

Full length article

A generalized solution for anisotropic compact star model in $F(Q)$ gravityJitendra Kumar^a, Sat Paul^{a,*}, S.K. Maurya^{b,c,*}, Sourav Chaudhary^a, Sweeti Kiroriwal^a^a Department of Mathematics, Central University of Haryana, Jant-Pali, Mahendergarh 123029, Haryana, India^b Department of Mathematical and Physical Sciences, College of Arts and Sciences, University of Nizwa, P.O. Box 33, Nizwa 616, Sultanate of Oman^c Research Center of Astrophysics and Cosmology, Khazar University, Baku, AZ 1096, 41 Mehseti Street, Azerbaijan

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ABSTRACT

In this work, we investigate an anisotropic compact star's physical properties and stability in $F(Q)$ gravity. The study focuses on the significance of $F(Q)$ gravity on the structure and stability of compact star, considering non-perfect fluid. Buchdahl ansatz along with transformation used to solve the Einstein field equations. We investigate the physical parameters of the 4U1820-30 compact star using a static spherical metric in the interior region and a Schwarzschild (anti) de-sitter metric in the exterior region. We investigate the behaviour of energy density(ρ), radial pressure(p_r), tangential pressure(p_t), anisotropy(Δ), metric potentials, energy state parameter and energy requirements in the interior of the proposed stellar object. The equilibrium state of this star is analysed using the Tolman–Oppenheimer–Volkoff(TOV) equation and their stability is determined using the Necessary and physical existence requirements, causality condition, Harrison-Zeldovich-Novikov condition, the adiabatic index(Γ) method and Herrera cracking method.

1. Introduction

Most galaxies have an erratic distribution of stars formed in clouds of gas and dust with an uneven matter distribution. All active stars ultimately come to a stage in their evolution where the constant gravitational pull is no longer resisted by the radiation pressure originating from the nuclear fusions occurring within it. According to [1], stellar death occurs when a star collapses under its weight. This process produces an extremely dense and compact stellar remnant, known as a compact star. Particularly, compact stars are the last phases of the evolution of ordinary stars including white dwarfs, neutron stars, black holes and quark stars. The evolution of compact stars is related to the phase separations that occurred in the early universe. Compact stars are identified by the stop of nuclear processes in their interiors, in addition to their extreme density. This is the reason they are unable to resist the force of gravity. Gravity in white dwarfs and neutron stars is resisted by the pressure of degenerate gases. In black holes, gravity dominates over other forces, compressing star material to infinite density [2]. The outer layers are expelled and when a star like the Sun runs out of its nuclear fuel, the core collapses to produce a compact and white star known as the white dwarf. When inert iron cores of stars with masses equal to or greater than ten times that of the Sun collapse, they form an extremely dense neutron star or even a black hole. Gravastars, theoretical substitutes for black holes, have intrigued astrophysicists because of their distinctive characteristics. The exact solution of an innovative gravastar model derived from the Mazur–Mottola (2004) approach within the

context of general relativity, particularly through incorporating the cloud of strings and quintessence in paper [3] Physicists began utilizing stars as “Physics Laboratories” in the mid-1920s to study how matter transforms under tremendous pressures and densities a phenomenon that is not possible to observe on Earth. In 1917 Bonolis provided details on how the concept of compact stars evolved. Oppenheimer initiated the initial conversation regarding the potential for gravitational collapse. In 1915, white dwarfs offered a novel means of testing Einstein's theory of general relativity (GR) [4], but the neutron star hypothesis, when combined with observations of supernovae, produced an even more extensive test of the theory. Eddington went into great depth about white dwarfs in 1924. Additionally, Friedrich divided the ultra-relativistic electron gas in the very dense stars into two groups: non-relativistic and ultra-relativistic. He made an accurate estimate of the mass of a stable star, which is limited to a specific maximum $M(\geq M_\odot)$ and exists in a relativistic degenerate form. Some basic equations have been established to regulate the configuration of a star in radiative equilibrium. In 1926, Fowler [5] explored the issue of degenerate dense matter in white dwarf stars. Oppenheimer used a variety of theoretical frameworks to contribute to the relativistic gravitational collapse of a neutron star towards the end of the 1930s. These studies initiated a new line of inquiry into the relationship between GR and compact objects. In 1926, Fowler discovered that the connection between white dwarfs' density, energy and temperature

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could be determined by studying them as a gas of non-relativistic, non-interacting nuclei that follow Fermi–Dirac statistics [5,6]. In 1929 stoner utilized the Fermi gas model to determine the mass, radius, and density of white dwarf stars, assuming they were homogeneous spheres of electrons. Wilhelm Anderson modified this model using relativity and proposed a theory in December 1928 that said “the mass of a star must have a maximum value” [7]. Stoner deduced the internal energy density equation of state for Fermi gases in 1930 and he was lastly able to approach the mass–radius equation in a completely relativistic manner as a result, and he came to the conclusion that its limiting mass is around 2.19×10^{30} kg [8]. Chandrasekhar published many papers [9–14] between 1931 and 1935. In these publications, he solved both the hydrostatic and nonrelativistic Fermi gas equations of state. Additionally, he contributed to the relativistic Fermi gas scenario, which resulted in the value of the first predetermined boundary. The Chandrasekhar limit is the name given to this new limit. The prevailing consensus regarding the Chandrasekhar 1.4 solar masses is the upper limit [15]. In 1938 Zwicky discovered the idea of neutron star [16,17] and made a fresh effort using GR in this direction. He established a more strong link between GR and the idea that the mass of a star of a given density cannot exceed a particular crucial value known as the Schwarzschild limit [18]. He presented his ideas and critiqued the Schwarzschild solution [19]. In his conclusion, he states that any star that reaches the Schwarzschild limiting configuration must be viewed as an object that effectively prevents physical connection with the rest of the universe. As a result, it is difficult to see any physical states in stellar bodies that have crossed the Schwarzschild limit [18]. By accounting for the effects of nuclear forces [20], we were able to arrive at a fair estimate of the lowest mass for which such an interior would be stable (about $0.1 M_{\odot}$). In 1939, Oppenheimer and Volkoff discovered that if the total mass of neutrons exceeds around $0.7 M_{\odot}$, the general relativistic field equations cannot offer a static solution for a spherical distribution of cold neutrons. Oppenheimer [21] proved that a star under these conditions would collapse due to its gravitational field. The TOV limit is the value that sets a limit on the mass of a cool, non-rotating neutron star and this limit was comparable to the white dwarf star mass at the Chandrasekhar limit. Given the high nuclear repulsion forces that exist between neutrons, current research suggests that this limit may be anywhere from 1.5 and $3 M_{\odot}$ [22]. Nonetheless, it was observed that rotation could raise the maximum mass [23–26]. Scientists Snyder and Oppenheimer had studied gravitational collapse. The whole ramifications of Einstein’s theory of gravitation are included in this work [27]. A small group of scientists had begun working on the issue of what would happen to a compact star interior composed only of neutrons by 1939. Due to these contributions, it became clear that only Einstein’s theory of gravity could account for such extraordinarily dense materials, while on the one hand, nuclear matter and particle physics were becoming increasingly important in characterizing matter at such high densities. The science of relativistic astrophysics becomes understandable. Though revolutionary, GR has faced difficulties in understanding the Universe’s late-time acceleration, the existence of dark matter, and the quantization of gravitational interaction. The existence of dark matter and the quantization of gravitational interaction are two of the many unresolved problems that must be addressed to describe the Universe’s late-time acceleration. There are disadvantages to using the affine connection in GR. This discrepancy in the study can be explained by selecting different affine connections with various gravity descriptions [28–30]. This implies alternate theories of gravity, such as the modified theory of gravity. Some modified gravity theories, like $F(R)$ gravity, $F(R, T)$ gravity, $F(Q)$ gravity, teleparallel gravity, and uni modular gravity so on. Modified metrics can reveal different circumstances, which can be explored further in future research [31–38].

One of the more plausible ideas to address the shortcomings of GR is the $F(R)$ theory. Scalar curvature in the Lagrangian can be substituted to provide $F(R)$ gravity. The usefulness of this idea in tackling dark energy is demonstrated in various research paper [39–41]. Similar to how $F(T)$ gravity can be created, the affine connection in GR allows the requirement that the curvature (R) can be non zero while allowing the values of nonmetricity (Q) and torsion (T) to vanish. It is possible to build a wide range of theories and geometries with Q , R , and T in many mergers, such as zero or non zero. As was previously mentioned, when we choose zero curvature and nonmetricity in the case of $F(T)$ gravity, we have a symmetric teleparallel formation (STGR). Various studies in the field of (STGR) have been conducted and are included in these papers [42–54]. An arbitrary function of Q is contained in the gravitational Lagrangian to create $F(Q)$ gravity.

In the present study, the main focus was on $F(Q)$ gravity. The modified gravity provides a straightforward composition of the classical GR, which contributes to improving inertial gravitational interaction, without taking into accounting for the affine space–time structure. The special feature of $F(Q)$ gravity is its capacity to interpret the Universe’s late-time acceleration without demanding alterations to scalar fields. [48,55,56]. The viability of $F(Q)$ gravity model has been proved in a wide range of domains, including wormhole geometry [57–63] and dynamical analysis [64], which covers the cosmological aspects [65] and has been presented in relation to $F(Q)$ gravity. The mathematical representation of $F(Q) = Q + mQ^n$ has been used to investigate holographic dark energy [66] which reveals the behaviour of the deceleration parameter changes from negative to positive, indicating that the universe is transitioning from an early phase of deceleration to the current phase of acceleration. In the framework of extended non-metric theories of gravity, Capozziello et al. [67] explored the possibility of cosmological slow-roll inflation. Specifically, they found that standard conformal structure is recovered in a high-energy regime while it breaks in the weak field limit, as demonstrated by generic functions $F(Q)$, particularly $F(Q) = \alpha Q + \beta Q^m$. Regardless of the extensively accepted, the teleparallel version of GR shares a formulation with both STGR and GR [68]. The primary contrast lies in that gravity is allocated to the non-metricity (Q), whereas curvature and torsion are regarded as zero in STGR. Compatible with the non-metricity and zero value of the torsion restriction, a coincident gauge is always applicable [47]. This method is unique for the reason that it eliminates the affine connections. Nevertheless, the development of metrics will differ in different coordinate systems if we continue with the incident gauge [64] and treat the metric as the only fundamental variable in modified STGR theories, such as the $F(Q)$ theory. While research on extended gravity theories is still in its early stages, the symmetric teleparallel, or $F(Q)$, theory of gravity has been receiving a lot of attention. Due to these novel features of $F(Q)$ gravity, several studies have been conducted so far to examine various aspects of the cosmos, including cosmology, energy requirements, covariant formulation, spherical symmetric structure and signatures that highlight the significance of gravity. The idea behind working in the $F(Q)$ theory of gravity is that on comparing to the $F(R)$ theory, which involves differential equations of the fourth order, the field equations of $F(Q)$ gravity are of the second order [69], $F(Q)$ gravity extends the theoretical foundation for modelling compact stars by giving new approaches to examining their structure, stability and observational signature. It offers a promising path for addressing issues concerning severe astrophysical conditions and challenging the limits of our understanding of gravity, which is a significant advantage.

In a self-gravitating, non-perfect fluid system, the anisotropy factor $\Delta = (p_t - p_r)$, defines the dissimilarity in pressures, particularly the radial pressure (p_r) and the tangential pressure (p_t). The origins of anisotropy in high-density compact stars is an interesting field of study, as it connects directly to the extreme physics of these objects. The

origin of anisotropy in high-density compact stars is because of a strong gravitational field, phase transition, stellar rotation and magnetic field, Spinning effects, variations in viscosity or shear stresses within the star's matter can also lead to anisotropy. The intricate structure of a neutron star's crust and its interactions with the core also contribute to anisotropy. The value of anisotropy might be positive or negative, each with distinct implications. The positive anisotropy factor produces an outward force, resulting in the stellar system experiencing a repulsive anisotropic force that opposes the gravitational gradient. Conversely, when the anisotropy factor is negative, it produces an inward force, resulting in an attractive anisotropic force within the system. The value of anisotropy must be zero in the centre of the star structure. Anisotropy is vital in the study of compact star models because it strongly impacts the structure, stability, and observable aspects of these objects. Anisotropic fluids are more important than isotropic fluids in self-gravitating systems because they provide a more accurate representation of the many physical events, particularly under non-uniform conditions like rotation, magnetic fields, or relativistic effects. While isotropic fluids are easier to examine, they fail to include the complexity necessary for understanding several astrophysical and cosmic systems. Anisotropy affects the macroscopic features of a star, including mass, radius, tidal deformability and nonradial oscillation [70–81]. Herrera and Santos [82] thoroughly studied anisotropic fluid spheres.

