



Optimal classes of estimators for population mean using higher order moments

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Abstract

This paper considers some optimal classes of difference and ratio type estimators for the estimation of population mean using higher order moments viz variance of auxiliary variable with the aim of improvement over its entrants existing till date. The bias and mean square error of the considered estimators are derived using Taylor series method up to the first order of approximation. The theoretical results have been determined and appraised with a computational study using real and artificially generated data sets. The computational results are turned out to be rather advance providing better improvement over the contemporary estimators.

Keywords Class of estimators · Efficiency · Higher order moments

Mathematic Subject Classification 62D05

1 Introduction

In the theory of sample surveys, it is well known fact that the primary knowledge of auxiliary information namely standard deviation S_x , mean, coefficient of skewness $\beta_1(x)$, coefficient of kurtosis $\beta_2(x)$ meliorate the performance of the estimators. Literature contains a wide range of ratio, regression and exponential type estimators consist of different auxiliary information. [1] utilized the auxiliary information and suggested regression estimator. [2] considered the ratio estimator using mean of auxiliary variable. [3] envisaged ratio estimator with the help of auxiliary information. [4] suggested ratio estimator using supplementary information. [5] utilized auxiliary information and investigated ratio estimator. [6] introduced difference type estimator using simple random sampling (SRS). [7] suggested exponential estimator using auxiliary information. [8] suggested a family of estimator using auxiliary information. [9] suggested improved estimator under SRS whereas [10] suggested exponential ratio type estimator under SRS. [11] utilized auxiliary information and suggested an efficient class of estimators for population mean. Some log-type classes of estimators based on auxiliary information were suggested by [12]. [13] introduced a generalized ratio-cum-product estimator

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for estimating population mean, while, a new class of estimator for estimating finite population mean was introduced by [14]. [15] developed an efficient family of estimator for finite population mean under SRS, while, a new efficient class of finite population mean estimators were designed and analyzed by [16]. [17] examined an improved predictive approach for estimating the population mean. [18] performed a simulation study for a new logarithmic ratio type estimator of population mean under SRS. [19] advocated mean estimation utilizing an efficient class of estimators based on SRS. [20] suggested an enhanced estimation of population mean utilizing SRS. [21] created an enhanced direct and synthetic estimators for domain mean along with the simulation and applications. Some direct and synthetic logarithmic estimators were introduced for small area estimation by [22]. [23] devised an improved mean estimation for a finite population utilizing bivariate auxiliary data under SRS. [24] introduced an elevated computational and simulation approach for estimating the population mean on current occasion. Some enhanced optimization problems are recently solved employing different optimizers by [25] and [26].

Consider a finite population $\Omega = (\Omega_1, \Omega_2, \dots, \Omega_N)$ based on N distinguishable units. Let a sample of size n be measured from the finite population Ω of size N using simple random sampling without replacement scheme. Let y_i and x_i be the values of the study and auxiliary variables for the unit $i, i = 1, 2, \dots, N$ of the population. Further, let $(\bar{Y} = N^{-1} \sum_{i=1}^N Y_i; \bar{X} = N^{-1} \sum_{i=1}^N X_i)$ and $(\bar{y} = n^{-1} \sum_{i=1}^n y_i; \bar{x} = n^{-1} \sum_{i=1}^n x_i)$ be the population and sample means of the study variable y and auxiliary variable x , respectively. The population mean square of the study variable y is given by $S_y^2 = (N-1)^{-1} \sum_{i=1}^N (y_i - \bar{Y})^2$ and auxiliary variable x is given by $S_x^2 = (N-1)^{-1} \sum_{i=1}^N (x_i - \bar{X})^2$. The sample mean square of the study variable y is given by $s_y^2 = (n-1)^{-1} \sum_{i=1}^n (y_i - \bar{y})^2$ and auxiliary variable x is given by $s_x^2 = (n-1)^{-1} \sum_{i=1}^n (x_i - \bar{x})^2$. The variation coefficients for the study and auxiliary variables are denoted by C_y and C_x , respectively. The correlation coefficient between variables y and x is given by ρ_{xy} .

In order to compare the performance of the proposed estimators, we consider the review of the well-known estimators of population mean under SRS.

The usual estimator of population mean $\bar{Y}$ of the study variable y is given as

$$T_m = \bar{y}$$

The classical ratio estimator for the estimation of population mean $\bar{Y}$ of the study variable y under SRS is given as

$$T_r = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)$$

The classical regression estimator under SRS is given as

$$T_{lr} = \bar{y} + \beta(\bar{X} - \bar{x})$$

where $\beta = \rho_{xy}(S_y/S_x)$ is the regression coefficient of y on x .

[2] introduced the power ratio estimator under SRS as

$$T_{s1} = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^\Theta$$

where Θ is a suitably chosen constant.

[5] introduced an improved ratio estimator under SRS as

$$T_p = k\bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)$$

where k is an optimizing scalar.

On the lines of [5], an improved form of [2] estimator is mentioned here under SRS as

$$T_{s_2} = \Theta_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{\Delta_1}$$

where Θ_1 and Δ_1 are suitably chosen scalars.

[6] introduced difference type estimator under SRS as

$$T_R = \alpha_1 \bar{y} + \alpha_2 (\bar{X} - \bar{x})$$

where α_1 and α_2 are optimizing scalars.

[8] envisaged a family of estimators under SRS as

$$T_k = \bar{y} \left(\frac{\bar{X}^*}{\alpha \bar{x}^* + (1 - \alpha) \bar{X}^*} \right)^g$$

where α is a constant and $\bar{x}^* = a\bar{x} + b$, $\bar{X}^* = a\bar{X} + b$, a and b are either real values or function of known parameters of auxiliary variable x .

On the lines of [27], the generalized form of [8] family of estimators under SRS is defined as

$$T_{kk} = \lambda_k \bar{y} \left(\frac{\bar{X}^*}{\alpha \bar{x}^* + (1 - \alpha) \bar{X}^*} \right)^g$$

[10] proposed the difference-cum-exponential type estimators under SRS as

$$T_{sg} = \{k_1 \bar{y} + k_2 (\bar{X} - \bar{x})\} \exp \left\{ \frac{(\bar{X} - \bar{x})}{(\bar{X} + \bar{x} + 2N\bar{X})} \right\}$$

where k_1 and k_2 are optimizing scalars.

[11] considered a class of estimators under SRS as

$$T_{ss_1} = w_1 \bar{y} \left(\frac{\bar{X}^*}{\alpha \bar{x}^* + (1 - \alpha) \bar{X}^*} \right)^g + w_2 \bar{y} \exp \left\{ \frac{h(\bar{X}^* - \bar{x}^*)}{(\bar{X}^* + \bar{x}^*)} \right\}$$

where w_1 and w_2 are suitably chosen scalars to be determine later.

[13] suggested ratio-cum-product type class of estimators of the population mean under SRS as

$$T_{ss_2} = \bar{y} \left(\frac{a\bar{X} + b}{a\bar{x} + b} \right)^g \left(\frac{a\bar{x} + b}{a\bar{X} + b} \right)^{(1-g)} = \left(\frac{a\bar{X} + b}{a\bar{x} + b} \right)^{2g-1}$$

[14] suggested the following class of estimator under SRS as

$$T_{ss_3} = \bar{y} \left(\frac{a\bar{X} + b\bar{x}}{c\bar{x} + d\bar{X}} \right)$$

where a , b , c , d are the function of known parameters of the auxiliary variable x .

On the lines of [28], we define a regression type estimator using higher order moment such as variance of auxiliary variable x under SRS as

$$T_{ms} = \bar{y} + \beta_2 (\bar{X} - \bar{x}) + \theta_2 (S_x^2 - s_x^2)$$

where β_i , $i = 1, 2$, θ_i are optimizing scalars.

Remark 1.1 It is to be noted that the ratio estimator based on higher order moment such as variance of auxiliary variable x furnishes equal efficiency as the regression estimator T_{ms} defined on the lines of [28] under the optimal conditions.

[15] introduced a class of difference-cum-exponential type estimator under SRS as

$$T_{p1} = \left[\bar{y} \left\{ \alpha \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + (1 - \alpha) \exp \left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right\} + \eta_1 (\bar{X} - \bar{x}) + \eta_2 \bar{y} \right] \\ \times \left\{ \delta \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + (1 - \delta) \exp \left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right\}$$

where α and δ are constants such that $0 \leq (\alpha, \delta) \leq 1$. Also, η_1 and η_2 are suitably chosen scalars.

