5}	0.0077	102.20	0.0060	130.77	0.0043	182.59	0.0021	364.16
t_{y_6}	0.0077	102.23	0.0060	130.77	0.0043	182.52	0.0021	363.47
t_{y_7}	0.0077	102.20	0.0060	130.77	0.0043	182.54	0.0021	363.56
t_{y_8}	0.0077	102.31	0.0060	130.77	0.0043	182.48	0.0021	363.38
t_{y_9}	0.0077	102.19	0.0060	130.77	0.0043	182.60	0.0021	364.32
t_{by}	0.0073	108.02	0.0060	130.78	0.0040	194.31	0.0014	537.55
t_{a_1}	0.0055	143.57	0.0049	158.94	0.0039	203.10	0.0009	823.88
t_{a_2}	0.0055	143.88	0.0049	158.72	0.0039	202.11	0.0008	947.30
t_{a_3}	0.0055	143.88	0.0049	158.72	0.0039	202.04	0.0008	970.31
t_{a_4}	0.0055	143.69	0.0049	158.89	0.0039	202.86	0.0009	861.64
t_{a_5}	0.0055	143.69	0.0049	158.89	0.0039	202.83	0.0008	882.05
t_{a_6}	0.0055	143.60	0.0049	158.92	0.0039	202.94	0.0009	866.47
t_{a_7}	0.0055	143.58	0.0049	158.93	0.0039	203.06	0.0009	835.60
t_{a_8}	0.0055	144.11	0.0050	158.34	0.0039	201.42	0.0008	980.99
t_{a_9}	0.0055	143.58	0.0049	158.93	0.0040	197.95	0.0009	838.57
$t_{a_{10}}$	0.0055	144.11	0.0050	158.34	0.0039	201.53	0.0008	957.43

Table 3
MSE and PRE of estimators for simulated normal population.

ρ_{xy}	0.3		0.5		0.7		0.9	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	0.0021	100.00	0.0021	100.00	0.0021	100.00	0.0021	100.00
t_r	0.0021	99.36	0.0018	119.54	0.0013	155.25	0.0007	276.01
t_{lr}	0.0020	105.27	0.0018	119.89	0.0013	160.44	0.0006	304.80
t_{sd}	0.0021	101.25	0.0018	119.88	0.0014	151.36	0.0009	239.64
t_{bt}	0.0021	105.27	0.0018	119.44	0.0016	130.38	0.0013	164.72
t_{us_1}	0.0021	103.04	0.0020	107.09	0.0019	111.89	0.0019	112.49
t_{us_2}	0.0021	100.26	0.0021	100.63	0.0021	101.07	0.0021	101.07
t_{st}	0.0020	103.71	0.0019	109.33	0.0018	116.42	0.0018	116.23
t_s	0.0020	103.71	0.0019	109.33	0.0018	116.42	0.0018	116.23
t_{sn}	0.0020	105.27	0.0018	115.44	0.0016	130.38	0.0013	164.72
t_k	0.0020	105.27	0.0018	119.89	0.0013	160.44	0.0006	304.80
t_{y_1}	0.0021	99.36	0.0018	119.54	0.0013	155.24	0.0007	275.96
t_{y_2}	0.0021	99.36	0.0018	119.55	0.0013	155.23	0.0007	275.88
t_{y_3}	0.0021	99.37	0.0018	119.55	0.0013	155.20	0.0007	275.50
t_{y_4}	0.0021	99.38	0.0018	119.56	0.0013	155.13	0.0007	274.61
t_{y_5}	0.0021	99.36	0.0018	119.55	0.0013	155.24	0.0007	275.87
t_{y_6}	0.0021	99.38	0.0018	119.55	0.0013	155.21	0.0007	275.58
t_{y_7}	0.0021	99.36	0.0018	119.55	0.0013	155.22	0.0007	275.64
t_{y_8}	0.0021	99.46	0.0018	119.56	0.0013	155.18	0.0007	275.50
t_{y_9}	0.0021	99.36	0.0018	119.54	0.0013	155.24	0.0007	275.96
t_{by}	0.0020	105.27	0.0018	119.89	0.0013	160.44	0.0006	304.80
t_{a_1}	0.0017	127.14	0.0015	139.68	0.0012	171.49	0.0005	397.21
t_{a_2}	0.0016	128.56	0.0015	137.27	0.0012	173.55	0.0006	350.45
t_{a_3}	0.0016	127.72	0.0015	139.92	0.0012	171.12	0.0005	395.68
t_{a_4}	0.0016	128.96	0.0015	138.83	0.0012	172.12	0.0006	320.37
t_{a_5}	0.0017	127.15	0.0015	139.69	0.0012	171.48	0.0005	397.26
t_{a_6}	0.0016	127.21	0.0015	139.74	0.0012	171.43	0.0005	397.78
t_{a_7}	0.0017	127.17	0.0015	139.71	0.0012	171.46	0.0005	397.56
t_{a_8}	0.0016	128.68	0.0015	139.99	0.0012	170.75	0.0005	394.70
t_{a_9}	0.0017	127.14	0.0015	139.68	0.0012	171.49	0.0005	397.23
$t_{a_{10}}$	0.0016	127.79	0.0015	135.71	0.0012	175.06	0.0005	381.42

Table 4
MSE and PRE of estimators for real data sets.