Based on the preceding literature study, we will broaden our search for new physical solutions to Einstein's field equations for static and spherically symmetric anisotropic fluid distribution. The approach assumes an uncharged fluid sphere, highlighting the importance of anisotropy in modelling. We used metric potentials as generating functions and then calculated energy density and pressure. To compare with the observational data, we addressed the compact object 4U1820-30 in the structure of the model.

The present work has been designed as follows:

Section 2: we write the field equation of $F(Q)$ Gravity.

Section 3: we discuss Buchdahl metric.

Section 4: we discuss the behaviour of metric potential functions.

Section 5: we commenced our proposed model for an anisotropic star and obtained expression for energy density, radial pressure, tangential pressure and anisotropy factor.

Section 6: we discuss necessary and physical existence requirements in $F(Q)$ gravity for a well-behaved model.

Section 7: we discuss boundary conditions which are very important to calculating the value of constants.

Section 8: we analysed the model physically as well as graphically.

Section 9: we discuss the adiabatic index of the compact star.

Section 10: we discuss the stability of compact star through the Harrison–Zeldovich–Novikov condition.

Section 11: The research work is concluded.

2. Field equations of $F(Q)$ gravity

The Levi-Civita affine connection is crucial in explaining and analysing general relativity. Its solution on the space–time manifold is compatible with the metric. Different manifolds and affine connections can be employed and several theories for gravity can be offered [30,48]. The Levi-Civita connection highlights the importance of nonmetricity (Q) and torsion (T), with curvature. Relaxing these constraints allows for non-Riemannian geometry theories with non-vanishing curvature, torsion, and nonmetricity. The $F(Q)$ gravity model was developed using these conditions, resulting in a gravitational Lagrangian with an arbitrary nonmetric function (Q). The extension of $F(Q)$ gravity is extremely important in the expansion of the Universe. Jimenez et al. [48] was the first to coin the term symmetric teleparallel gravity, generally known as the $F(Q)$ gravity. Following [83] work on

$F(Q)$ gravity, we investigate a generic metric space–time with independent metric tensors $g_{\theta\nu}$ and $\Gamma_{\theta\nu}^\sigma$. The equation below demonstrates the nonmetricity of the aforementioned connection:

$$Q_{\alpha\theta\nu} = \nabla_\alpha g_{\theta\nu} = \partial_\alpha g_{\theta\nu} - \Gamma_{\alpha\theta}^\sigma g_{\sigma\nu} - \Gamma_{\alpha\nu}^\sigma g_{\theta\sigma}. \quad (1)$$

These three components, in their generic form, are obtained by disassembling this affine relationship into independent components:

$$\Gamma_{\theta\nu}^\sigma = \{\overset{\sigma}{\theta\nu}\} + \zeta_{\theta\nu}^\sigma + L_{\theta\nu}^\sigma, \quad (2)$$

where $\{\overset{\sigma}{\theta\nu}\}$ denotes the Levi-Civita connection, $\zeta_{\theta\nu}^\sigma$ denote the Contortion tensor and $L_{\theta\nu}^\sigma$ denotes the disformation. $\{\overset{\sigma}{\theta\nu}\}$ is solved by using the metric potential $g_{\theta\nu}$:

$$\{\overset{\sigma}{\theta\nu}\} \equiv \frac{1}{2} g^{\sigma\beta} (\partial_\theta g_{\beta\nu} + \partial_\nu g_{\beta\theta} - \partial_\beta g_{\theta\nu}), \quad (3)$$

$\zeta_{\theta\nu}^\sigma$ is written as:

$$\zeta_{\theta\nu}^\sigma \equiv \frac{1}{2} T^\sigma_{\theta\nu} + T_{(\theta}^{\sigma}{}_{\nu)}, \quad (4)$$

and $L_{\theta\nu}^\sigma$ is written as:

$$L_{\theta\nu}^\sigma \equiv \frac{1}{2} Q_{\theta\nu}^\sigma - Q_{(\theta}^{\sigma}{}_{\nu)}. \quad (5)$$

The nonmetricity conjugate is defined as:

$$H_{\theta\nu}^\alpha = -\frac{1}{4} Q_{\theta\nu}^\alpha + \frac{1}{2} Q_{(\theta\nu)}^\alpha + \frac{1}{4} (Q^\alpha - \tilde{Q}^\alpha) g_{\theta\nu} - \frac{1}{4} \delta_{(\theta\nu)}^\alpha. \quad (6)$$

The independent traces of the preceding equations are as follows:

$$Q_\alpha \equiv Q_{\alpha\theta}{}^\theta, \quad \tilde{Q}_\alpha \equiv \tilde{Q}_{\alpha\theta}{}^\theta, \quad (7)$$

now, the nonmetricity scalar is defined as:

$$Q = - Q_{\alpha\theta}{}^\theta H^{\alpha\theta\nu}. \quad (8)$$

The $F(Q)$ gravity is defined by the following action using Lagrange multipliers:

$$S = \int \sqrt{-g} d^4x \left[\frac{1}{2} F(Q) + \sigma_\alpha^{\beta\theta\nu} \mathcal{R}_{\beta\theta\nu}^\alpha + \sigma_\alpha^{\theta\nu} T_{\theta\nu}^\alpha + \mathcal{L}_n \right]. \quad (9)$$

In Eq. (9), g represents the determinant metric, $F(Q)$ denotes the nonmetricity of Q , $\mathcal{L}_n$ represents the Lagrange density, and $\sigma_\alpha^{\beta\theta\nu}$ represents the Lagrange multipliers. The action given in Eq. (9) has been used to derive the field equations concerning the metric given below

$$T_{\theta\nu} = \frac{2}{\sqrt{-g}} A_\alpha (\sqrt{-g} F_Q H_{\theta\nu}^\alpha) + \frac{1}{2} g_{\theta\nu} F + F_Q (H_{\theta\alpha\beta} Q_{\nu}^{\alpha\beta} - 2 Q_{\alpha\beta\theta} H_{\nu}^{\alpha\beta}). \quad (10)$$

In equation Eq. (10), the subscript Q denote the derivative of the function $F(Q)$ with respect to Q , the energy momentum tensor is defined as:

$$T_{\theta\nu} \equiv \frac{2}{\sqrt{g}} \frac{\delta(\sqrt{-g}) \mathcal{L}_n}{\delta g^{\theta\nu}}. \quad (11)$$

we get the following equations when we change Eq. (9) concerning affine connection:

$$\nabla_\rho \sigma_\alpha^{\nu\theta\rho} + \sigma_\alpha^{\theta\nu} = \sqrt{-g} F_Q H_{\theta\nu}^\alpha + J_\alpha^{\theta\nu}. \quad (12)$$

The hyper momentum tensor density is represented by the following equation:

$$J_\alpha^{\theta\nu} = \frac{-1}{2} \frac{\delta \mathcal{L}_n}{\delta T_{\theta\nu}^\alpha}. \quad (13)$$

Now, we reduce the equation Eq. (12) to the following equation by taking into account the anti-symmetry property of ϑ and v in the Lagrangian multiplier coefficients:

$$\nabla_{\vartheta} \nabla_v (\sqrt{-g} F_Q H^{\vartheta v}_{\alpha} + J^{\vartheta v}_{\alpha}) = 0. \quad (14)$$

if we consider $\nabla_{\vartheta} \nabla_v J^{\vartheta v}_{\alpha} = 0$ we get :

$$\nabla_{\vartheta} \nabla_v (\sqrt{-g} F_Q H^{\vartheta v}_{\alpha}) = 0. \quad (15)$$

The affine connection without torsion or curvature has the following form:

$$\Gamma^{\alpha}_{\vartheta v} = \left(\frac{\partial z^k}{\partial \xi^{\sigma}} \right) \partial_{\vartheta} \partial_v \chi^{\sigma}. \quad (16)$$

If we consider a specific coordinate selection that is well-known as a coincident gauge, for which $\Gamma^{\alpha}_{\vartheta v} = 0$. After that, the nonmetricity becomes:

$$Q_{\alpha\vartheta v} = \partial_{\alpha} g_{\vartheta v}. \quad (17)$$

The metric for a general static and spherically symmetric space is expressed as :

$$ds^2 = -e^{v(r)} dt^2 + e^{\sigma(r)} dr^2 + r^2 d\omega^2. \quad (18)$$

In the above equation $d\omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$. we get Eq. (19) by putting Eq. (18) in Eq. (8), then Q is expressed in terms of r ,

$$Q(r) = -\frac{2e^{-\sigma}}{r} \left(v' + \frac{1}{r} \right), \quad (19)$$

where prime(r) signifies the derivative with respect to r . An anisotropic fluid with spherically symmetric geometry has the following energy-momentum tensor:

$$T_{\vartheta v} = (\rho + p_t) u_{\vartheta} u_v + p_t g_{\vartheta v} + (p_r - p_t) v_{\vartheta} v_v, \quad (20)$$

where, u_{ϑ} denotes four-velocity, and v_{ϑ} denotes the unitary space-like vector in the radial direction that fulfils $u^{\vartheta} u_{\vartheta} = -1, v^{\vartheta} v_{\vartheta} = 1, u^{\vartheta} u_{\vartheta} = -1, u^{\vartheta} v_v = 0$. $\rho(r)$ represents energy density, $p_r(r)$ represents the radial pressure in the direction of v_{ϑ} , while the orthogonal represents the tangential pressure (v_{ϑ}) is denoted by $p_t(r)$. The following are the independent components of the equations of motion Eq. (10) for anisotropic fluid Eq. (12):

$$\rho = -\frac{F}{2} + F_Q \left[Q + \frac{1}{r^2} + \frac{e^{-\sigma}}{r} (v' + \sigma') \right], \quad (21)$$

$$p_r = \frac{F}{2} - F_Q \left[Q + \frac{1}{r^2} \right], \quad (22)$$

$$p_t = \frac{F}{2} - F_Q \frac{Q}{2} - F_Q e^{-\sigma} \frac{Q}{2} - e^{-\sigma} F_Q \left[\frac{v''}{2} + \left(\frac{v'}{4} + \frac{1}{2r} \right) (v' - \sigma') \right], \quad (23)$$

$$0 = \frac{\cot \theta}{2} Q' F_{QQ}. \quad (24)$$

Assuming a zero affine connection in the coordinate system, $F(Q)$ gravity Eq. (9) can be expressed as follows:

$$\frac{\cot \theta}{2} Q' F_{QQ} = 0, \quad (25)$$

Combining Eq. (10) and motion equations yields $F_{QQ} = 0$. This leads us to the conclusion that the function $F(Q)$ must be linear. Choosing nonlinear values for the function $F(Q)$ might result in conflicting equations of motion and solutions, especially for $F(Q) = Q^2$. Using a nonlinear function of $F(Q)$ gravity will give inconsistent values for the equation of motion. To investigate and solve the nonlinear function

of $F(Q)$ gravity, we require a more extended spherically symmetric metric for coincident gauges. Please visit [83] for a more in-depth study of this topic. In this investigation, we used a linear function with $F_{QQ} = 0$ and found that the spherically symmetric coordinate system (18) corresponds to the affine connection $\Gamma^{\alpha}_{\vartheta v} = 0$. To formulate a realistic compact stellar model, we need to discover a useful function of $F(Q)$. According to [31], matching the compact star solution with the Schwarzschild (Anti-) de sitter solution on the exterior boundary requires selecting a function that assumes $F_{QQ} = 0$. Then, to determine the field equations, we utilize the function $F(Q)$, which is expressed as follows:

$$F_{QQ} = 0 \Rightarrow F(Q) = \beta_1 Q + \beta_2. \quad (26)$$

Using equations Eqs. (25) and (26) with β_1 and β_2 as integration constants, the field equations for $F(Q)$ gravity can be stated as follows:

$$\rho = \frac{1}{2r^2} \left[2\beta_1 + 2e^{-\sigma} \beta_1 (r\sigma' - 1) - r^2 \beta_2 \right], \quad (27)$$

$$p_r = \frac{1}{2r^2} \left[-2\beta_1 + 2e^{-\sigma} \beta_1 (rv' + 1) + r^2 \beta_2 \right], \quad (28)$$

$$p_t = \frac{e^{-\sigma}}{4r} \left[2e^{\sigma} r \beta_2 + \beta_1 (2 + rv')(v' - \sigma') + 2r \beta_1 v'' \right]. \quad (29)$$