[16] envisaged the following class of estimators under SRS as

$$T_{p2} = \left[K_1 \bar{y} \left(\frac{\bar{X}^*}{\alpha \bar{x}^* + (1 - \alpha) \bar{X}^*} \right)^g \exp \left\{ \frac{h(\bar{X}^* - \bar{x}^*)}{(\bar{X}^* + \bar{x}^*)} \right\} \right. \\ \left. + K_2 \{ \alpha_1 \bar{y} + (\alpha_2 \bar{y} + \alpha_3)(\bar{X} - \bar{x}) \} \left(\frac{\bar{X}^*}{\eta \bar{x}^* + (1 - \eta) \bar{X}^*} \right)^\delta \exp \left\{ \frac{\phi(\bar{X}^* - \bar{x}^*)}{(\bar{X}^* + \bar{x}^*)} \right\} \right]$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha, g, h, \eta, \delta, \phi$ are the scalars assuming real values for generating different estimators and K_1, K_2 are the suitably chosen scalars. The mean square error (MSE) of the above reviewed estimators is given in Appendix A.

The rest of the paper is divided into following sections. Section 2 considers the suggested optimal classes of estimators based on higher order moments under SRS along with their properties. Section 3 is devoted to the optimality conditions under which the proposed classes of estimators dominate their contemporary estimators. The optimality conditions are evaluated numerically using real and simulated populations and the results are interpreted under Sect. 4. Finally, the paper is concluded in Sect. 5.

2 Suggested classes of estimators

In survey sampling, the primary objective of the researchers is to develop more efficient estimators than the existing ones. With this objective of improvement over the estimators discussed in previous section, motivated by [29, 30], and [31], we propose some optimal classes of difference and ratio type estimators for the estimation of population mean $\bar{Y}$ using higher order moments namely variance of auxiliary variable x as

$$T_{y1} = \zeta_1 \bar{y} + \delta_1 (\bar{x} - \bar{X}) + \phi_1 (s_x^2 - S_x^2) \\ T_{y2} = \zeta_2 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{\delta_2} \left(\frac{S_x^2}{s_x^2} \right)^{\phi_2} \\ T_{y3} = \zeta_3 \bar{y} \left(\frac{\bar{X}}{\bar{X} + \delta_3 (\bar{x} - \bar{X})} \right) \left(\frac{S_x^2}{S_x^2 + \phi_3 (s_x^2 - S_x^2)} \right)$$

where ζ_i, δ_i and $\phi_i, i = 1, 2, 3$ are suitably chosen optimizing scalars.

Theorem 2.1 The bias and minimum MSE of the proposed class of estimators T_{y1} are given by

$$Bias(T_{y1}) = \bar{Y} (\zeta_1 - 1) \quad (2.1)$$

$$min.MSE(T_{y1}) = \bar{Y}^2 (1 - \zeta_1(opt)) \quad (2.2)$$

Proof Refer to appendix B for definitions of parameters and outline of derivations. $\square$

Theorem 2.2 The bias and minimum MSE of the proposed class of estimators T_{y_i} , $i = 2, 3$ are given up to first order approximation by

$$\text{Bias}(T_{y_i}) \approx \bar{Y} (\zeta_i Q_i - 1) \quad (2.3)$$

$$\min.MSE(T_{y_i}) \approx \bar{Y}^2 \left(1 - \frac{Q_i^2}{P_i} \right) \quad (2.4)$$

Proof Refer to appendix B for outline of the derivations and the definition of the parametric functions P_i and Q_i , $i = 2, 3$. $\square$

Corollary 2.1 The proposed class of ratio type estimators T_{y_i} , $i = 2, 3$ dominate the proposed class of difference type estimator T_{y_1} , iff

$$\frac{Q_i^2}{P_i} > \zeta_{1(opt)} \quad (2.5)$$

and vice versa. Otherwise both are equally efficient in case of equality holds in (2.5).

Proof On comparing the minimum MSE of these estimators, we can obtain (2.5). $\square$

3 Optimality conditions

In this section, we compare the minimum MSE of the proposed class of estimators T_{y_i} , $i=1,2,3$ from (2.2) and (2.4) with the minimum MSE of existing estimators from (A.1), (A.2), (A.5), (A.7), (A.9), (A.11), (A.14), (A.15), (A.17), (A.18), (A.20), (A.22) and (A.24), we obtain the following optimality conditions.

$$MSE(T_{y_1}) < V(T_m) \implies \zeta_{1(opt)} > 1 - \gamma C_y^2 \quad (3.1)$$

$$MSE(T_{y_i}) < V(T_m), i = 2, 3 \implies \frac{Q_i^2}{P_i} > 1 - \gamma C_y^2 \quad (3.2)$$

$$MSE(T_{y_1}) < MSE(T_r) \implies \zeta_{1(opt)} > 1 - \gamma(C_y^2 + C_x^2 - 2\rho_{xy}C_xC_y) \quad (3.3)$$

$$MSE(T_{y_i}) < MSE(T_r), i = 2, 3 \implies \frac{Q_i^2}{P_i} > 1 - \gamma(C_y^2 + C_x^2 - 2\rho_{xy}C_xC_y) \quad (3.4)$$

$$MSE(T_{y_1}) < MSE(T) \implies \zeta_{1(opt)} > 1 - \gamma C_y^2(1 - \rho_{xy}^2) \quad (3.5)$$

$$MSE(T_{y_i}) < MSE(T), i = 2, 3 \implies \frac{Q_i^2}{P_i} > 1 - \gamma C_y^2(1 - \rho_{xy}^2) \quad (3.6)$$

$$\begin{aligned} MSE(T_{y_1}) < MSE(T_p) &\implies \zeta_{1(opt)} \\ &> 1 - [\gamma(k^{*2}C_x^2 + C_y^2 - 2k^*\rho_{xy}C_xC_y) + (k^* - 1)^2] \end{aligned} \quad (3.7)$$

$$\begin{aligned} MSE(T_{y_i}) < MSE(T_p), i = 2, 3 \\ \implies \frac{Q_i^2}{P_i} &> 1 - [\gamma(k^{*2}C_x^2 + C_y^2 - 2k^*\rho_{xy}C_xC_y) + (k^* - 1)^2] \end{aligned} \quad (3.8)$$

$$MSE(T_{y_1}) < MSE(T_{s_2}) \implies \zeta_{1(opt)} > \frac{B_s^2}{A_s} \quad (3.9)$$

$$MSE(T_{y_i}) < MSE(T_{s_2}) \implies \frac{Q_i^2}{P_i} > \frac{B_s^2}{A_s} \quad (3.10)$$

$$MSE(T_{y_1}) < MSE(T_R) \implies \zeta_{1(opt)} > \alpha_{1(opt)} \quad (3.11)$$

$$MSE(T_{y_i}) < MSE(T_R), i = 2, 3 \implies \frac{Q_i^2}{P_i} > \alpha_{1(opt)} \quad (3.12)$$

$$MSE(T_{y_1}) < MSE(T_{sg}) \implies \zeta_{1(opt)} > 1 - \left\{ 1 - \frac{\gamma C_x^2}{4(N+1)^2} \right\} + \frac{\left\{ 1 - \frac{\gamma C_x^2}{8(N+1)^2} \right\}^2}{\{1 + \gamma C_y^2(1 - \rho_{xy}^2)\}} \quad (3.13)$$

$$MSE(T_{y_i}) < MSE(T_{sg}), i = 2, 3 \implies \frac{Q_i^2}{P_i} > 1 - \left\{ 1 - \frac{\gamma C_x^2}{4(N+1)^2} \right\} + \frac{\left\{ 1 - \frac{\gamma C_x^2}{8(N+1)^2} \right\}^2}{\{1 + \gamma C_y^2(1 - \rho_{xy}^2)\}} \quad (3.14)$$

$$MSE(T_{y_1}) < MSE(T_{ss_1}) \implies \zeta_{1(opt)} > \frac{(H_2 H_4^2 + H_1 H_5^2 - 2H_3 H_4 H_5)}{(H_1 H_2 - H_3^2)} \quad (3.15)$$

$$MSE(T_{y_i}) < MSE(T_{ss_1}), i = 2, 3 \implies \frac{Q_i^2}{P_i} > \frac{(H_2 H_4^2 + H_1 H_5^2 - 2H_3 H_4 H_5)}{(H_1 H_2 - H_3^2)} \quad (3.16)$$

$$MSE(T_{y_1}) < MSE(T_{ss_2}) \implies \zeta_{1(opt)} > 1 - \gamma C_y^2(1 - \rho_{xy}^2) \quad (3.17)$$