ρ_{xy} Estimators	Data set 1		Data set 2		Data set 3		Data set 4	
	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_m	285549	100.00	1376058	100.00	272	100.00	2523	100.00
t_r	140440	203.32	613176	224.41	319	85.47	156	1612.72
t_{lr}	140345	203.46	460606	298.74	259	105.20	58	4358.36
t_{sd}	140441	203.32	613212	224.40	319	85.48	157	1607.32
t_{bt}	178535	159.93	914474	150.47	263	103.56	945	266.91
t_{us_1}	167563	170.41	918088	149.88	261	104.2	67	3742.37
t_{us_2}	276202	103.38	1337022	102.91	268	101.52	1339	188.39
t_{st}	140441	203.32	613191	224.40	319	85.48	157	1605.61
t_s	140449	203.31	613674	224.23	318	85.54	158	1592.25
t_{sn}	178535	159.93	914474	150.47	263	103.56	945	266.91
t_{y_1}	140440	203.32	613176	224.41	319	85.47	156	1612.72
t_{y_2}	140440	203.32	613176	224.41	319	85.47	156	1612.72
t_{y_3}	140440	203.32	613176	224.41	319	85.47	156	1612.71
t_{y_4}	140440	203.32	613176	224.41	319	85.47	156	1612.72
t_{y_5}	140440	203.32	613176	224.41	319	85.47	156	1612.48
t_{y_6}	140440	203.32	613177	224.41	319	85.47	156	1612.54
t_{y_7}	140440	203.32	613177	224.41	319	85.47	156	1612.54
t_{y_8}	140440	203.32	613177	224.41	319	85.48	156	1612.53
t_{y_9}	140440	203.32	613176	224.41	319	85.47	156	1612.40
t_{d_1}	139167	205.18	450309	305.58	233	116.97	57	4415.97
t_{d_2}	139147	205.21	450299	305.58	233	117.00	56	4471.19
t_{d_3}	139166	205.18	450308	305.58	233	116.98	56	4430.39
t_{d_4}	139155	205.20	450304	305.58	233	116.98	56	4489.61
t_{d_5}	139167	205.18	450309	305.58	233	116.97	57	4421.00
t_{d_6}	139167	205.18	450308	305.58	233	116.97	56	4434.97
t_{d_7}	139167	205.18	450309	305.58	233	116.97	56	4441.29
t_{d_8}	139166	205.18	450308	305.58	233	116.99	56	4430.56
t_{d_9}	139167	205.18	450309	305.58	233	116.97	57	4422.59
$t_{d_{10}}$	139138	205.22	450296	305.58	233	117.08	56	4471.84

6. Applications

Some real data sets are delineated below that are utilized to exemplify the uses of the suggested estimators.

- (1). The **data set 1** is obtained from ref. [21], which includes the quantity of apples produced (y) and apple trees (x) in Turkey's Black Sea region. The necessary summary statistics are as follows: $N = 204$, $n = 20$, $\bar{X} = 26441.72$, $\bar{Y} = 966.95$, $M_x = 13650$, $C_x = 1.71$, $C_y = 2.47$, and $\rho_{xy} = 0.71$.
- (2). The **data set 2** is obtained from ref. [21], which includes the quantity of apples produced (y) and apple trees (x) in Turkey's Marmara area. The necessary summary statistics are as follows: $N = 106$, $n = 30$, $\bar{X} = 24375.59$, $\bar{Y} = 1536.77$, $M_x = 8600$, $C_x = 2.01$, $C_y = 4.18$, and $\rho_{xy} = 0.81$.
- (3). The **data set 3** is chosen from ref. [22], page number 663-665, which is consisting of the population (in million) of 1983 (y) and import (in millions of U.S. dollars) during 1983 (x) of the CO124 population of 124 countries. The necessary summary statistics are as follows: $N = 124$, $n = 50$, $\bar{X} = 14276.03$, $\bar{Y} = 36.65$, $M_x = 2291$, $C_x = 2.20$, $C_y = 3.18$, and $\rho_{xy} = 0.22$.
- (4). The **data set 4** is selected from ref. [23], page number 228, which is consisting of the number of workers (y) and fixed capital (x) for eighty factories in an area. The necessary summary statistics are as follows: $N = 80$, $n = 29$, $\bar{X} = 1126.46$, $\bar{Y} = 285$, $M_x = 757.50$, $C_x = 0.75$, $C_y = 0.94$, and $\rho_{xy} = 0.98$.

For these data sets, the results are calculated in terms of MSE and PRE. The PRE of the estimators is calculated against the basic mean estimator t_m utilizing the expression given in (6.1).

$$PRE = \frac{MSE(t_m)}{MSE(T)} \times 100. \quad (6.1)$$

The findings for each data set are shown in Table 4, which indicates how the member specified in Table 1 of the suggested estimators t_a outperforms the current estimators.

7. Conclusion

We have suggested an efficient class of population mean estimators utilizing SRS in the current study. We have determined the MSE of the suggested class of estimators. The efficiency conditions are generated by likening the MSEs of the suggested and the ordinary estimators. The efficiency conditions are further examined with a comprehensive simulation study. Certain popular real data sets are also analyzed to demonstrate the execution of the suggested estimators. The numerical results obtained from simulation study and real data illustration confirmed that the suggested estimators dominates the existing estimators with least MSE and highest PRE. Therefore, survey experts may take into consideration the suggested estimators for mean estimate for real-world issues.

The best possible future extensions of the suggested estimators are as follows:

- The suggested estimators may be examined for the mean estimation utilizing stratified random sampling, for more details, see [24], [25], [26], [27], [28], [29], and [30].
- The suggested estimators may be examined for the mean estimation utilizing ranked set sampling (RSS), for more details, see [31], [32], [33].
- The suggested estimators may be examined for the mean estimation utilizing stratified RSS, for more details, see [34].
- The suggested estimators may be examined for the mean estimation utilizing median RSS and double median RSS, for more details, see [35], [36], [37], and [38].
- The proposed estimators may be examined for the mean estimation utilizing adaptive cluster sampling, for more details, see [39] and [40].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

Data availability

No potential competing interest was reported by the authors.

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