In Eqs. (27)–(29) prime (r) denotes the differentiation with respect to radial coordinate r and ρ, p_r, p_t, σ and v are unknowns which we are going to solve to get our desired result. For static and spherically symmetric fluid configurations, the general expression of the anisotropic factor, denoted as $\Delta = (p_t - p_r)$, can be stated as follows:

$$\Delta(r) = \frac{\beta_1}{8\pi} \left[e^{-\sigma} \left(\frac{v''}{2} - \frac{\sigma' v'}{4} + \frac{(v')^2}{4} - \frac{v' + \sigma'}{2r} - \frac{1}{r^2} \right) + \frac{1}{r^2} \right]. \quad (30)$$

3. Buchdahl metric

$e^{\sigma} = \frac{\zeta(1+\xi r^2)}{\zeta+\xi r^2}$ Buchdahl [84] proposed this metric potential form, based on two parameters ζ and ξ , to identify a viable spherically symmetric perfect fluid solution. This metric potential is non-singular in the centre of a fluid configuration and fulfils the necessary physical conditions of a relativistic compact star. This section will demonstrate how the spheroidal parameter ζ can be increased analytically to take into account both positive and negative values. The Schwarzschild interior solution can be found at $\zeta = 0$, and the hypersurfaces ($t = \text{constant}$) seem to be flat at $\zeta = 1$. By setting $\xi = -\zeta/R^2$, we derive the metric function considered by Tikekar and Vaidya [85] and also obtain Durgapal and Bannerji solution [86] with $\zeta = -2$. Numerous viable models for charged/uncharged, isotropic/anisotropic compact stars have been successfully developed utilizing this metric potential form [87–93]. In the present investigation, we conduct an analysis using $\xi > 0$ and $\zeta < 0$ to verify the validity of the solution.

4. Metric potentials

The behaviour of potential functions e^{σ} and e^v : For developing a compact star model we need two metric potential functions. Here, we choose $e^{\sigma} = \frac{\zeta(1+\xi r^2)}{\zeta+\xi r^2}$, where ξ is some positive constant and the value of ζ is less than zero. Buchdahl [84] proposed this metric potential form, based on two parameters ζ and ξ , to identify a viable spherically symmetric perfect fluid solution. Numerous viable models for charged/uncharged, isotropic/anisotropic compact stars have been successfully developed utilizing this metric potential form [87–93] and find $e^v = \left[A_1 (1 - Y_1^2)^{1/4} \frac{\alpha + \beta Y_1}{Y_1} \left[\frac{\alpha}{\beta^3} F(Y_1) + \frac{B_1}{A_1} \right] \right]^2$, where $Y_1 = \sqrt{\frac{\zeta+\xi}{\zeta-1}}$,

$F(Y_1) = \frac{\sec^2 t - \cos^2 t}{2} + \log \cos^2 t$ with $t = \tan^{-1} \sqrt{\frac{\beta Y_1}{\alpha}}$. Here, A_1 and B_1 are arbitrary constants of integration and α and β are non zero parameters.

For a physically viable model, [94] established that the metric potentials e^σ and e^ν must be regular and monotonically increasing with respect to an increase in radius r . It must be positive and free of singularity at the centre, with $\sigma'(0) = 0$ and $\nu'(0) = 0$. In our model, the value of e^σ at $r = 0$ equals 1 and the value of e^ν at $r = 0$ equals 0.2897. We have a positive value of e^σ and e^ν at $r = 0$ for non zero, α and β , indicates that the metric potential is stable at the centre of the compact star. The accurate behaviour of the metric potentials has been shown in Fig. 1. The graph of metric potentials e^σ and e^ν are finite and regular throughout the radius of the star and both the graph of e^σ and e^ν are monotonically increasing towards the surface and the graph of σ' and ν' is also monotonically increasing towards the surface with zero value at the centre ($r = 0$). From these characteristics of metric potentials, we can conclude that metric potentials are competent to produce the model for the anisotropic compact star.

5. Solution for new model

The five functions that describe fluid and its gravitation are σ , ν , ρ , p_r and p_t and they are dependent on the radius r , as was mentioned in the above section. Since there are only three fundamental field equations, two of the characteristics above referred to as the generating function must be provided and the remaining three fluid characteristics must be discovered by the use of field equations. Various solutions can be found based on the provided pair of functions. This does not preclude the possibility of overlap in the solutions derived from several examples. The situation involving knowing ρ and Δ is entirely generic. Since ρ and Δ in this instance are generating functions, we have control over them and may select them to be regular, positive, and very simple. However, in this scenario, the following three functions are typically more complicated and not always physically realistic. Thus, it would be preferable if we choose σ and ν beforehand. Even though this is the simplest scenario, we are forced to proceed by trial and error because we have no control over pressure or density. To obtain analytical solutions, it is necessary to describe a metric function that relates pressure and density. This article presents a reasonable approach to calculating the metric potential based on Buchdahl [84]. Here we use the metric potential in the following form

$$\sigma = \ln(\zeta(1+x)) - \ln(\zeta+x) \quad (31)$$

and

$$\nu = 2 \ln \eta \quad (32)$$

where $x = \xi r^2$ and η is function of r . The value of ξ is positive and the value ζ is less than zero. It is not novel to use the Buchdahl ansatz to calculate the metric potential. Although simple, the model meets the physical limitations of an actual star and maintains regularity and finite conditions at the centre of the sphere. Vaidya and Tikekar [85] proposed a metric function where $\xi = \frac{\zeta}{R^2}$. This allowed for a unique geometric interpretation of the model, breaking away from the constraints of 3-space geometry. Nevertheless, [94] has required that the metric coefficients e^σ and e^ν be regular and increasing with the increase in r to have a physically valid model. It ought to be positive, devoid of singularity in the centre, with $\nu'(0) = 0$ and $\sigma'(0) = 0$.

With the help of Eq. (30), Eq. (31) and (32), we find the expression for anisotropy factor Δ given below:

$$\Delta = \frac{\beta_1}{8\pi} \left[\frac{\zeta+x}{\zeta(1+x)} \left[\frac{\eta''}{\eta} - \sqrt{\frac{\xi}{x}} \frac{\eta'}{\eta} + \frac{\sqrt{\xi x}(\zeta-1)}{(\zeta+x)(1+x)} \varpi \right] \right]. \quad (33)$$

where $\varpi = \left(\sqrt{\xi x} - \frac{\eta'}{\eta} \right)$. Now by using transformation $Y_1 = \sqrt{\frac{\xi+x}{\zeta-1}}$ and $\eta = (1-Y_1^2)^{1/4} X$ in Eq. (33), We get a simpler form of second order differential equation that is

$$X'' + \frac{1}{Y_1^2 - 1} \left[1 - \zeta + \frac{8\pi \Delta \zeta (1+x)^2}{\xi x} + \frac{3Y_1^2 + 2}{4(1-Y_1^2)} \right] X = 0. \quad (34)$$

Table 1

The numerical value of the parameters β , α , ξ and ζ for different values of β_1 and β_2 .

β_1	β_2 (km ⁻²)	β	α	ξ (km ⁻²)	ζ	Compact star
0.8	0.00001	0.93	0.2	0.002955930	-0.62	4U1820-30
0.9	0.00001	0.93	0.2	0.002955930	-0.62	4U1820-30
1.0	0	0.93	0.2	0.002955930	-0.62	4U1820-30
1.1	0.00001	0.93	0.2	0.002955930	-0.62	4U1820-30
1.2	0.00001	0.93	0.2	0.002955930	-0.62	4U1820-30

To obtain the solution of Eq. (34), We assume the anisotropy factor Δ which is described as

$$\Delta = \frac{\xi x}{8\pi \zeta (1+x)^2} \beta_1 \left[\frac{5}{4} \frac{1}{(1-Y_1^2)} - \frac{2\alpha(1-Y_1^2)}{Y_1^2(\alpha+\beta Y_1)} + \zeta - \frac{7}{4} \right], \quad (35)$$

where α and β are non zero parameters. Putting this value of Δ in Eq. (34) we get,

$$X'' - \frac{2\alpha}{Y_1^2(\alpha+\beta Y_1)} X = 0. \quad (36)$$

Solving this we get,

$$X = A_1 \frac{\alpha + \beta Y_1}{Y_1} \left[\frac{\alpha}{\beta^3} F(Y_1) + \frac{B_1}{A_1} \right]. \quad (37)$$

By using this obtained value of X we get η as,

$$\eta = A_1 (1-Y_1^2)^{1/4} \frac{\alpha + \beta Y_1}{Y_1} \left[\frac{\alpha}{\beta^3} F(Y_1) + \frac{B_1}{A_1} \right], \quad (38)$$

where $F(Y_1) = \frac{\sec^2 t - \cos^2 t}{2} + \log \cos^2 t$ with $t = \tan^{-1} \sqrt{\frac{\beta Y_1}{\alpha}}$. Here, A_1 and B_1 are arbitrary constants of integration and $\zeta < 0$.

Using Eq. (31), Eq. (32) and (38) in Eqs. (27)–(29) we get the expression for energy density, radial pressure, and tangential pressure given below:

$$\rho = \frac{\beta_1 \xi (\zeta-1)(3+x)}{8\pi \zeta (1+x)^2} - \frac{\beta_2}{16\pi}, \quad (39)$$

$$p_r = \frac{-2\xi \beta_1 + \beta_2(1+x) + 2(1+x)\xi \beta_1 \psi_1}{8\pi 2(1+x)\psi_2 \psi_3} + \frac{2\xi^2 r^2 \beta_1 \psi_4}{8\pi \zeta (1+x)^2 \psi_5 \psi_6}, \quad (40)$$

$$p_t = \frac{2B_1 \beta^3 (\zeta-1) \psi_7 + A_1 \beta \zeta^2 \beta_2 \psi_8}{8\pi \psi_{11} \psi_{12}} + \frac{2A_1 (\zeta-1) \alpha \psi_9 \psi_{10}}{8\pi \psi_{11} \psi_{12}}. \quad (41)$$

The value of $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8, \psi_9, \psi_{10}, \psi_{11}, \psi_{12}$ are given in the Appendix.

6. Necessary and physical existence requirements in $F(Q)$ gravity

The following are the necessary and physical existence requirements for the $F(Q)$ gravity model; a well-behaved model must satisfy these requirements:

- Singularities must not exist in the solution, meaning that the metric functions, density and pressures must all be non-negative for $0 \leq r \leq R$.
- The density and pressures ought to be positive, monotonically decreasing towards the surface, with the centre having the largest value.
- At the border, radial pressure should disappear.