$$MSE(T_{y_i}) < MSE(T_{ss_2}), i = 2, 3 \implies \frac{Q_i^2}{P_i} > 1 - \gamma C_y^2(1 - \rho_{xy}^2) \quad (3.18)$$

$$MSE(T_{y_1}) < MSE(T_{ss_3}) \implies \zeta_{1(opt)} > 1 - \gamma[C_y^2 + (\psi_1 - \psi_2)^2 C_x^2 - 2(\psi_1 - \psi_2)\rho_{xy}C_x C_y] \quad (3.19)$$

$$MSE(T_{y_i}) < MSE(T_{ss_3}), i = 2, 3 \implies \frac{Q_i^2}{P_i} > 1 - \gamma[C_y^2 + (\psi_1 - \psi_2)^2 C_x^2 - 2(\psi_1 - \psi_2)\rho_{xy}C_x C_y] \quad (3.20)$$

$$MSE(T_{y_1}) < MSE(T_{p_1}) \implies \zeta_{1(opt)} > 1 - \left[\gamma(C_y^2 + \theta^2 C_x^2 + 2\theta\rho_{xy}C_x C_y) - \frac{(D_2 D_4^2 + D_1 D_5^2 - 2D_3 D_4 D_5)}{D_1 D_2 - D_3^2} \right] \quad (3.21)$$

$$MSE(T_{y_i}) < MSE(T_{p_1}), i = 2, 3 \implies \frac{Q_i^2}{P_i} > 1 - \left[\gamma(C_y^2 + \theta^2 C_x^2 + 2\theta\rho_{xy}C_x C_y) - \frac{(D_2 D_4^2 + D_1 D_5^2 - 2D_3 D_4 D_5)}{D_1 D_2 - D_3^2} \right] \quad (3.22)$$

$$MSE(T_{y_1}) < MSE(T_{p_2}) \implies \zeta_{1(opt)} > \frac{(E_2 E_4^2 + E_1 E_5^2 - 2E_3 E_4 E_5)}{(E_1 E_2 - E_3^2)} \quad (3.23)$$

$$MSE(T_{y_i}) < MSE(T_{p_2}), i = 2, 3 \implies \frac{Q_i^2}{P_i} > \frac{(E_2 E_4^2 + E_1 E_5^2 - 2E_3 E_4 E_5)}{(E_1 E_2 - E_3^2)} \quad (3.24)$$

$$MSE(T_{y_1}) < MSE(T_{ms}) \implies \zeta_{1(opt)} > 1 - \gamma C_y^2 \left[1 - \rho_{xy}^2 - \frac{(\lambda_{12} - \rho_{xy}\lambda_{03})^2}{\lambda_{04} - 1 - \lambda_{03}^2} \right] \quad (3.25)$$

$$MSE(T_{y_i}) < MSE(T_{ms}), i = 2, 3$$

$$\Rightarrow \frac{Q_i^2}{P_i} > 1 - \gamma C_y^2 \left[1 - \rho_{xy}^2 - \frac{(\lambda_{12} - \rho_{xy} \lambda_{03})^2}{\lambda_{04} - 1 - \lambda_{03}^2} \right] \quad (3.26)$$

The above efficiency conditions are important because under these conditions the proposed estimators will dominate the existing ones.

4 Computational study

The computational study is further divided in two heads namely, numerical study using real data and simulation study using artificially generated data.

4.1 Numerical study

In order to assess the merits of the proposed class of estimators over the existing estimators, we have carried out a numerical study using three different populations, which are given below:

- (1). The data are taken from [32, p. 185], concerns the values of total cultivated area and of area under wheat in two consecutive years. The necessary parameters of the population are:

Y_i =area under wheat in 1937,

X_i =total cultivated area in 1931.

$N=34, n=9, \bar{Y}=201.4118, \bar{X}=765.3529, S_y^2=23154.8, S_x^2=222931.4, \lambda_{03}=0.7083,$

$\lambda_{04}=2.6618, \lambda_{12}=0.7452, \rho_{xy}=0.8992$

- (2). The data are taken from [33, p. 652] with the following descriptions:

Y_i =total number of seats in municipality council in 1982,

X_i =number of conservative seats in municipality council in 1982.

$N=284, n=80, r=50, \bar{Y}=46.0704, \bar{X}=9.0950, S_y^2=158.5533, S_x^2=24.3690, \lambda_{03}=1.2412,$

$\lambda_{04}=5.3688, \lambda_{12}=0.7729, \rho_{xy}=0.6878$

- (3). The data are taken from [34, pp. 399], concerning to the area under wheat in a region in 1963–1964. The necessary parameters are given below:

Y_i =area under wheat in 1964,

X_i =area under wheat in 1963.

$N=80, n=25, \bar{Y}=199.4412, \bar{X}=208.8824, S_y^2=22564.56, S_x^2=22652.05, \lambda_{03}=0.9206,$

$\lambda_{04}=2.8266, \lambda_{12}=0.9405, \rho_{xy}=0.98$

The outcomes of numerical study are displayed in Table 1 for each population.

4.2 Simulation study

To verify the optimality conditions, we have accomplished a simulation study. A bivariate normal population of size $N = 500$ is generated artificially from R software with parameters $\bar{Y} = 15, \bar{X} = 20, \sigma_y = 20, \sigma_x = 25$, and different values of correlation coefficient $\rho_{xy} = \pm 0.9, \pm 0.8, \pm 0.7$ and ± 0.6 . We have drawn 10 random samples each of size 100 unit from the above population. Using 20,000 iterations, MSE of the proposed class of estimators are computed using the following expression.

$$MSE(T_i) = \frac{1}{20,000} \sum_{i=1}^{20,000} (T_i - \bar{Y})^2$$

Table 1 MSE and PRE of different estimators

Estimators	Population 1		Population 2		Population 3	
	MSE	PRE	MSE	PRE	MSE	PRE
T_m	1891.73	100.000	1.42363	100.000	620.525	100
T_r	375.071	504.3661	3.14894	45.2097	24.9111	2490.958
T_{lr}/T_{s1}	362.152	522.3586	0.75015	189.7781	24.5728	2525.2530
T_p	378.535	499.7506	3.15705	45.0935	24.9425	2487.8210
T_{s2}	374.403	505.2656	0.74930	189.9935	26.7967	2315.6740
T_R	358.947	527.0219	0.74989	189.8452	24.5576	2526.8130
T_{kk}	358.947	527.0219	0.74989	189.8449	24.5576	2526.8130
T_{sg}	359.162	526.7068	0.74998	189.8223	24.6206	2520.3450
T_{ss1}	360.572	524.6468	0.74958	189.9236	24.5679	2525.7480
T_{ss3}	1304.35	145.0319	0.76417	186.2955	446.517	138.9701
T_{p1}	464.345	407.3979	0.74707	190.5608	39.6561	1564.7650
T_{p2}	357.408	529.2909	0.74982	189.8626	24.3830	2544.9100
T_{ms}	343.027	551.4812	0.74686	190.6131	23.6425	2624.6130
T_{y1}	340.186	556.0871	0.74660	190.6802	23.6285	2626.1640
T_{y2}	339.004	558.0262	0.74629	190.7601	23.6120	2627.9990
T_{y3}	341.626	553.7434	0.74657	190.6869	23.6423	2624.6310

where $T_i = T_m, T_r, T_{lr}, T_{s1}, T_p, T_{s2}, T_R, T_k, T_{kk}, T_{sg}, T_{ssi}, i = 1, 2, 3, T_{ms}$ and $T_{yi}, i = 1, 2, 3$. We have reported the simulation outcomes from Tables 2, 3, 4, 5, 6, 7, 8 and 9 for different values of correlation coefficient ρ_{xy} .

4.3 Interpretation of computational outcomes

We construed the elucidation of computational outcomes as follows:

- Table 1 based on the results of real populations 1 to 3 show the dominance of the proposed classes of estimators $T_{yi}, i = 1, 2, 3$ in terms of lesser MSE and greater PRE over the usual mean estimator T_m , classical ratio estimator T_r , classical regression estimator T_{lr} , [2] estimator T_{s1} , [5] estimator T_p , improved form of [2] estimator T_{s2} , [6] estimator T_R , [8] estimator T_k , generalized form of [8] estimator T_{kk} , [10] estimator T_{sg} , [11] estimator T_{ss1} , [13] estimator T_{ss2} , [14] estimator T_{ss3} , higher order moments based regression estimator T_{ms} envisaged on the lines of [15, 28] estimator T_{p1} , and [16] estimator T_{p2} .
- Table 2 consists of the results of artificially generated normal population for $\rho_{xy} = 0.9$ exhibit the dominance of the proposed classes of estimators $T_{yi}, i = 1, 2, 3$ over their conventional counterparts in each sample by least MSE.
- The simulation outcomes of Tables 3, 4, 5, 6, 7, 8 and 9 for different values of correlation coefficient show the similar proclivity as discussed in point (ii).
- Moreover, the proposed estimator T_{y2} is found to be best among the proposed classes in both real population and artificially generated population.