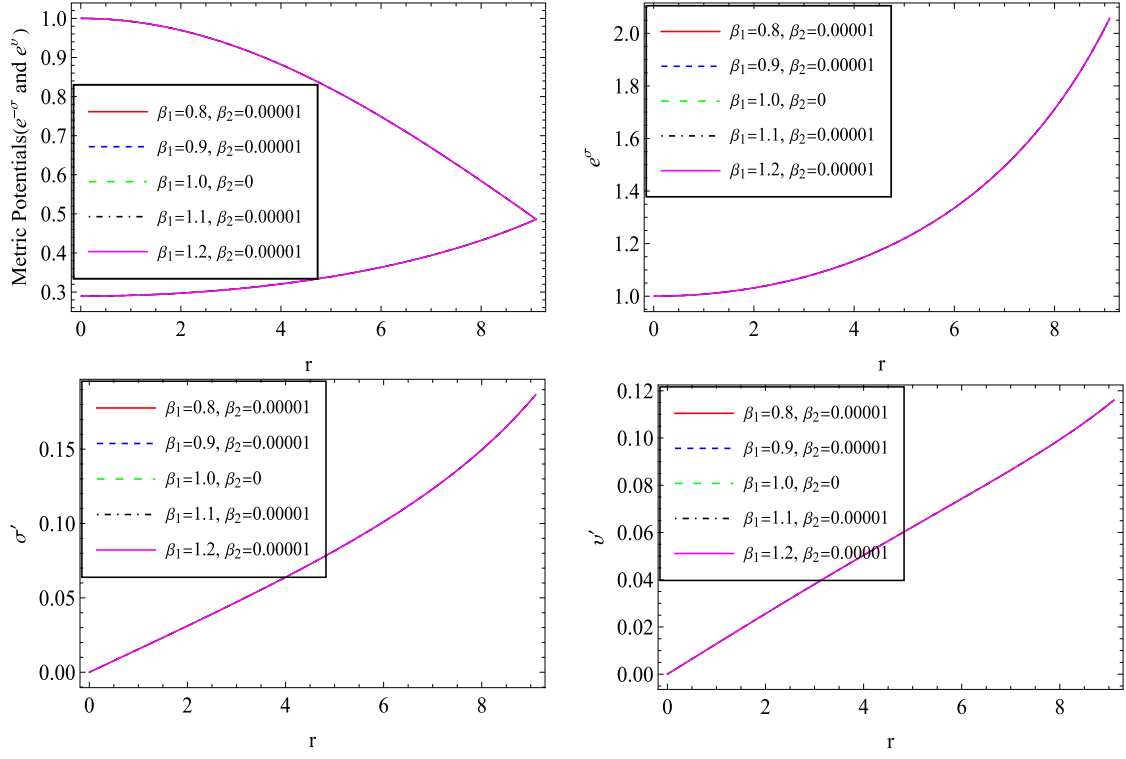


Fig. 1. The behaviour of $e^{-\sigma}$ and e^{ν} at top left in this graph decreasing line represent $e^{-\sigma}$ and increasing line is for e^{ν} , e^{σ} at top right, σ' at bottom left and ν' at bottom right versus the radius $r(\text{km})$ of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$.

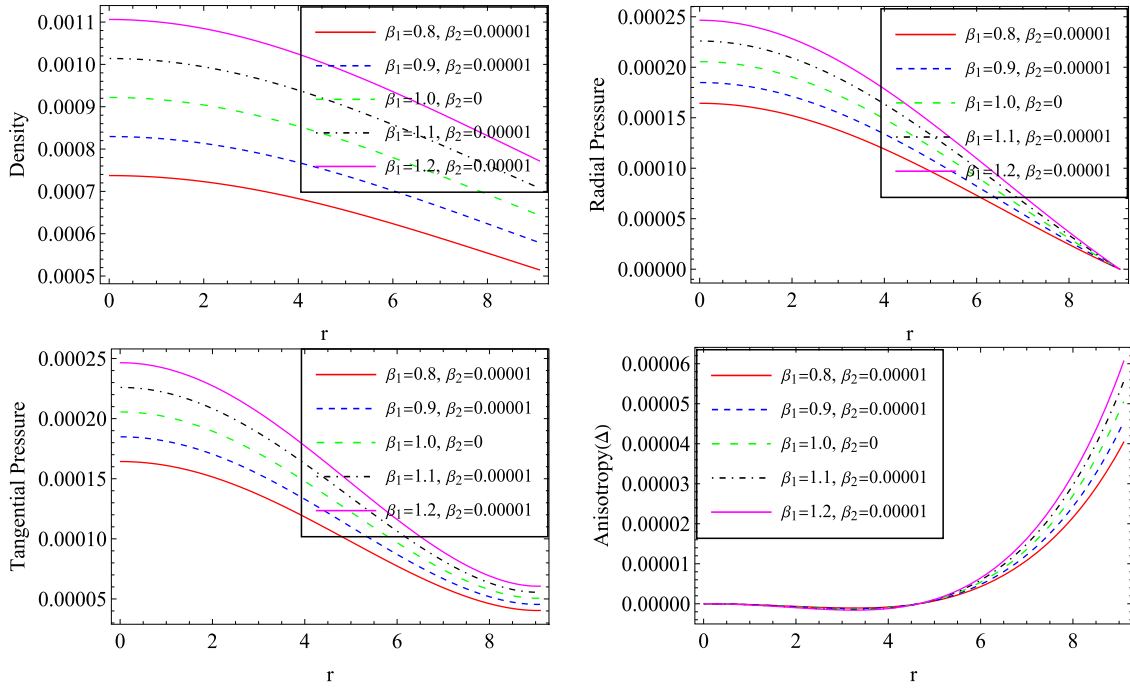


Fig. 2. The behaviour of energy density $\rho \text{ (km}^{-2}\text{)}$, radial pressure $p_r \text{ (km}^{-2}\text{)}$, tangential pressure $p_t \text{ (km}^{-2}\text{)}$ and anisotropy $\Delta \text{ (km}^{-2}\text{)}$ versus the radius $r(\text{km})$ of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$.

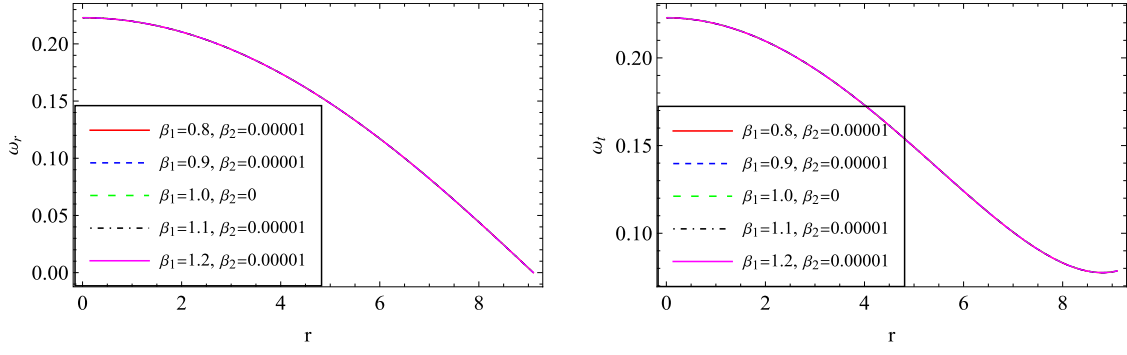


Fig. 3. The behaviour of energy state parameters(ω_r, ω_t) versus the radius $r(\text{km})$ of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$.

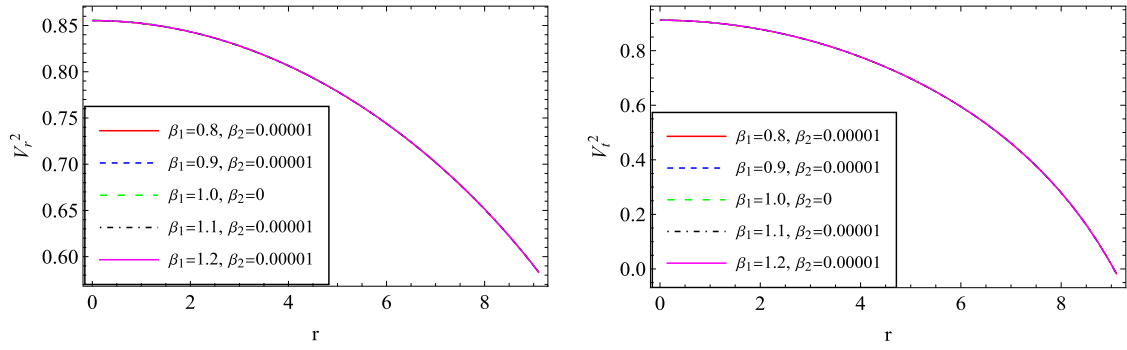


Fig. 4. The behaviour of radial velocity (V_r^2) and Tangential velocity (V_t^2) versus the radius $r(\text{km})$ of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$.

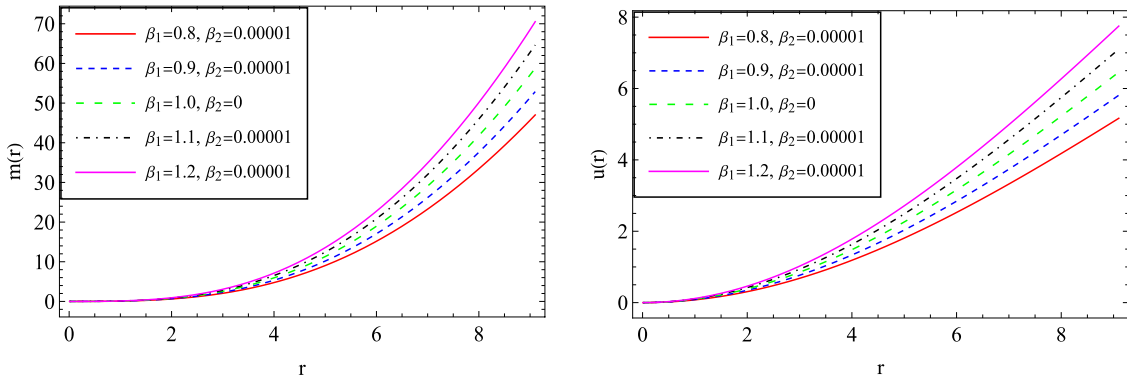


Fig. 5. The behaviour of mass $m(\text{kg})$ and compactness parameter $u(r)$ versus the radius (km) of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$.

Table 2

The numerical values of some physical quantities (central density ρ_c , surface density ρ_s and central pressure p_c) of the compact star for different values of β_1 .

β_1	Central density (gm/cm^3)	Surface density (gm/cm^3)	Central pressure (dyne/cm^2)
0.8	3.95769×10^{13}	2.76555×10^{13}	0.792776×10^{34}
0.9	4.45710×10^{13}	3.10923×10^{13}	0.894290×10^{34}
1.0	4.951140×10^{13}	3.45828×10^{13}	0.995804×10^{34}
1.1	5.445180×10^{13}	3.80196×10^{13}	1.092484×10^{34}
1.2	5.939220×10^{13}	4.14564×10^{13}	1.193998×10^{34}

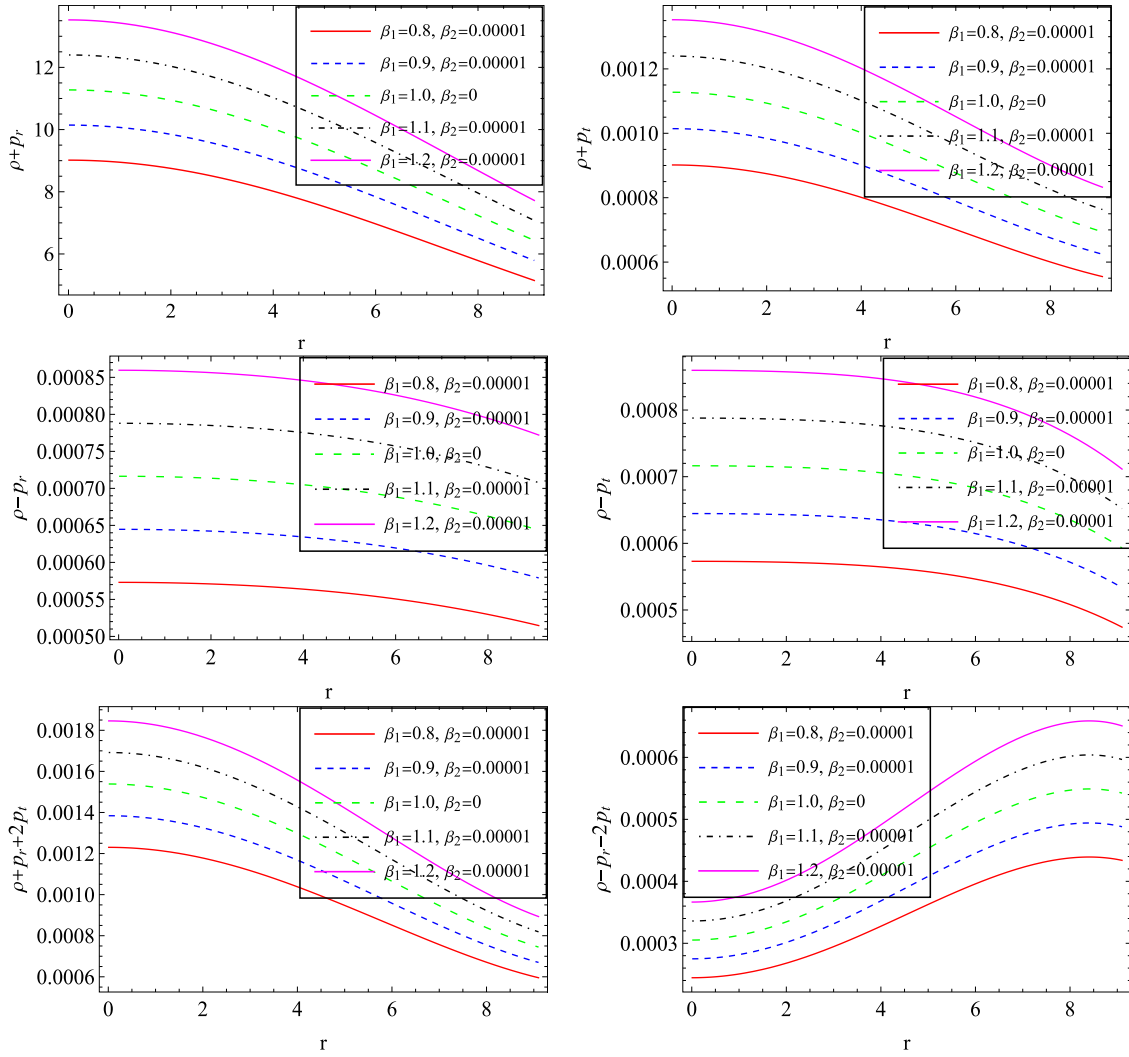


Fig. 6. The behaviour of WEC (km^{-2}), DEC (km^{-2}), SEC (km^{-2}) and TEC (km^{-2}) versus the radius $r(\text{km})$ of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$.

Table 3

The numerical value of the mass M , radius R , mass-radius ratio $\frac{M}{R}$ and constants A_1 and B_1 for the compact star.

Compact star	$M(M_\odot)$	R (km)	$\frac{M}{R} (< \frac{4}{9})$	A_1	B_1
4U1820-30	1.58446	9.1	0.1741164	2.08102	2.54774
4U1820-30	1.58452	9.1	0.1741230	2.07918	2.54787
4U1820-30	1.58499	9.1	0.1741747	2.06445	2.54893
4U1820-30	1.58460	9.1	0.1741318	2.07650	2.54807
4U1820-30	1.58464	9.1	0.1741362	2.07550	2.54814

- The ratio of pressures to the density of the star i.e $\frac{p}{\rho}$ must be positive and have a value greater than zero and less than 1.

$$0 < \frac{p}{\rho} < 1. \quad (42)$$

- At the centre of the star, the causality requirements must be satisfied. i.e $0 < \left(\frac{dp}{d\rho}\right)_{r=0} \leq 1$.

- The value of $\frac{dp}{d\rho}$ must be positive and decrease monotonically as the value of r increases.

$$\frac{d}{dr} \left(\frac{dp}{d\rho} \right)_{r=0} < 0. \quad (43)$$

- As the value of r increases the solution value of $\frac{p}{\rho}$ are monotonically decreasing.

$$\frac{d}{dr} \left(\frac{p}{\rho} \right)_{r=0} < 0. \quad (44)$$

7. Boundary requirements

The boundary requirement for solving the Einstein-field equations in $F(Q)$ gravity is presented in this section. In order to accomplish this, we compare the solution on the exterior and interior spacetimes. There are numerous indications in more recent works, such as [31], that the feasible solution in $F(Q)$ gravity is solvable using Schwarzschild (Anti-) di sitter solution. The solution is provided as follows:

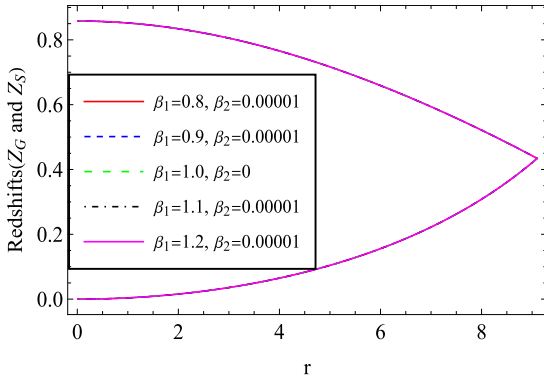


Fig. 7. The behaviour of redshifts(Z_G and Z_S) versus the radius r (km) of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$. In this graph decreasing line represent Z_G and increasing line is for Z_S .

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (45)$$

Taking $r = R$, the Schwarzschild (Anti-) di sitter metric has been solved at the surface using the requirement that Darmois–Israel derived in their research [95]

$$\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right) = e^{\nu(R)}, \quad (46)$$

$$\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right) = e^{-\sigma(R)}, \quad (47)$$

$$p_r(R) = 0. \quad (48)$$

where the notations M and Λ denote the mass and cosmological constant. It is noteworthy that the cosmological constant Λ is contingent upon the constants β_1 and β_2 , which are expressed as follows $\Lambda = \frac{\beta_2}{2\beta_1}$. In this scenario, the evidence for the cosmological constant is zero, hence its value does not affect the current stellar model. Normally, the cosmological constant would have a significant value to influence the mathematical formulation of the problem. The cosmological constant, which is zero in the current study, is estimated to be 10^{-46} km^{-2} , based on numerical evidence. For this project, β_2 has been set to 0.00001 for the entire problem. Eqs. (46)–(48) are boundary conditions with the help of these boundary conditions we find the values of the constants A_1 and B_1 that is

$$A_1 = -\frac{2B_1(-1 + \zeta)\beta^3\psi_{13}}{(\zeta + \xi R^2)\beta\psi_{14} + 2(-1 + \zeta)\alpha Z}, \quad (49)$$

$$B_1 = \frac{\psi_{15}}{(1 + \xi R^2)(\alpha + Y_2\beta)\psi_{16}}. \quad (50)$$

The value of ψ_{13} , ψ_{14} , ψ_{15} , and ψ_{16} are given in Appendix

8. Physical analysis of the model

We conducted several physical experiments using various parameter values to show the physical acceptability of our proposed model. We have included the graphical representations using the parameter values from Tables 1 and 3 for clarity's sake.

8.1. Energy density, pressures and anisotropy

In $F(Q)$ gravity, the pressure and density of a compact star decrease towards the surface, the radial pressure goes to zero on the boundary ($r = R$), and the tangential pressure remains positive throughout the star. It is also noteworthy that the radial and tangential pressures as well as the energy density, are significantly influenced by the parameter β_1 . As the value of β_1 increases, the value of radial and tangential pressures as well as energy density also increases. The Fig. 2 makes this completely evident. The star's interior is more compact than its outer layer, as seen by the behaviour where the density is highest in the centre and decreases towards the surface. Furthermore, we observe that the curves of p_r and p_t for various β_1 values are monotonically decreasing towards the surface. It follows that the pressure-induced force is positive and exerts an outward force. An essential prerequisite for a stable, realistic stellar object is the presence of a force induced by pressure characteristic that counteracts the gravitational pull to avoid gravitational collapse. In addition, we have also analysed the celestial body's anisotropy in this section. The two opposing forces affect a celestial body's anisotropy: an attracting force is induced if anisotropy i.e $p_t - p_r$ is negative and a repulsive force is induced if anisotropy i.e $p_t - p_r$ is positive. While the matter is drawn towards the body by the attraction force, it is pushed away from it by the repulsive force. At the centre of the celestial body's anisotropy i.e $p_t - p_r = 0$ is zero. There are isotropic conditions, the two forces balance and there is no net force. As one moves out from the compact star's centre, the anisotropy develops monotonically with a positive slope. This rise results from the repulsive force becoming more powerful the further one goes from the compact star. The graph in Fig. 2 illustrates the same. Throughout the star proximity, the anisotropy graph must have a rising behaviour and be positive. The graph that is being shown and Table 2 both show the same behaviour. The mass–radius ratio($\frac{M}{R}$) is shown in Table 3, which also provides the mathematical value of density at the centre and surface and the value of pressure at the centre. The compact star exhibits good behaviour, as evidenced by the increasing values of β_1 and the corresponding increases in the mathematical value of density at the centre and surface and pressure at the centre of the star.

8.2. Energy state parameter

The energy state parameters in radial and transversal form can be shown as:

$$\omega_r = \frac{p_r}{\rho}, \quad \omega_t = \frac{p_t}{\rho}. \quad (51)$$

We know that ω_r and ω_t must satisfy the inequality $0 < \omega_r, \omega_t < 1$ for the model to be viable. Fig. 3 shows a graphical representation of energy state parameters. The energy state parameters have been created and are completely fulfilled. Thus, our model is physically possible.

8.3. Stability analysis

By examining the model for any possible cracks or overturns, we can determine if the star is possibly stable or unstable. At both extremities of the star, the fluid components accelerate to one another. The stability condition of the compact star explained by Herrera is the possibility of the star splitting at some points within the star, provided that the star does not compress or expand. According to this requirement, the model's radial velocity(V_r^2) and tangential velocity(V_t^2) must both be smaller than 1. When we look at Fig. 4, we can see that the values of V_r^2 and V_t^2 fall between 0 and 1. As a result, the star is steady and Herrera's stability requirement is satisfied.

$$V_r^2 = \frac{dp_r}{d\rho} < 1, \quad V_t^2 = \frac{dp_t}{d\rho} < 1, \quad (52)$$

Fig. 4 shows a graphical depiction of V_r^2 and V_t^2 and complies with the stability requirements stated by Abreu et al. [96]. A stability region was

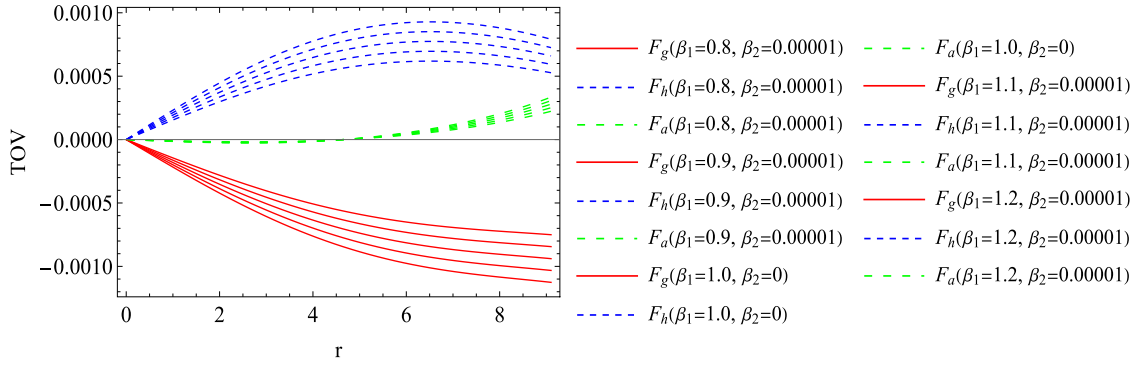


Fig. 8. The behaviour of F_g (km^{-3}), F_h (km^{-3}) and F_a (km^{-3}) versus the radius r (km) of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$.

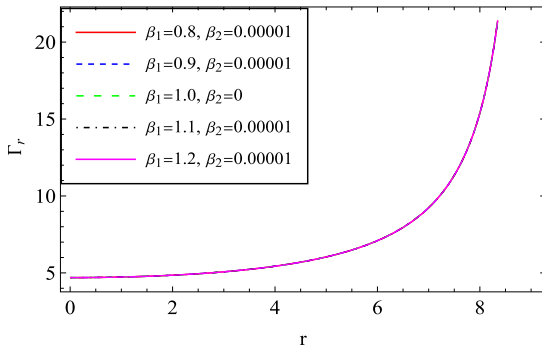


Fig. 9. The behaviour of Adiabatic index Γ_r versus the radius r (km) of the proposed compact star using $\xi = 0.002955930 \text{ km}^{-2}$.

identified by this investigation, when $V_r^2 > V_t^2$ and the nature of the difference between these velocities must be positive. This shows that the compact star is stable. A generalization of this claim was proposed by Andreasson [97,98] who also introduced the idea of no cracking, which indicates the stability of the compact star. The zone that is free of cracks is denoted by $0 < |V_t^2 - V_r^2| < 1$. The illustrations demonstrate the compact star's good mathematical and physical behaviour. We may also examine how β_1 affects on V_t^2 and V_r^2 . Fig. 4 shows that the value of V_t^2 and V_r^2 decrease monotonically as the value of β_1 increases.