Table 2 MSE of different estimators at $\rho_{xy} = 0.9$

Sample Estimators	1	2	3	4	5	6	7	8	9	10
T_m	4.34603	3.22225	3.04413	2.80478	3.15322	3.13114	3.80303	2.60932	3.68780	3.15435
T_r	6.69652	5.67711	4.61440	5.22667	5.27704	5.80874	6.93136	5.31150	5.68155	4.70060
T_{lr}/T_{s1}	4.34426	3.21418	3.01374	2.78383	3.14952	3.11867	3.79492	2.57973	3.66396	3.10533
T_p	6.64431	5.62967	4.58335	5.18037	5.24499	5.77504	6.88261	5.27735	5.60834	4.66580
T_{s2}	4.27718	3.16825	2.96968	2.74468	3.11182	3.07837	3.73911	2.55444	3.57849	3.05374
T_R	4.27815	3.16620	2.97269	2.74135	3.11284	3.08061	3.74135	2.55143	3.57327	3.05803
T_{kk}	4.27831	3.16634	2.97277	2.74154	3.11293	3.08071	3.74153	2.55152	3.57358	3.05813
T_{sg}	4.27815	3.16620	2.97269	2.74135	3.11284	3.08061	3.74135	2.55143	3.57327	3.05803
T_{ss1}	4.29269	3.18461	2.97834	2.76118	3.12245	3.09198	3.75759	2.56737	3.60152	3.06298
T_{ss3}	4.35082	3.21420	3.06937	2.78450	3.15760	3.20528	3.89557	2.59212	3.66655	3.21993
T_{p1}	4.32102	3.19280	2.99694	2.76359	3.13211	3.09254	3.76139	2.56343	3.63968	3.08587
T_{p2}	4.21974	3.12579	2.91724	2.70876	3.06806	3.00994	3.65436	2.52684	3.53559	2.98838
T_{ms}	4.17366	3.16837	2.92208	2.69594	3.09804	3.04015	3.67542	2.55288	3.46405	2.99493
T_{y1}	4.11299	3.12177	2.88363	2.65624	3.06258	3.00406	3.62535	2.52518	3.38389	2.95114
T_{y2}	4.10166	3.11816	2.87223	2.65355	3.05714	2.99417	3.61559	2.52483	3.37530	2.93970
T_{y3}	4.11257	3.12167	2.88316	2.65614	3.06256	3.00371	3.62520	2.52517	3.38289	2.95049

Table 3 MSE of different estimators at $\rho_{xy} = 0.8$

Sample Estimators	1	2	3	4	5	6	7	8	9	10
T_m	3.57472	2.97374	3.41184	2.82411	3.75125	3.20720	2.81512	2.51487	3.88758	3.62092
T_r	5.44149	6.05725	5.82767	4.27417	7.81166	4.65332	4.68086	5.31303	7.96132	7.00608
T_{lr}/T_{s1}	3.57394	2.96716	3.40210	2.81707	3.66551	3.15980	2.78767	2.51422	3.72659	3.61458
T_p	5.40496	6.02579	5.76464	4.23782	7.74931	4.62246	4.65364	5.27691	7.89818	6.93764
T_{s2}	3.52736	2.93979	3.33890	2.76778	3.62540	3.11276	2.75168	2.47967	3.69084	3.55274
T_R	3.52687	2.93844	3.33601	2.76947	3.61808	3.11643	2.75442	2.47909	3.68147	3.55019
T_{kk}	3.52697	2.93853	3.33628	2.76958	3.61828	3.11653	2.75448	2.47923	3.68164	3.55048
T_{sg}	3.52687	2.93844	3.33601	2.76947	3.61808	3.11643	2.75442	2.47909	3.68147	3.55019
T_{ss1}	3.53851	2.95264	3.35983	2.77803	3.64815	3.12059	2.76081	2.49597	3.71252	3.57853
T_{ss3}	3.58160	2.96755	3.40574	2.84305	3.69094	3.23015	2.85666	2.57155	3.85000	3.65918
T_{p1}	3.55945	2.94794	3.37549	2.80007	3.63961	3.14373	2.76940	2.48641	3.70610	3.57655
T_{p2}	3.49525	2.90154	3.28660	2.72141	3.58712	3.05827	2.69746	2.41983	3.66661	3.47518
T_{ms}	3.53271	2.92725	3.26592	2.69783	3.54528	3.05239	2.70133	2.42469	3.47806	3.51917
T_{y1}	3.48673	2.89931	3.20537	2.65448	3.50110	3.01210	2.67022	2.39218	3.43951	3.45829
T_{y2}	3.48084	2.89679	3.20283	2.64639	3.49826	3.00179	2.66200	2.38514	3.43937	3.45396
T_{y3}	3.48671	2.89924	3.20406	2.65421	3.50139	3.01194	2.67008	2.39190	3.43900	3.45791

Table 4 MSE of different estimators at $\rho_{xy} = 0.7$

Sample Estimators	1	2	3	4	5	6	7	8	9	10
T_m	2.68930	2.68165	2.84179	3.67366	3.52362	3.81966	3.07149	3.12694	3.88311	3.78644
T_r	5.95967	5.28363	4.16837	4.96807	9.07690	6.50757	6.29118	6.44732	5.13751	6.81021
T_{lr}/T_{s1}	2.67269	2.67926	2.77782	3.67355	3.50598	3.81945	3.05401	3.10768	3.87090	3.78464
T_p	5.92673	5.25760	4.13188	4.91958	9.04115	6.45844	6.25010	6.40905	5.09461	6.76915
T_{s2}	2.64712	2.65078	2.72145	3.59098	3.47408	3.76503	3.02107	3.07736	3.79348	3.73991
T_R	2.64476	2.65159	2.72694	3.59072	3.47106	3.76537	3.01838	3.07475	3.79586	3.74076
T_{kk}	2.64487	2.65166	2.72707	3.59089	3.47124	3.76551	3.01851	3.07486	3.79600	3.74087
T_{sg}	2.64476	2.65159	2.72694	3.59072	3.47106	3.76537	3.01838	3.07475	3.79586	3.74076
T_{ss1}	2.66140	2.66212	2.73056	3.60390	3.49561	3.78153	3.03726	3.09252	3.80343	3.75495
T_{ss3}	2.67285	2.70885	2.97702	3.67368	3.54058	3.83352	3.05410	3.11039	3.88145	3.80273
T_{p1}	2.65126	2.65873	2.75666	3.65700	3.46678	3.79416	3.03147	3.08664	3.85482	3.75974
T_{p2}	2.60770	2.60158	2.64468	3.55236	3.40085	3.70578	2.97939	3.03880	3.74582	3.68022
T_{ms}	2.62106	2.61594	2.66623	3.62892	3.43138	3.74194	2.95756	3.00046	3.80458	3.64849
T_{y1}	2.59424	2.58962	2.61964	3.54812	3.39798	3.69011	2.92427	2.96990	3.73216	3.60791
T_{y2}	2.59251	2.58361	2.60492	3.54321	3.39397	3.68383	2.92005	2.96570	3.72082	3.59460
T_{y3}	2.59412	2.58956	2.61880	3.54806	3.39799	3.69002	2.92422	2.96963	3.73215	3.60754

Table 5 MSE of different estimators at $\rho_{xy} = 0.6$

Sample Estimators	1	2	3	4	5	6	7	8	9	10
T_m	3.06341	3.20302	3.29863	2.66986	3.27634	3.46224	2.22123	3.67937	2.79321	3.38999
T_r	8.92690	5.15655	8.73841	5.78075	5.85732	6.61223	5.52643	6.38866	4.67762	7.39661
T_{lr}/T_{s1}	3.02683	3.15641	3.28885	2.65842	3.26592	3.45706	2.21164	3.67217	2.74751	3.38983
T_p	8.90004	5.12071	8.70427	5.75758	5.80455	6.57090	5.51988	6.33764	4.64443	7.35546
T_{s2}	3.00222	3.10639	3.25303	2.63960	3.21628	3.41912	2.19473	3.62136	2.70085	3.34608
T_R	2.99800	3.11084	3.25041	2.63820	3.21373	3.41769	2.19601	3.61940	2.70544	3.34640
T_{kk}	2.99817	3.11096	3.25062	2.63824	3.21388	3.41783	2.19606	3.61955	2.70558	3.34657
T_{sg}	2.99800	3.11084	3.25041	2.63820	3.21373	3.41769	2.19601	3.61940	2.70544	3.34640
T_{ss1}	3.02209	3.11764	3.27747	2.64963	3.23462	3.43477	2.20413	3.63878	2.71229	3.36722
T_{ss3}	3.06278	3.30978	3.40955	2.67048	3.26671	3.45714	2.29321	3.67226	2.95140	3.50801
T_{p1}	2.98752	3.13242	3.24108	2.64330	3.24205	3.43439	2.18920	3.64930	2.72203	3.35042
T_{p2}	2.93413	3.03072	3.16013	2.61077	3.16957	3.37312	2.14068	3.57520	2.62108	3.25771
T_{ms}	2.93833	3.00732	3.14692	2.58160	3.19893	3.35893	2.14751	3.55859	2.63559	3.38570
T_{y1}	2.91125	2.96632	3.11198	2.56259	3.14892	3.32188	2.13282	3.50921	2.59712	3.34237
T_{y2}	2.90947	2.95660	3.10296	2.56049	3.14606	3.31392	2.12905	3.50359	2.58517	3.34000
T_{y3}	2.91119	2.96616	3.11169	2.56234	3.14822	3.32176	2.13280	3.50905	2.59690	3.34236

Table 6 MSE of different estimators at $\rho_{xy} = -0.6$

Sample Estimators	1	2	3	4	5	6	7	8	9	10
T_m	3.37344	3.60654	2.74931	2.15006	2.77696	2.93322	2.45803	3.84688	3.00450	3.01015
T_r	7.00341	5.94362	4.65359	4.64049	6.10379	4.75236	4.64939	5.00260	4.94998	5.55668
T_{lr}/T_{s1}	3.37292	3.60597	2.72935	2.14325	2.76305	2.92544	2.45280	3.84657	2.97290	2.98984
T_p	6.96168	5.88443	4.62627	4.62248	6.06839	4.71434	4.61960	4.93232	4.89559	5.51445
T_{s2}	3.33266	3.53747	2.69443	2.11941	2.73482	2.87809	2.42573	3.71204	2.91901	2.95342
T_R	3.33215	3.53815	2.69678	2.12079	2.73251	2.87993	2.42452	3.71268	2.91428	2.95051
T_{kk}	3.33228	3.53838	2.69685	2.12087	2.73263	2.88005	2.42460	3.71303	2.91448	2.95064
T_{sg}	3.33215	3.53815	2.69679	2.12079	2.73251	2.87993	2.42452	3.71268	2.91427	2.95051
T_{ss1}	3.35112	3.55623	2.70384	2.13014	2.75021	2.88995	2.43683	3.72860	2.93706	2.96848
T_{ss3}	3.41274	3.64852	2.78648	2.23547	2.76540	2.96911	2.45299	3.85851	2.98350	2.99622
T_{p1}	3.34175	3.57694	2.71112	2.12082	2.73966	2.90502	2.43724	3.82330	2.95442	2.97139
T_{p2}	3.26511	3.46811	2.64251	2.06429	2.69092	2.82369	2.39458	3.65351	2.88871	2.92020
T_{ms}	3.24605	3.51121	2.56874	2.08479	2.65837	2.83682	2.36889	3.75678	2.86936	2.85664
T_{y1}	3.20850	3.44706	2.54032	2.06359	2.63026	2.79416	2.34263	3.62924	2.81498	2.82102
T_{y2}	3.20269	3.43845	2.52760	2.05925	2.62557	2.78552	2.33860	3.61627	2.80875	2.81488
T_{y3}	3.20827	3.44676	2.53996	2.06356	2.63016	2.79406	2.34251	3.62895	2.81497	2.82081

Table 7 MSE of different estimators at $\rho_{xy} = -0.7$

Sample Estimators	1	2	3	4	5	6	7	8	9	10
T_m	3.03131	3.27387	4.39243	2.51565	2.53475	3.34452	3.82205	3.26115	2.62331	3.49995
T_r	6.66129	6.05575	7.20658	5.91581	4.30530	6.84227	7.69698	5.71146	4.03751	5.90846
T_{lr}/T_{s1}	3.02667	3.26337	4.39012	2.51246	2.51926	3.32748	3.82097	3.23334	2.60298	3.49854
T_p	6.63212	6.02374	7.15025	5.88249	4.25697	6.78975	7.63393	5.64715	4.01440	5.86837
T_{s2}	2.99029	3.22577	4.32359	2.48091	2.47222	3.28486	3.76100	3.17501	2.57113	3.45554
T_R	2.99179	3.22765	4.32478	2.47958	2.46902	3.28159	3.75999	3.17007	2.57310	3.45482
T_{kk}	2.99191	3.22775	4.32496	2.47973	2.46919	3.28176	3.76026	3.17033	2.57315	3.45494
T_{sg}	2.99179	3.22765	4.32478	2.47957	2.46902	3.28159	3.75999	3.17007	2.57310	3.45482
T_{ss1}	3.00744	3.23856	4.34159	2.49890	2.48828	3.30553	3.78700	3.19681	2.57785	3.46895
T_{ss3}	3.16847	3.31614	4.41419	2.58731	2.51929	3.33281	3.90887	3.23367	2.62348	3.49910
T_{p1}	2.99180	3.23943	4.36178	2.48016	2.50063	3.29826	3.77837	3.20856	2.59049	3.48000
T_{p2}	2.90719	3.16423	4.25404	2.41505	2.43837	3.22985	3.66923	3.13192	2.53367	3.41552
T_{ms}	2.87427	3.20124	4.28742	2.43950	2.48011	3.06999	3.69272	3.13081	2.52384	3.45226
T_{y1}	2.84314	3.16692	4.22523	2.40859	2.43145	3.03195	3.63599	3.07169	2.49585	3.40972
T_{y2}	2.83605	3.16082	4.21548	2.40541	2.42920	3.01845	3.62925	3.06851	2.48903	3.40625
T_{y3}	2.84277	3.16692	4.22502	2.40844	2.43148	3.03066	3.63560	3.07168	2.49559	3.40966

Table 8 MSE of different estimators at $\rho_{xy} = -0.8$

Sample Estimators	1	2	3	4	5	6	7	8	9	10
T_m	3.65234	3.28330	3.03492	2.75481	3.53644	2.87502	3.92077	2.86527	4.05857	3.65556
T_r	6.57725	5.87356	6.22548	3.90041	7.68810	5.56936	7.76514	4.89181	5.78491	6.05960
T_{lr}/T_{s1}	3.65226	3.27611	3.03186	2.74165	3.51762	2.86754	3.91793	2.86481	4.05624	3.64035
T_p	6.52967	5.83533	6.18336	3.86229	7.63480	5.53310	7.68461	4.85880	5.73317	6.00330
T_{s2}	3.60320	3.23952	2.99361	2.68299	3.47540	2.83552	3.84316	2.82801	3.97893	3.57029
T_R	3.60301	3.23797	2.99235	2.68550	3.47187	2.83389	3.84117	2.82840	3.98010	3.57377
T_{kk}	3.60316	3.23805	2.99248	2.68563	3.47209	2.83399	3.84153	2.82848	3.98028	3.57402
T_{sg}	3.60301	3.23797	2.99235	2.68550	3.47187	2.83389	3.84117	2.82840	3.98010	3.57378
T_{ss1}	3.62028	3.25302	3.01123	2.69244	3.49801	2.84932	3.87478	2.83919	3.99283	3.58831
T_{ss3}	3.66681	3.28059	3.05618	2.78347	3.53061	2.86755	4.03553	2.87109	4.06252	3.77640
T_{p1}	3.62601	3.25819	3.00393	2.72513	3.48373	2.84802	3.86791	2.84745	4.03582	3.60723
T_{p2}	3.54427	3.20363	2.93555	2.63381	3.41212	2.79699	3.73688	2.78675	3.92655	3.48019
T_{ms}	3.56749	3.17004	2.95912	2.67782	3.41728	2.73509	3.70331	2.79290	3.93015	3.38420
T_{y1}	3.52059	3.13447	2.92155	2.62435	3.37422	2.70473	3.63554	2.75837	3.85891	3.32786
T_{y2}	3.51530	3.12941	2.91722	2.61454	3.37316	2.70010	3.62352	2.75228	3.84957	3.31274
T_{y3}	3.52040	3.13429	2.92146	2.62401	3.37376	2.70440	3.63468	2.75821	3.85855	3.32670