8.4. Mass function and the compactness parameter

The compact star's compactness and mass are shown in Fig. 5. The impact of parameter β_1 on the compactness and mass of the compact star is visible. When the value of β_1 increases, the value of mass and compactness also increases. The values of mass and compactness are zero at the centre of the compact star, and they monotonically increase towards the surface. It is significant to note that there is a mass-radius ratio bound for any stellar structure, which is provided by Buchdahl and is $\frac{2M}{R} \leq \frac{8}{9}$ [84]. This bound has been computed for the compact star in our model and is displayed in Table 3. This bound is less than $\frac{4}{9}$, demonstrating that our model satisfies Buchdahl's requirements. The functions $m(r)$, $u(r)$ of radial coordinate r are used to represent the compact star mass and compactness are given below:

$$m(r) = \frac{\kappa}{2} \int_0^r \rho r^2 dr, \quad (53)$$

$$u(r) = \frac{\kappa}{2r} \int_0^r \rho r^2 dr. \quad (54)$$

The following is the expression for the mass and compactness of the compact star:

$$m(r) = -\frac{44r^3 (-6\beta_1\xi(\xi-1) + \beta_2\xi\xi r^2 + \beta_2\xi)}{7(\xi(3\xi r^2 + 3))}, \quad (55)$$

$$u(r) = -\frac{44r^2 (-6\beta_1\xi(\xi-1) + \beta_2\xi\xi r^2 + \beta_2\xi)}{7(\xi(3\xi r^2 + 3))}. \quad (56)$$

The star obtained here is inside the star's limit of being a compact star, with a solar mass of 1.58 for a value of $\beta_1 = 1$. The graph's growing tendency suggests that the star's mass rises gradually as the parameter β_1 rises. In this scenario, the constant β_2 is supposed to be 0.00001, therefore it has minimal impact on the mass and compactness of the compact star.

8.5. Energy requirements

When analysing data, all energy constraints must be met for the model to be feasible. Here NEC represents the null energy condition, WEC represents the weak energy condition, DEC represents the dominant energy condition, SEC represents the strong energy condition, and TEC depicts the trace energy condition.

$$NEC : \rho + p_r \geq 0, \rho + p_t \geq 0 \quad (57)$$

$$WEC : \rho \geq 0, \rho + p_r \geq 0, \rho + p_t \geq 0, \quad (58)$$

$$DEC : \rho - p_r \geq 0, \rho - p_t \geq 0, \quad (59)$$

$$SEC : \rho + p_r + 2p_t \geq 0, \quad (60)$$

$$TEC : \rho - p_r - 2p_t \geq 0. \quad (61)$$

Fig. 6 represents the energy requirements graphically; all energy requirements needs have been identified and are fully met. Therefore, our solutions are physically possible.

8.6. Gravitational and surface redshift

A phenomenon known as gravitational redshift occurs when electromagnetic waves or photons, lose energy when they emerge from a gravitational well. Electromagnetic radiation expands its wavelength and photon energy to escape when it moves out of a gravitational well. It loses energy not through a change in speed but rather through a change in frequency because it must travel simultaneously at the speed of light. When energy is lost, the photon's wave frequency decreases and its wavelength increases, resulting in a shift towards the red end of the electromagnetic spectrum, commonly known as redshift. The profile of interior density is similarly presented by the behaviour of

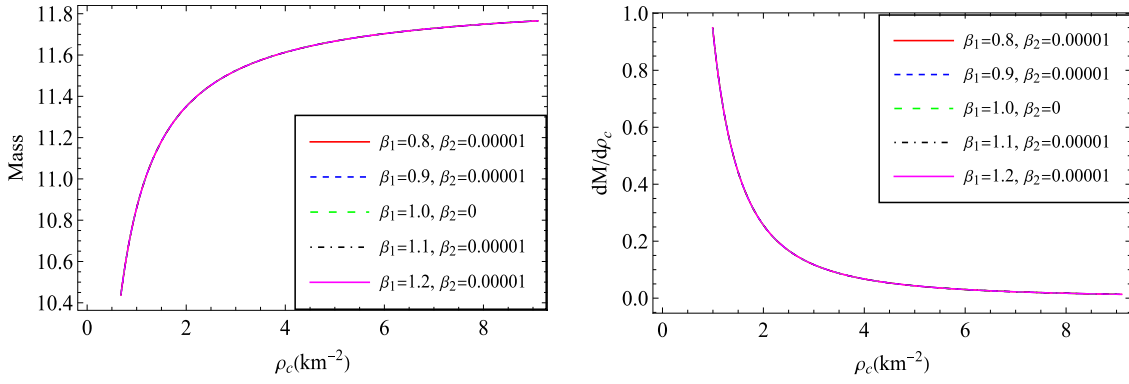


Fig. 10. The behaviour of Mass $M(\text{kg})$ and $\frac{dM}{d\rho_c}$ for a fixed value of $\delta_2 = 0.00001$ versus δ_1 and the central density ρ_c proposed compact star.

interior redshift. A photon must travel a greater distance and through a denser region the compact star's interior if it is to leave the centre and reach the surface. This greater dispersion causes a significant energy loss. A photon that rises close to the surface, will travel a shorter distance through a less dense zone, resulting in less dispersion and energy loss. This means that the interior redshift is highest in the centre and lowest towards the surface. The surface redshift depends on the surface gravity, which is defined by the stellar object's radius and total mass. Surface gravity increases as a result of a modest increase in radius, similar to the increase in mass. As a result, the surface redshift increases monotonically towards the surface of star. Gravitational and surface redshifts are defined as:

$$Z_G = \frac{1}{\sqrt{|e^{v(r)}|}} - 1, \quad (62)$$

and the value of $e^{v(r)}$ is seen in Appendix.

$$Z_S = e^{-u(R)/2} - 1 = e^{\sigma(R)/2} - 1 = \frac{1}{\sqrt{1 - 2u(r)}} - 1, \quad (63)$$

where $e^{\frac{\sigma(R)}{2}} = \sqrt{\frac{\zeta(1+\xi R^2)}{\zeta+\xi R^2}}$ and $u(r) = \frac{m(r)}{r}$.

The graph of gravitational redshift is monotonically decreasing towards the surface and the graph of surface redshift is monotonically increasing towards the surface both of these coincide at the surface of the stellar configuration as shown in Fig. 7.

8.7. Tolman–Volkoff–Oppenheimer equation for equilibrium compact objects

The equilibrium of an anisotropic stellar object is maintained by the collaborative influence of gravitational force(F_g), hydrostatic force(F_h) and anisotropic force(F_a). At this stable point, the Tolman–Volkoff–Oppenheimer(TOV) equation [20,99] defines the balancing force equation.

$$F_g + F_h + F_a = 0. \quad (64)$$

The equation below represents the forces in the TOV equation.

$$F_g = -\frac{v'}{2}(\rho + p_r), F_h = -\frac{dp_r}{dr}, F_a = \frac{2}{r}(p_t - p_r). \quad (65)$$

Fig. 8 illustrates the dominance of (F_g), which is countered by the collaborative effects of (F_h) and (F_a). This shows that our solutions for anisotropic fluid distributions are in equilibrium.

9. Adiabatic index

The relativistic adiabatic index is crucial when evaluating the stability of any stellar fluid sphere. The adiabatic index comprises the fundamental properties of the equation of state in relation to instability formulas and serves as a connection between the relativistic structure of a spherical static object and the equation of state of the interior fluid [100]. The relativistic adiabatic index Γ for an isotropic fluid sphere was introduced by Chan et al. [101] and is expressed as follows:

$$\Gamma = \left(\frac{\rho + p}{p} \right) \cdot \left(\frac{dp}{d\rho} \right). \quad (66)$$

Given that we have examined spherically symmetric configurations permitting just radial displacements, the aforementioned definition of the adiabatic index for an anisotropic fluid sphere is expressed as follows:

$$\Gamma = \left(\frac{\rho + p_r}{p_r} \right) \cdot \left(\frac{dp_r}{d\rho} \right). \quad (67)$$

The criterion for the stability of a Newtonian isotropic sphere is $\Gamma > \frac{4}{3}$ [102]. This criterion alters for a relativistic anisotropic fluid. The criterion for the collapse of the specific heat ratio in an anisotropic fluid distribution is given in [101],

$$\Gamma < \frac{4}{3} \left[1 - \max \left(\frac{p_r - p_t}{r|p'_r|} \right) \right]. \quad (68)$$

The unstable range of Γ is clearly contingent upon the nature of anisotropy. It will increase when $\Delta > 0$ and decrease when $\Delta < 0$. Therefore, we may conclude that the anisotropy in pressure increases or decreases the instability of the collapsing system. It is feasible that Δ may alter its sign inside the system. If it changes sign, it will indicate the presence of both stable and unstable zones inside the object, which could lead to the fragmentation of the sphere [101]. Given that radial pressure increases the effective density of the system, in the post-Newtonian approximation, the aforementioned inequality assumes a more precise form.

$$\Gamma < \frac{4}{3} + \max \left(-\frac{4}{3} \frac{p_r - p_t}{r|p'_r|} + \frac{8\pi}{3} \frac{r\rho p_r}{|p'_r|} \right). \quad (69)$$

The aforementioned expression indicates that the unstable range of Γ increases as a result of the relativistic influence of pressure. Nevertheless, as radiation adversely affects the total energy, the unstable range Γ will decrease when a radiative system is taken into account [101].

Moustakidis [100] has established a more stringent constraint on the adiabatic index for the stable area. He stated that the critical value of the adiabatic index ($\Gamma_{critical}$) is contingent upon the compactness and the amplitude of the Lagrangian displacement from equilibrium. He has determined the value of this restriction as

$$\Gamma_{critical} = \frac{4}{3} + \frac{19}{21} \frac{M}{R}. \quad (70)$$

Any stable configuration must satisfy $\Gamma_r \geq \Gamma_{critical}$. Fig. 9 confirms that this criterion is satisfied for the proposed model. Furthermore, Γ_r is a monotonically increasing function of r .

10. Harrison–Zeldovich–Novikov stability condition

Harrison [103] and Zeldovich–Novikov [104] proposed stability criteria for the compact star model, contingent upon the star's mass and centre density. According to these observations, they suggested that for a stable configuration, $\frac{dM}{d\rho_c} > 0$, where M and ρ_c represent the mass and central density of the compact star, respectively. With the central density, we will define the mass function of our static anisotropic fluid solution as

$$M(\rho_c) = \frac{R}{2} \left(1 - \frac{(6(\zeta - 1) + 2\rho_c + \beta_2 R^2)}{(6(\zeta - 1) + 2\zeta\rho_c + \zeta\beta_2 R^2)} \right) \quad (71)$$

When differentiating Eq. (71) with respect to ρ_c , we obtain

$$\frac{dM}{d\rho_c} = -\frac{R}{2} \left(\frac{\zeta(-12\zeta + 12)}{(6(\zeta - 1) + 2\zeta\rho_c + \zeta\beta_2 R^2)^2} \right) \quad (72)$$

Fig. 10 demonstrates that our anisotropic models fulfil the Harrison–Zeldovich–Novikov stability criterion. The stability of our structures increases with increasing radii and remains constant after the central densities attain their maximum values.

11. Conclusion

The study of compact stars has been a fascinating topic of research in astronomy and theoretical physics for the last twenty years. This work introduces the formation of a stable compact star using $F(Q)$ gravity, incorporating a non-perfect fluid distribution of matter. It is crucial to highlight that the formation of a compact star under $F(Q)$ gravity behaves equivalently to that under GR. As a result, any solution derived in GR can also be obtained using $F(Q)$ gravity. To derive stable results, we utilized the reliable Buchdhal metric potential to produce a different solution within the framework of $F(Q)$ gravity theory. The preceding section provides a thorough explanation and investigation of the conclusions of computations conducted within the framework of $F(Q)$ gravity. Various values of β_1 and β_2 were explored and their impacts on the numerical outcomes of the model were examined as depicted in the graphs presented earlier. The model's thermodynamic quantities have been investigated in detail, and the figures presented the findings of this analysis. Fig. 2 illustrates the variations in thermodynamic quantities of the model corresponding to the constant $\xi = 0.002955930$. Table 2 provides the numerical values of the thermodynamic quantities derived from these observations. It is important to recognize that the thermodynamic properties vary with the parameter β_1 , which was tested at values of 0.8, 0.9, 1.0, 1.1, and 1.2. As β_1 increases, there is a consistent increase in these properties. The figures show smooth behaviour and all necessary requirements are satisfied. In this work, we examined V_r^2 , V_t^2 and $\frac{p}{\rho}$ of the model under consideration. Figs. 4 and 3 depict these relationships graphically for the constant $\xi = 0.002955930$. Graphs for all these quantities were plotted using varying values of β_1 as previously listed. As β_1 increased, there was a consistent downward trend in the values depicted on the graphs. These quantities vary directly with β_1 . The trajectories of V_r^2 and

V_t^2 decrease monotonically, ensuring that neither velocity exceeds the speed of light at any point, which fulfils the causality requirement. The pressure density ratio, specifically the ratio of radial pressure to density i.e $\frac{p_r}{\rho}$, exhibits a decreasing trend in its graph. For the ratio of tangential pressure to density i.e $\frac{p_t}{\rho}$, the graph initially decreases until $r = 8.82$ and then shows a gradual increase. Furthermore, the graphs exhibited smooth behaviour, indicating stability in the model during this specific analysis. Fig. 6 examines the energy requirements of a compact star. The positive trends observed in the thermodynamic quantities depicted in Fig. 2 declare the validity of the SEC, WEC, NEC, and DEC for the compact star. The graph of DEC is plotted in Fig. 6, demonstrating the stable characteristics of the compact star. Both DEC and TEC maintain positive values across the region surrounding the compact star. Figs. 5, 9 and 7 illustrate the Mass, Adiabatic index, and redshift of the compact star. As the values of β_1 increase, there is a corresponding increase in the mass of the compact star. The adiabatic index shows a rising trend as compact star mass. If the value of β_1 rises, the gravitational redshift graph consistently decreases throughout the vicinity of the star and the graph of surface redshift is increasing towards the surface. Fig. 8 presents a visual depiction of various forces within the compact star model. The graph confirms the satisfaction of the TOV equation. F_g , acts inwards, suggesting attractiveness in nature. Conversely, F_a and F_h exert outward pressure, suggesting they are repulsive in nature. Importantly, the sum of these forces is zero, expressed as $F_g + F_h + F_a = 0$. Fig. 10 shows the graphical behaviour of mass and gradient of mass with respect to central density ρ_c with $\frac{dM}{d\rho_c} > 0$. In summary, this work introduces a formation of compact stars within $F(Q)$ gravity, focusing on the study of non-perfect fluid distribution. It is noted that the formation in $F(Q)$ gravity yields identical results to those in GR, implying that solutions obtained in GR are also obtained within the framework of $F(Q)$ gravity. By using the Buchdahl metric, the study produced a new solution within the $F(Q)$ gravity theory, ensuring the attainment of stable solutions. Overall, $F(Q)$ gravity is useful for producing anisotropic solutions in GR and modified gravity theories.

CRedit authorship contribution statement

Jitendra Kumar: Writing – review & editing, Supervision, Formal analysis, Conceptualization. **Sat Paul:** Writing – original draft, Software, Resources, Methodology, Investigation, Data curation. **S.K. Maurya:** Writing – review & editing, Visualization, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Sourav Chaudhary:** Software, Investigation. **Sweeti Kiroriwal:** Software.

Declaration of competing interest

The authors declare that they have no conflict of interest or personal relationships that could have appeared to influence the work reported in this article.

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Appendix

$$Y_1 = \sqrt{\frac{\zeta + x}{\zeta - 1}}$$

$$Y_2 = \sqrt{\frac{\zeta + \xi R^2}{\zeta - 1}}$$

$$Y_3 = \sqrt{\frac{\zeta + \xi R^2}{\zeta + \xi \zeta R^2}}.$$

$$\psi_1 = 2A_1\beta^3Y_1 + \frac{2\beta}{(\zeta - 1)(x + 1)Y_1(\alpha + \beta Y_1)} - \frac{2}{(x + 1)(\zeta + x)} + \frac{1}{\zeta x + \zeta} + \frac{4}{(x + 1)^2}$$

$$\psi_2 = (1 + x)(\alpha + Y_1\beta)$$

$$\psi_3 = \beta \left(2A_1(-1 + \zeta)Y_1\alpha + A_1(\zeta + x)\beta + 2B_1(-1 + \zeta)\beta^2(\alpha + Y_1\beta) \right) + 2A_1(-1 + \zeta)\alpha(\alpha + Y_1\beta) \log\left[\frac{\alpha}{\alpha + Y_1\beta}\right]$$

$$\psi_4 = A_1\beta^3(x + 1)Y_1 + \frac{\beta(x + 1)}{(\zeta - 1)Y_1(\alpha + \beta Y_1)} - \frac{x + 1}{\zeta + x} + 2$$

$$\psi_5 = \alpha + \beta Y_1$$

$$\psi_6 = \beta \left(A_1\beta(\zeta + x) + 2\alpha A_1(\zeta - 1)Y_1 + 2\beta^2 B_1(\zeta - 1)(\alpha + \beta Y_1) \right) + 2\alpha A_1(\zeta - 1)(\alpha + \beta Y_1) \log\left(\frac{\alpha}{\alpha + \beta Y_1}\right)$$

$$\psi_7 = \xi\zeta\psi_{74} + \beta^2\xi x^4\psi_{72} + \xi x^3\psi_{73} + \xi x^2\psi_{75} + \xi x\psi_{76} + \beta 2\zeta^2\psi_{71}$$

$$\psi_{71} = \alpha^3(\zeta - 1) + 3\alpha\beta^2\zeta + 3\alpha^2\beta(\zeta - 1)Y_1 + \beta^3\zeta Y_1$$

$$\psi_{72} = 3\alpha(6\beta_1 + \beta_2\zeta r^2) + \beta Y_1(8\beta_1 + \beta_2\zeta r^2)$$

$$\psi_{73} = \alpha^3(\zeta - 1)(2\beta_1 + \beta_1\zeta r^2) + \alpha\beta^2(\beta_1(38\zeta + 36) + 3\beta_2\zeta(2\zeta + 3)r^2) + 3\alpha^2\beta(\zeta - 1)Y_1(4\beta_1 + \beta_2\zeta r^2) + \beta^3Y_1(2\beta_1(9\zeta + 7) + \beta_2\zeta(2\zeta + 3)r^2)$$

$$\psi_{74} = \alpha^3(\zeta - 1)(\beta_1(6\zeta - 2) + \beta_1(3\zeta + 1)r^2) + \alpha\beta^2\zeta(2\beta_1(9\zeta + 1) + 3\beta_2(3\zeta + 2)r^2) + \alpha^2\beta(\zeta - 1)Y_1(2\beta_1(9\zeta - 1) + 3\beta_2(3\zeta + 1)r^2) + \beta^3\zeta Y_1(\beta_1(6\zeta + 2) + \beta_2(3\zeta + 2)r^2).$$

$$\psi_{15} = 4(1 - \zeta)Y_2Y_3$$

$$\psi_{75} = \alpha\beta^2(\beta_1(26\zeta^2 + 82\zeta + 6) + 3\beta_2\zeta(\zeta^2 + 6\zeta + 3)r^2) + \alpha^2\beta(\zeta^2 + 2\zeta - 3)Y_1(10\beta_1 + 3\beta_2\zeta r^2) + \beta^3Y_1(2\beta_1(6\zeta^2 + 17\zeta + 1) + \beta_2\zeta(\zeta^2 + 6\zeta + 3)r^2) + \alpha^3(\zeta - 1)(8\beta_1 + \beta_2\zeta(\zeta + 3)r^2)$$

$$\psi_{76} = \alpha^3(\zeta - 1)(2\beta_1(\zeta^2 + 3\zeta + 1) + 3\beta_1\zeta(\zeta + 1)r^2) + \alpha\beta^2\zeta(\beta_1(6\zeta\zeta^2 + 64\zeta + 8) + 3\beta_2(3\zeta^2 + 6\zeta + 1)r^2) + \alpha^2\beta(\zeta - 1)Y_1(\beta_1(6\zeta^2 + 32\zeta + 6) + 9\beta_2\zeta(\zeta + 1)r^2) + \beta^3\zeta Y_1(2\beta_1(\zeta^2 + 13\zeta + 2) + \beta_2(3\zeta^2 + 6\zeta + 1)r^2)$$

$$\psi_8 = \psi_{81} + \xi x^5\beta^3(18\beta_1 + \zeta r^2\beta_2) + \xi x^4\beta\psi_{82} + \xi\zeta\psi_{83} + \xi x^3\psi_{84} + \xi x\psi_{85} + \xi x^2\psi_{86}$$

$$\psi_{81} = \beta^3\zeta^2 + 5\alpha^2\beta(\zeta - 1)\zeta + 2\alpha^3(\zeta - 1)^2Y_1 + 4\alpha\beta^2(\zeta - 1)\zeta Y_1$$

$$\psi_{82} = \alpha^2(\zeta - 1)(22\beta_1 + 5\beta_2\zeta r^2) + \beta^2(28\beta_1(2\zeta + 1) + 3\beta_1\zeta(\zeta + 1)r^2) + 4\alpha\beta(\zeta - 1)Y_1(9\beta_1 + \beta_2\zeta r^2)$$

$$\psi_{83} = 3\beta^3\zeta^2(\zeta + 1)(2\beta_1 + \beta_2r^2) + \alpha^2\beta(\zeta - 1)\zeta(\beta_1(30\zeta - 2) + 5\beta_2(3\zeta + 2)r^2) + 2\alpha^3(\zeta - 1)^2Y_1(\beta_1(6\zeta - 2) + \beta_2(3\zeta + 1)r^2) + 4\alpha\beta^2(\zeta - 1)\zeta Y_1(\beta_1(6\zeta + 2) + \beta_2(3\zeta + 2)r^2)$$

$$\psi_{84} = 3\beta^3(\beta_1(20\zeta^2 + 30\zeta + 2) + \beta_2\zeta(\zeta^2 + 3\zeta + 1)r^2) + \alpha^2\beta(\zeta - 1)(\beta_1(42\zeta + 52) + 5\beta_2\zeta(2\zeta + 3)r^2) + 2\alpha^3(\zeta - 1)^2Y_1(2\beta_1 + \beta_2\zeta r^2) + 4\alpha\beta^2(\zeta - 1)Y_1(\beta_1(19\zeta + 16) + \beta_2\zeta(2\zeta + 3)r^2)$$

$$\psi_{85} = \alpha^2\beta(\zeta - 1)\zeta(2\beta_1(5\zeta^2 + 44\zeta + 4) + 5\beta_2(3\zeta^2 + 6\zeta + 1)r^2) + \beta^3\zeta^2(2\beta_1(\zeta^2 + 23\zeta + 9) + 3\beta_2(\zeta^2 + 3\zeta + 1)r^2) + 2\alpha^3(\zeta - 1)^2Y_1(2\beta_1(\zeta^2 + 3\zeta + 1) + 3\beta_2\zeta(\zeta + 1)r^2) + 4\alpha\beta^2(\zeta - 1)\zeta Y_1(\beta_1(2\zeta^2 + 26\zeta + 5) + \beta_2(3\zeta^2 + 6\zeta + 1)r^2)$$

$$\psi_{86} = 5\alpha^2\beta(\zeta - 1)(\beta_1(6\zeta^2 + 22\zeta + 2) + \beta_2\zeta(\zeta^2 + 6\zeta + 3)r^2)$$