Table 9 MSE of different estimators at $\rho_{xy} = -0.9$

Sample Estimators	1	2	3	4	5	6	7	8	9	10
T_m	3.12039	2.86793	3.11369	2.96688	3.09314	2.64437	2.68063	2.88714	2.44025	2.76425
T_r	5.20969	5.03743	8.75658	5.54564	6.08771	4.48408	7.46887	5.06457	6.28538	5.04267
T_{lr}/T_{s1}	3.11854	2.86277	3.10983	2.93116	3.07634	2.63685	2.65276	2.88674	2.44024	2.75061
T_p	5.15765	4.99311	8.73542	5.48201	6.06219	4.44319	7.44489	5.03415	6.27004	5.00151
T_{s2}	3.05816	2.81913	3.07610	2.88010	3.04387	2.58775	2.63222	2.85507	2.41802	2.71376
T_R	3.05933	2.81749	3.07446	2.87444	3.04610	2.58984	2.62908	2.85475	2.41797	2.71131
T_{kk}	3.05953	2.81761	3.07467	2.87472	3.04619	2.59000	2.62918	2.85483	2.41806	2.71142
T_{sg}	3.05933	2.81749	3.07446	2.87444	3.04610	2.58984	2.62908	2.85475	2.41797	2.71131
T_{ss1}	3.07489	2.83434	3.10020	2.90238	3.05610	2.60132	2.64795	2.86571	2.43186	2.72832
T_{ss3}	3.17796	2.86447	3.31570	2.93116	3.15880	2.72534	2.66558	2.88683	2.52125	2.75146
T_{p1}	3.09137	2.84239	3.05720	2.90552	3.05135	2.61251	2.62311	2.87121	2.41087	2.73223
T_{p2}	2.99121	2.77752	2.97064	2.83741	2.97861	2.52362	2.57988	2.82080	2.35520	2.67930
T_{ms}	3.01319	2.81598	3.00038	2.88598	2.94049	2.47476	2.56625	2.84325	2.30392	2.65383
T_{y1}	2.95813	2.77221	2.96761	2.83104	2.91308	2.43398	2.54419	2.81224	2.28433	2.61741
T_{y2}	2.95120	2.76937	2.96277	2.82937	2.90367	2.42733	2.54349	2.80773	2.28055	2.61136
T_{y3}	2.95781	2.77219	2.96745	2.83124	2.91283	2.43302	2.54410	2.81219	2.28401	2.61732

5 Conclusion

In this paper, we have suggested some optimal classes of difference and ratio type estimators for the estimation of population mean using higher order moments such as variance of auxiliary variable x with the objective of improvement over its prominent competitors existing till date. The bias and MSE of the proposed classes of estimators are studied up to first order approximation. It has been demonstrated under some optimality conditions that the proposed classes of estimators T_{y_i} , $i = 1, 2, 3$ perform better than usual mean estimator T_m , classical ratio estimator T_r , classical regression estimator T_{lr} , [2] estimator T_{s_1} , [5] estimator T_p , improved form of [2] estimator T_{s_2} , [6] estimator T_R , [8] estimator T_k , generalized form of [8] estimator T_{kk} , [10] estimator T_{sg} , [11] estimator T_{ss_1} , [13] estimator T_{ss_2} , [14] estimator T_{ss_3} , higher order moments based regression estimator T_{ms} envisaged on the lines of [15, 28] estimator T_{p_1} , and [16] estimator T_{p_2} . Further, the optimality conditions have been assessed with a computational study using real and simulated data sets. The results are found to be quite advance and furnishing better advancement over all discussed work. Thus, taking theoretical as well as empirical results into consideration, we enthusiastically prefer the proposed classes of estimators in usual practice.

In forthcoming studies, the proposed estimators may be examined under ranked set sampling (see, [35–38]), stratified ranked set sampling (see, [39] and [40]), and multistage ranked set sampling (see, [41, 42], and [43]).

Appendix A

The MSE and optimum values of scalars of the existing estimators discussed in Sect. 1 are given below.

$$V(T_m) = \gamma \bar{Y}^2 C_y^2 \quad (\text{A.1})$$

$$MSE(T_r) \approx \gamma \bar{Y}^2 (C_x^2 + C_y^2 - 2\rho_{xy} C_x C_y) \quad (\text{A.2})$$

$$MSE(T_{lr}) \approx \gamma \bar{Y}^2 (\beta^2 C_x^2 + C_y^2 - 2\beta \rho_{xy} C_x C_y) \quad (\text{A.3})$$

$$MSE(T_{s_1}) \approx \gamma \bar{Y}^2 (\Theta^2 C_x^2 + C_y^2 + 2\Theta \rho_{xy} C_x C_y) \quad (\text{A.4})$$

$$\min.MSE(T) \approx \gamma \bar{Y}^2 C_y^2 (1 - \rho_{xy}^2), \text{ where } T = T_{lr}, T_{s_1} \quad (\text{A.5})$$

$$MSE(T_p) \approx \gamma \bar{Y}^2 [\gamma (k^2 C_x^2 + C_y^2 - 2k \rho_{xy} C_x C_y) + (k - 1)^2] \quad (\text{A.6})$$

$$\min.MSE(T_p) \approx \gamma \bar{Y}^2 [\gamma (k^{*2} C_x^2 + C_y^2 - 2k^* \rho_{xy} C_x C_y) + (k^* - 1)^2] \quad (\text{A.7})$$

$$MSE(T_{s_2}) \approx \bar{Y}^2 \left[\begin{aligned} &1 + \Theta_1^2 \left(1 + \gamma (\Delta_1 (2\Delta_1 + 1) C_x^2 + C_y^2 - 4\Delta_1 \rho_{xy} C_x C_y) \right) \\ &- 2\Theta_1 \left(1 - \gamma (\Delta_1 \rho_{xy} C_x C_y - \frac{\Delta_1 (\Delta_1 + 1)}{2} C_x^2) \right) \end{aligned} \right] \quad (\text{A.8})$$

$$\min.MSE(T_{s_2}) \approx \bar{Y}^2 \left[1 - \frac{B_s^2}{A_s} \right] \quad (\text{A.9})$$

$$MSE(T_R) \approx \left[\bar{Y}^2 (\alpha_1 - 1)^2 + \gamma (\bar{Y}^2 \alpha_1^2 C_y^2 + \alpha_2^2 \bar{X}^2 C_x^2 + 2\alpha_1 \alpha_2 \bar{X} \bar{Y} \rho_{xy} C_x C_y) \right] \quad (\text{A.10})$$

$$\min.MSE(T_R) \approx \bar{Y}^2(1 - \alpha_{1(opt)}) \quad (A.11)$$

$$MSE(T_{ss1}) \approx \bar{Y}^2 \left[1 + w_1^2 H_1 + w_2^2 H_2 + 2w_1 w_2 H_3 - 2w_1 H_4 - 2w_2 H_5 \right] \quad (A.12)$$

$$MSE(T_{sg}) \approx \left[\bar{Y}^2 \left[\begin{aligned} & (k_1 - 1)^2 + k_1^2 \gamma \left\{ C_y^2 + \frac{C_x^2}{(N+1)^2} - \frac{2\rho_{xy}C_xC_y}{(N+1)} \right\} \\ & - 2k_1 \gamma \left\{ \frac{3C_x^2}{8(N+1)^2} - \frac{\rho_{xy}C_xC_y}{2(N+1)} \right\} \\ & + k_2^2 \bar{X}^2 \gamma C_x^2 - k_2 \bar{X} \bar{Y} \frac{\gamma C_x^2}{(N+1)} + 2k_1 k_2 \bar{X} \bar{Y} \gamma \left\{ \frac{C_x^2}{(N+1)} - \rho_{xy}C_xC_y \right\} \end{aligned} \right] \right] \quad (A.13)$$

$$\min.MSE(T_{sg}) \approx \bar{Y}^2 \left[\left\{ 1 - \frac{\gamma C_x^2}{4(N+1)^2} \right\} - \frac{\left\{ 1 - \frac{\gamma C_x^2}{8(N+1)^2} \right\}^2}{\left\{ 1 + \gamma C_y^2(1 - \rho_{xy}^2) \right\}} \right] \quad (A.14)$$

$$\min.MSE(T_{ss1}) \approx \bar{Y}^2 \left[1 - \frac{(H_2 H_4^2 + H_1 H_5^2 - 2H_3 H_4 H_5)}{(H_1 H_2 - H_3^2)} \right] \quad (A.15)$$