$$\begin{aligned}
& +4\alpha\beta^2(\zeta-1)Y_1(3\beta_1(4\zeta^2+12\zeta+1)+\beta_2\zeta(\zeta^2+6\zeta+3)r^2) \\
& +\beta^3\zeta(6\beta_1(4\zeta^2+17\zeta+3)+\beta_2(\zeta^3+9\zeta^2+9\zeta+1)r^2)+2\alpha^3(\zeta-1)^2Y_1(8\beta_1+\beta_2\zeta(\zeta+3)r^2) \\
\psi_9 & =\zeta^2\beta^2\psi_{91}+\xi x^4\beta^2\psi_{92}+\xi x^3\psi_{93}+\xi\zeta\psi_{94}+\xi x^2\psi_{95}+\xi x\psi_{96} \\
\psi_{91} & =\alpha^3(\zeta-1)+3\alpha\beta^2\zeta+3\alpha^2\beta(\zeta-1)Y_1+\beta^3\zeta Y_1. \\
\psi_{92} & =3\alpha(6\beta_1+\beta_2\zeta r^2)+\beta Y_1(8\beta_1+\beta_2\zeta r^2) \\
\psi_{93} & =\alpha^3(\zeta-1)(2\beta_1+\beta_2\zeta r^2)+\alpha\beta^2(\beta_1(38\zeta+36)+3\beta_2\zeta(2\zeta+3)r^2)+3\alpha^2\beta(\zeta-1)Y_1(4\beta_1+\beta_1\zeta r^2) \\
& +\beta^3Y_1(2\beta_1(9\zeta+7)+\beta_2\zeta(2\zeta+3)r^2) \\
\psi_{94} & =\alpha^3(\zeta-1)(\beta_1(6\zeta-2)+\beta_2(3\zeta+1)r^2)+\alpha\beta^2\zeta(2\beta_1(9\zeta+1)+3\beta_1(3\zeta+2)r^2) \\
& +\alpha^2\beta(\zeta-1)Y_1(2\beta_1(9\zeta-1)+3\beta_2(3\zeta+1)r^2)+\beta^3\zeta Y_1(\beta_1(6\zeta+2)+\beta_2(3\zeta+2)r^2) \\
\psi_{95} & =\alpha\beta^2(\beta_1(26\zeta^2+82\zeta+6)+3\beta_2\zeta(\zeta^2+6\zeta+3)r^2)+\alpha^2\beta(\zeta^2+2\zeta-3)Y_1(10\beta_1+3\beta_2\zeta r^2) \\
& +\beta^3Y_1\beta_1((6\zeta^2+17\zeta+1)+\beta_2\zeta(\zeta^2+6\zeta+3)r^2)+\alpha^3(\zeta-1)(8\beta_1+\beta_2\zeta(\zeta+3)r^2) \\
\psi_{96} & =\alpha^3(\zeta-1)(2\beta_1(\zeta^2+3\zeta+1)+3\beta_2\zeta(\zeta+1)r^2)+\alpha\beta^2\zeta(\beta_1(6\zeta^2+64\zeta+8)+3\beta_2(3\zeta^2+6\zeta+1)r^2) \\
& +\alpha^2\beta(\zeta-1)Y_1(\beta_1(6\zeta^2+32\zeta+6)+9\beta_2\zeta(\zeta+1)r^2)+\beta^3\zeta Y_1(2\beta_1(\zeta^2+13\zeta+2)+\beta_2(3\zeta^2+6\zeta+1)r^2) \\
\psi_{10} & =\log\left(\frac{\alpha}{\alpha+\beta Y_1}\right) \\
\psi_{11} & =2(\zeta-1)\zeta(x+1)^3(\zeta+x)(\alpha+\beta Y_1)^2 \\
\psi_{12} & =\beta(A_1\beta(\zeta+x)+2\alpha A_1(\zeta-1)Y_1+2\beta^2 B_1(\zeta-1)(\alpha+\beta Y_1))+2\alpha A_1(\zeta-1)(\alpha+\beta Y_1)\log\left(\frac{\alpha}{\alpha+\beta Y_1}\right) \\
\psi_{13} & =\left(\zeta\beta_2\psi_{131}+\xi^3 R^4\beta\psi_{132}+\xi^2 R^2\psi_{133}+\xi\psi_{134}\right) \\
\psi_{131} & =2\alpha\beta\zeta+\beta^2\zeta Y_2+(\zeta-1)Y_2\alpha^2 \\
\psi_{132} & =\alpha(2\beta_2\zeta R^2-4\beta_1(\zeta-4))+\beta Y_2(\beta_2\zeta R^2-2\beta_1(\zeta-5)). \\
\psi_{133} & =\beta^2 Y_2(\beta_2\zeta(\zeta+2)R^2-2\beta_1(\zeta^2-8\zeta-1))+2\alpha\beta\zeta(\beta_2(\zeta+2)R^2-2\beta_1(\zeta-7)) \\
& +\alpha^2(\zeta-1)Y_2(\beta_2\zeta R^2-2\beta_1(\zeta-3)) \\
\psi_{134} & =2\alpha\beta\zeta(6\beta_1\zeta+2\beta_2\zeta R^2+\beta_2 R^2)+2\alpha^2(\zeta-1)Y_2(\beta_1(3\zeta-1)+\beta_2\zeta R^2) \\
& +\beta^2\zeta Y_2(\beta_1(6\zeta+2)+\beta_2(2\zeta+1)R^2) \\
\psi_{14} & =\left(\zeta\beta^2\psi_{141}+\xi^3 R^4\beta^2(-2(-7+\zeta)\beta_1+\zeta\beta^2\beta_2)+\xi^2 R^2\psi_{142}+\xi\psi_{143}\right) \\
\psi_{141} & =2\alpha\beta\zeta+\alpha^2(\zeta-1)Y_2+\beta^2\zeta Y_2 \\
\psi_{142} & =\beta^2(\beta_1(-2\zeta^2+20\zeta+6)+\beta_2\zeta(\zeta+2)R^2)-2\alpha^2(\zeta-1)(2\beta_1(\zeta-3)-\beta_2\zeta R^2) \\
& +\alpha\beta(\zeta-1)Y_2(\beta_1(26-6\zeta)+3\beta_2\zeta R^2) \\
\psi_{143} & =4\alpha^2(\zeta-1)(\beta_1(3\zeta-1)+\beta_2\zeta R^2)+\beta^2\zeta(6\beta_1(\zeta+1)+\beta_2(2\zeta+1)R^2) \\
& +2\alpha\beta(\zeta-1)Y_2(\beta_1+9\beta_1\zeta+3\beta_2\zeta R^2) \\
Z & =\left(\zeta\beta^2 Z_1+\xi^3 R^4\beta Z_2+\xi^2 R^2 Z_3+\xi Z_4\right)\left(\log\left[\frac{\alpha}{\alpha+Y_2}\right]\right) \\
Z_1 & =2\alpha\beta\zeta+\alpha^2(\zeta-1)Y_2+\beta^2\zeta Y_2 \\
Z_2 & =\alpha(2\beta_2\zeta R^2-4\beta_1(\zeta-4))+\beta Y_2(\beta_2\zeta R^2-2\beta_1(\zeta-5)) \\
Z_3 & =\beta^2 Y_2(\beta_2\zeta(\zeta+2)R^2-2\beta_1(\zeta^2-8\zeta-1))+2\alpha\beta\zeta(\beta_2(\zeta+2)R^2-2\beta_1(\zeta-7)) \\
& +\alpha^2(\zeta-1)Y_2(\beta_2\zeta R^2-2\beta_1(\zeta-3)) \\
Z_4 & =2\alpha\beta\zeta(6\beta_1\zeta+2\beta_2\zeta R^2+\beta_2 R^2)+2\alpha^2(\zeta-1)Y_2(\beta_1(3\zeta-1)+\beta_2\zeta R^2)+\beta^2\zeta Y_2(\beta_1(6\zeta+2)+\beta_2(2\zeta+1)R^2) \\
\psi_{16} & =\left(1-\frac{\psi_{17}}{\psi_{18}}\right)
\end{aligned}$$

$$\begin{aligned}
\psi_{17} &= 2(-1 + \zeta)\alpha \frac{1}{2} \left(1 + \frac{Y_2\beta}{2} - \frac{\alpha}{\alpha + Y_2\beta} \right) \\
&\quad + 2(-1 + \zeta) \log \left[\frac{\alpha}{\alpha + Y_2\beta} \right] \left(\zeta\beta_2\psi_{171} + \xi^3 R^4 \beta\psi_{172} + \xi^2 R^2 \psi_{173} + \xi\psi_{174} \right) \\
\psi_{171} &= 2\alpha\beta\zeta + \alpha^2(\zeta - 1)Y_2 + \beta^2\zeta Y_2 \\
\psi_{172} &= \alpha(2\beta_2\zeta R^2 - 4\beta_1(\zeta - 4)) + \beta Y_2(\beta_2\zeta R^2 - 2\beta_1(\zeta - 5)) \\
\psi_{173} &= \beta^2 Y_2 (\beta_2\zeta(\zeta + 2)R^2 - 2\beta_1(\zeta^2 - 8\zeta - 1)) + 2\alpha\beta\zeta (\beta_2(\zeta + 2)R^2 - 2\beta_1(\zeta - 7)) \\
&\quad + \alpha^2(\zeta - 1)Y_2 (\beta_2\zeta R^2 - 2\beta_1(\zeta - 3)) \\
\psi_{174} &= 2\alpha\beta\zeta (6\beta_1\zeta + 2\beta_2\zeta R^2 + \beta_2 R^2) + 2\alpha^2(\zeta - 1)Y_2 (\beta_1(3\zeta - 1) + \beta_2\zeta R^2) \\
&\quad + \beta^2\zeta Y_2 (\beta_1(6\zeta + 2) + \beta_2(2\zeta + 1)R^2) \\
\psi_{18} &= (\zeta + \xi R^2)\beta \left(\zeta\beta_2\psi_{181} + \xi^3 R^4 \beta^2\psi_{182} + \xi^2 R^2 \psi_{183} + \xi\psi_{184} \right) \\
&\quad + 2(-1 + \zeta)\alpha \left(\zeta\beta_2\psi_{185} + \xi^3 R^3 \beta\psi_{186} + \xi^2 R^2 \psi_{187} + \xi\psi_{188} \log \left[\frac{\alpha}{\alpha + Y_2\beta} \right] \right) \\
\psi_{181} &= 2\alpha^2(\zeta - 1) + \beta^2\zeta + 3\alpha\beta(\zeta - 1)Y_2 \\
\psi_{182} &= \beta_2\zeta R^2 - 2\beta_1(\zeta - 7) \\
\psi_{183} &= \beta^2 (\beta_1(-2\zeta^2 + 20\zeta + 6) + \beta_2\zeta(\zeta + 2)R^2) - 2\alpha^2(\zeta - 1) (2\beta_1(\zeta - 3) - \beta_2\zeta R^2) \\
&\quad + \alpha\beta(\zeta - 1)Y_2 (\beta_1(26 - 6\zeta) + 3\beta_2\zeta R^2) \\
\psi_{184} &= 4\alpha^2(\zeta - 1) (\beta_1(3\zeta - 1) + \beta_2\zeta R^2) + \beta^2\zeta (6\beta_1(\zeta + 1) + \beta_2(2\zeta + 1)R^2) \\
&\quad + 2\alpha\beta(\zeta - 1)Y_2 (\beta_1 + 9\beta_1\zeta + 3\beta_2\zeta R^2) \\
\psi_{185} &= 2\alpha\beta\zeta + \alpha^2(\zeta - 1)Y_2 + \beta^2\zeta Y_2 \\
\psi_{186} &= \alpha (2\beta_2\zeta R^2 - 4\beta_1(\zeta - 4)) + \beta Y_2 (\beta_2\zeta R^2 - 2\beta_1(\zeta - 5)) \\
\psi_{187} &= \beta^2 Y_2 (\beta_2\zeta(\zeta + 2)R^2 - 2\beta_1(\zeta^2 - 8\zeta - 1)) + 2\alpha\beta\zeta (\beta_2(\zeta + 2)R^2 - 2\beta_1(\zeta - 7)) \\
&\quad + \alpha^2(\zeta - 1)Y_2 (\beta_2\zeta R^2 - 2\beta_1(\zeta - 3)) \\
\psi_{188} &= 2\alpha\beta\zeta (6\beta_1\zeta + 2\beta_2\zeta R^2 + \beta_2 R^2) \\
&\quad + 2\alpha^2(\zeta - 1)Y_2 (\beta_1(3\zeta - 1) + \beta_2\zeta R^2) + \beta^2\zeta Y_2 (\beta_1(6\zeta + 2) + \beta_2(2\zeta + 1)R^2) \\
e^{\nu(r)} &= \left((1 - Y_1^2)^{\frac{1}{4}} \left(\frac{\alpha + \beta \times Y_1}{Y_1} \left(\frac{\alpha}{\beta^3} A_1 \left(\frac{\sec t \times \sec t - \cos t \times \cos t}{2} \right) + \log \left[\cos t \times \cos t \right] \right) + B_1 \right) \right)^2
\end{aligned}$$

Data availability

No data was used for the research described in the article.

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