$$MSE(T_{ss2}) \approx \gamma \bar{Y}^2 \left[C_y^2 + (2g - 1)\theta \{ (2g - 1)\theta C_x^2 - 2\rho_{xy}C_xC_y \} \right] \quad (A.16)$$

$$\min.MSE(T_{ss2}) \approx \gamma \bar{Y}^2 C_y^2 [1 - \rho_{xy}^2] \quad (A.17)$$

$$MSE(T_{ss3}) \approx \gamma \bar{Y}^2 [C_y^2 + (\psi_1 - \psi_2)^2 C_x^2 - 2(\psi_1 - \psi_2)\rho_{xy}C_xC_y] \quad (A.18)$$

$$MSE(T_{p1}) \approx \bar{Y}^2 \left[\gamma (C_y^2 + \theta^2 C_x^2 + 2\theta\rho_{xy}C_xC_y) + \eta_1^2 D_1 + \eta_2^2 D_2 - 2\eta_1 \eta_2 D_3 \right] \quad (A.19)$$

$$\min.MSE(T_{p1}) \approx \bar{Y}^2 \left[\gamma (C_y^2 + \theta^2 C_x^2 + 2\theta\rho_{xy}C_xC_y) - \frac{(D_2 D_4^2 + D_1 D_5^2 - 2D_3 D_4 D_5)}{D_1 D_2 - D_3^2} \right] \quad (A.20)$$

$$MSE(T_{p2}) \approx \bar{Y}^2 [1 + K_1^2 E_1 + K_2^2 E_2 + 2K_1 K_2 E_3 - 2K_1 E_4 - 2K_2 E_5] \quad (A.21)$$

$$\min.MSE(T_{p2}) \approx \bar{Y}^2 \left[1 - \frac{(E_2 E_4^2 + E_1 E_5^2 - 2E_3 E_4 E_5)}{(E_1 E_2 - E_3^2)} \right] \quad (A.22)$$

$$MSE(T_{ms}) \approx \left[\gamma C_y^2 + \beta_1^2 \gamma C_x^2 + \theta_1^2 \gamma (\lambda_{04} - 1) - 2\beta_1 \gamma \rho_{xy} C_x C_y - 2\theta_1 \gamma C_y \lambda_{12} \right] \quad (A.23)$$

$$\min.MSE(T_{ms}) \approx \gamma \bar{Y}^2 C_y^2 \left[1 - \left\{ \rho_{xy}^2 + \frac{(\lambda_{12} - \rho_{xy} \lambda_{03})^2}{(\lambda_{04} - 1 - \lambda_{03}^2)} \right\} \right] \quad (A.24)$$

where

$$A_s = 1 + \gamma (C_y^2 + \rho_{xy}C_xC_y - 2\rho_{xy}^2 C_y^2)$$

$$B_s = 1 + \frac{\gamma \rho_{xy} C_x C_y}{2} - \gamma \rho_{xy}^2 C_y^2$$

$$\psi_1 = c(c + d)^{-1}$$

$$\psi_2 = b(a + b)^{-1}$$

$$H_1 = 1 + \gamma \left\{ C_y^2 + (\alpha^2 v^2 (2g^2 + g) - 4\alpha v g \rho_{xy} \frac{C_y}{C_x}) \right\}$$

$$\begin{aligned}
H_2 &= 1 + \gamma \left\{ C_y^2 + \frac{(\delta^2 + \delta)}{2} v^2 C_x^2 - 2\delta v \rho_{xy} C_x C_y \right\} \\
H_3 &= 1 + \gamma \left\{ C_y^2 + \frac{H_1}{8} v^2 C_x^2 - (2\alpha g + \delta) v \rho_{xy} C_x C_y \right\} \\
H_4 &= 1 + \gamma \left\{ C_y^2 + \frac{(g^2 + g)}{2} \alpha^2 v^2 C_x^2 - \alpha v g \rho_{xy} C_x C_y \right\} \\
H_5 &= 1 + \gamma \left\{ \frac{(\delta^2 + 2\delta)}{8} v^2 C_x^2 - \frac{\delta}{2} v \rho_{xy} C_x C_y \right\} \\
D_1 &= R^2 \gamma C_x^2 \\
D_2 &= 1 + \gamma \{ C_y^2 + 2(1 - 2\delta) \rho_{xy} C_x C_y + \delta^2 C_x^2 \} \\
D_3 &= \gamma R [\rho_{xy} C_x C_y + (1 - 2\delta) C_x^2] \\
D_4 &= \gamma R [\rho_{xy} C_x C_y + \theta C_x^2] \\
D_5 &= \gamma [C_y^2 + \{ 2\theta + (1 - 2\delta)/2 \} \rho_{xy} C_x C_y + \{ \alpha \delta + (1 - 2\delta)\theta/2 \} C_x^2] \\
E_1 &= 1 + \gamma \{ C_y^2 - 2v(2\alpha g + h) \rho_{xy} C_x C_y + (v^2/4) \{ (2\alpha g + h)^2 + A_1 \} C_x^2 \} \\
E_2 &= \alpha_1^2 (1 + \gamma C_y^2) + (\gamma/4) \{ (A_2^2 + \alpha_1 A_4) C_x^2 - 4\alpha_1 (A_2 + A_3) \rho_{xy} C_x C_y \} \\
E_3 &= \left[\alpha_1 (1 + \gamma C_y^2) + (\gamma/8) \{ (A_4 + 2v(2\alpha g + h) A_2 + v^2 A_1) C_x^2 \right. \\
&\quad \left. - 4(A_2 + A_3 + 2\alpha_1 v(2\alpha g + h)) \rho_{xy} C_x C_y \} \right] \\
E_4 &= 1 + (\gamma v/8) (v A_1 C_x^2 - 4(2\alpha g + h) \rho_{xy} C_x C_y) \\
E_5 &= \alpha_1 + (\gamma/8) (A_4 C_x^2 - 4A_3 \rho_{xy} C_x C_y) \\
A_1 &= (2\alpha g + h)^2 + 2(2\alpha^2 g + h) \\
A_2 &= 2(\alpha_2 \bar{X} + \alpha_3 R^{-1}) + \alpha_1 v(2\delta \eta + \phi) \\
A_3 &= 2\alpha_2 \bar{X} + \alpha_1 v(2\delta \eta + \phi) \\
A_4 &= \alpha_1 v^2 \{ (2\delta \eta + \phi)^2 + 2(2\eta^2 \delta + \phi) \} + 4v(2\delta \eta + \phi) (\alpha_2 \bar{X} + \alpha_3 R^{-1}) \\
R &= \frac{\bar{Y}}{\bar{X}}
\end{aligned}$$

The optimum values of scalars are given below as

$$\begin{aligned}
\Theta &= -\rho_{xy} \frac{C_y}{C_x} \\
\Theta_{1(opt)} &= \frac{\left(1 + \frac{\gamma \rho_{xy} C_x C_y}{2} - \gamma \rho_{xy}^2 C_y^2 \right)}{\left(1 + \gamma (C_y^2 + \rho_{xy} C_x C_y - 2\rho_{xy}^2 C_y^2) \right)} \\
\Delta_{1(opt)} &= \rho_{xy} \left(\frac{C_y}{C_x} \right) \\
\alpha_{1(opt)} &= \frac{1}{1 + \gamma C_y^2 (1 - \rho_{xy}^2)} \\
\alpha_{2(opt)} &= \rho_{xy} \left(\frac{C_y}{C_x} \right)
\end{aligned}$$

$$\begin{aligned}
k_{1(opt)} &= \frac{1 - \frac{\gamma C_x^2}{8(N+1)^2}}{1 + \gamma C_y^2(1 - \rho_{xy}^2)} \\
k_{2(opt)} &= \frac{\bar{Y}}{\bar{X}} \left[\frac{1}{2(N+1)} - k_{1(opt)} \left\{ \frac{1}{(N+1)} - \rho_{xy} \frac{C_y}{C_x} \right\} \right] \\
w_{1(opt)} &= \frac{(H_2 H_4 - H_3 H_5)}{(H_1 H_2 - H_3^2)} \\
w_{2(opt)} &= \frac{(H_1 H_5 - H_3 H_4)}{(H_1 H_2 - H_3^2)} \\
g_{(opt)} &= \frac{1}{2} \left(1 + \frac{\rho_{xy} C_y}{\theta C_x} \right) \\
\eta_{1(opt)} &= \frac{(D_2 D_4 - D_3 D_5)}{(D_1 D_2 - D_3^2)} \\
\eta_{2(opt)} &= \frac{(D_1 D_5 - D_3 D_4)}{(D_1 D_2 - D_3^2)} \\
K_{1(opt)} &= \frac{(E_2 E_4 - E_3 E_5)}{(E_1 E_2 - E_3^2)} \\
K_{2(opt)} &= \frac{(E_1 E_5 - E_3 E_4)}{(E_1 E_2 - E_3^2)} \\
\beta_{1(opt)} &= \frac{C_y \{\rho_{xy}(\lambda_{04} - 1) - \lambda_{12} \lambda_{03}\}}{C_x (\lambda_{04} - 1 - \lambda_{03}^2)} \\
\theta_{1(opt)} &= \frac{C_y \{\lambda_{12} - \rho_{xy} \lambda_{03}\}}{(\lambda_{04} - 1 - \lambda_{03}^2)} \\
\beta_{2(opt)} &= \frac{S_y \{\rho_{xy}(\lambda_{04} - 1) - \lambda_{12} \lambda_{03}\}}{S_x (\lambda_{04} - 1 - \lambda_{03}^2)} \\
\theta_{2(opt)} &= \frac{S_y \{\lambda_{12} - \rho_{xy} \lambda_{03}\}}{S_x^2 (\lambda_{04} - 1 - \lambda_{03}^2)}
\end{aligned}$$

Appendix B

This section considers the precis of the derivations of Theorem 2.1 and Theorem 2.2.

To obtain the properties of the proposed estimators, let us assume that $\bar{y} = \bar{Y}(1 + e_0)$, $\bar{x} = \bar{X}(1 + e_1)$, $s_x^2 = S_x^2(1 + e_2)$ such that $E(e_0) = E(e_1) = E(e_2) = 0$, $E(e_0^2) = \gamma C_y^2$, $E(e_1^2) = \gamma C_x^2$, $E(e_2^2) = \gamma(\lambda_{04} - 1)$, $E(e_0 e_1) = \gamma \rho_{xy} C_x C_y$, $E(e_0 e_2) = \gamma C_y \lambda_{12}$, $E(e_1 e_2) = \gamma C_x \lambda_{03}$, where $\gamma = (N - n)/Nn$, $\lambda_{cd} = \mu_{cd}/\mu_{20}^{c/2} \mu_{02}^{d/2}$, $\mu_{cd} = (N - 1)^{-1} \sum_1^N (y_i - \bar{Y})^c (x_i - \bar{X})^d$, $c, d \in \{m \in z | 0 \leq m \leq 4\}$, $C_y = S_y/\bar{Y}$, $C_x = S_x/\bar{X}$ and $\rho_{xy} = S_{xy}/S_x S_y$.

Using above notations, the MSE of the proposed class of estimator T_{y_1} is given by

$$MSE(T_{y_1}) = \left\{ (\zeta_1 - 1)^2 \bar{Y}^2 + \zeta_1^2 \gamma S_y^2 + \delta_1^2 \gamma S_x^2 + \phi_1^2 S_x^4 \gamma (\lambda_{04} - 1) \right. \\
\left. - 2\zeta_1 \delta_1 \gamma \rho_{xy} S_x S_y - 2\zeta_1 \phi_1 S_x^2 S_y \gamma \lambda_{12} + 2\delta_1 \phi_1 S_x^3 \gamma \lambda_{03} \right\} \quad (B.25)$$

On minimizing the $MSE(T_{y_1})$ w.r.t. scalars, we get

$$\zeta_{1(opt)} = \frac{\bar{Y}^2}{\left[\bar{Y}^2 + \gamma S_y^2 \left\{ 1 - \rho_{xy}^2 - \frac{(\lambda_{12} - \rho_{xy}\lambda_{03})^2}{(\lambda_{04} - 1 - \lambda_{03}^2)} \right\} \right]} \quad (B.26)$$

$$\delta_{1(opt)} = \zeta_{1(opt)} \frac{S_y \{ \rho_{xy}(\lambda_{04} - 1) - \lambda_{12}\lambda_{03} \}}{S_x (\lambda_{04} - 1 - \lambda_{03}^2)} \quad (B.27)$$

$$\phi_{1(opt)} = \zeta_{1(opt)} \frac{S_y \{ \lambda_{12} - \rho_{xy}\lambda_{03} \}}{S_x^2 (\lambda_{04} - 1 - \lambda_{03}^2)} \quad (B.28)$$

Putting the optimum values of scalars in the $MSE(T_{y_1})$, we get minimum MSE as

$$\min.MSE(T_{y_1}) = \frac{\bar{Y}^2 \gamma S_y^2 \left[1 - \rho_{xy}^2 - \frac{(\lambda_{12} - \rho_{xy}\lambda_{03})^2}{\lambda_{04} - 1 - \lambda_{03}^2} \right]}{\bar{Y}^2 + \gamma S_y^2 \left[1 - \rho_{xy}^2 - \frac{(\lambda_{12} - \rho_{xy}\lambda_{03})^2}{\lambda_{04} - 1 - \lambda_{03}^2} \right]} = \bar{Y}^2 (1 - \zeta_{1(opt)}) \quad (B.29)$$

The MSE of the proposed class of estimator T_{y_2} is given by

$$MSE(T_{y_2}) \approx \bar{Y}^2 (1 + \zeta_2^2 P_2 - 2\zeta_2 Q_2) \quad (B.30)$$

On minimizing the $MSE(T_{y_2})$ w.r.t. ζ_2 , we get

$$\zeta_{2(opt)} = \frac{Q_2}{P_2} \quad (B.31)$$

Putting the optimum value of ζ_2 in the $MSE(T_{y_2})$, we get minimum MSE as

$$\min.MSE(T_{y_2}) \approx \bar{Y}^2 \left(1 - \frac{Q_2^2}{P_2} \right) \quad (B.32)$$

The derivation of other estimator T_{y_3} can be done in similar lines. In general, we can write

$$MSE(T_{y_i}) \approx \bar{Y}^2 (1 + \zeta_i^2 P_i - 2\zeta_i Q_i) \quad (B.33)$$

It is worth mentioning that the simultaneous optimization w.r.t. ζ_i , δ_i and ϕ_i of the expression of MSE is not possible and we use optimum values of $\delta_i = \delta_{i(opt)}$ and $\phi_i = \phi_{i(opt)}$ when $\zeta_i = 1$ and use this within $\zeta_i = \zeta_{i(opt)}$ to obtain the derivations of MSE expressions of estimators T_{y_i} , $i = 2, 3$. The optimum values of the scalars involved in the estimators are given by

$$\zeta_{i(opt)} = \frac{Q_i}{P_i}, \quad i = 2, 3 \quad (B.34)$$

where

$$\begin{aligned} P_2 &= 1 + \gamma \left\{ C_y^2 + (2\delta_2^2 + \delta_2)C_x^2 + (2\phi_2^2 + \phi_2)(\lambda_{04} - 1) - 4\delta_2\rho_{xy}C_xC_y \right. \\ &\quad \left. - 4\phi_2C_y\lambda_{12} + 4\delta_2\phi_2C_x\lambda_{03} \right\} \\ Q_2 &= 1 + \gamma \left\{ \frac{\delta_2(\delta_2 + 1)}{2}C_x^2 + \frac{\phi_2(\phi_2 + 1)}{2}(\lambda_{04} - 1) \right. \\ &\quad \left. - \delta_2\rho_{xy}C_xC_y - \phi_2C_y\lambda_{12} + \delta_2\phi_2C_x\lambda_{03} \right\} \\ P_3 &= 1 + \gamma \left\{ C_y^2 + 3\delta_3^2C_x^2 + 3\phi_3^2(\lambda_{04} - 1) - 4\delta_3\rho_{xy}C_xC_y \right. \end{aligned}$$

$$Q_3 = 1 + \gamma \left\{ \delta_3^2 C_x^2 + \phi_3^2 (\lambda_{04} - 1) - \delta_3 \rho_{xy} C_x C_y - 4\phi_3 C_y \lambda_{12} + 4\delta_3 \phi_3 C_x \lambda_{03} \right\}$$

where $\delta_{i(opt)} = \frac{C_y}{C_x} \left(\frac{\rho_{xy}(\lambda_{04}-1) - \lambda_{12}\lambda_{03}}{\lambda_{04}-1-\lambda_{03}^2} \right)$ and $\phi_{i(opt)} = C_y \left(\frac{\lambda_{12}-\rho_{xy}\lambda_{03}}{\lambda_{04}-1-\lambda_{03}^2} \right)$ are used as the optimum values of these scalars when $\zeta_i=1$ is put in the corresponding estimators.

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