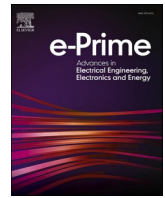




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Laplacian atom search optimization algorithm: development and application for harmonic estimator design

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ABSTRACT

Harmonics, are the major source of contaminations in fundamental voltage and current signals. For consumer satisfaction and for better equipment performance, these contaminations shall be identified and mitigated. Although, for accurate identification of harmonics, several methods have been proposed, yet metaheuristic based approaches have been used and in practice, since last two decades. The work reported here is based on a newly proposed optimization algorithm named as Atom Search Optimisation (ASO). In the first phase, a variant of ASO is proposed, based on Laplacian operator based position update mechanism named as Laplacian-ASO (L-ASO) for enhancing the performance of ASO and in the next phase, application of a newly developed L-ASO is carried out on harmonic estimator design problems. The improvisation of our proposed L-ASO is validated through conducted analyses and results showcased in the discussion section.

1. Introduction

In power system network consumer satisfaction plays an important role. Consumer satisfaction comes from the ability of power distributors to fulfill the demand of power by maintaining the reliability of power supply. To provide consumer satisfaction, engineers use power electronic device based storage units which helps to handle the shortage of power. Obviously, these power electronic devices are non-linear devices [1] which do not follow the Ohm's law. These electronic devices are not only helpful to provide a reliable supply of power but also have an adverse effect on power system networks such as heating problems, communication interference problems and they also disturb the stability of system. Harmonics [2] are nothing but, unwanted electrical signals, these unwanted harmonic signals are multiples of the fundamental frequency (50Hz, F) like $2 \times F$, $1.2 \times F$ and $0.15 \times F$, etc. These harmonics, signals are harmful to the power system network. For ensuring the protection and stability of the power system network, it is necessary to design a harmonic estimator that provides an exact estimation of amplitude and phase, of the power harmonics.

Harmonic component estimation is not a new problem for researchers. With the passing years, many experiments have been conducted by the researchers. In fact it is a challenging problem due to its

nonlinear nature. Hence, still this design problem attracts attention of researchers for exact estimation of phase and amplitude components of power harmonics. In the beginning of harmonic estimation, Discrete Fourier Transform (DFT) [3] and Fast Fourier Transform (FFT) are used due to aliasing and picket fence effect, it is not used for multiple frequency components. Wavelet Transform (WT) [4] also estimates components of harmonic signals, in this transform mother wavelets are randomly chosen, which affect the effectiveness of harmonic estimator. In recent years some approaches are adopted which are successfully implemented which are Extended Phase-Locked Loop (EPLL), Kalman Filtering, Artificial Neural Network (ANN), Radial Basis Function Based Neural Network (RBFNN) etc. Kalman filter method helps to estimate the harmonic components for online tracking application but it needed information on noise and state variable modelling which increases complexity during the length of signal increase. EPLL is also a suitable method because it has the adaptive nature but it provides slow dynamic response. ANN is also used to estimate the amplitude of phase, of harmonic signal, sometimes it loses its accuracy when the number of hidden layers increases. RBFNN has fast convergence and simple structure but it gives inaccurate estimation for harmonic estimator in presence of noise and transients. Harmonics problem is mostly solved by two methods: the first one is classical techniques and second one is meta-heuristic techniques. Classical techniques are known as conventional techniques,

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Nomenclature			
Z	Power Harmonics	M_i	Mass of Atom
K_x	Parameter of Harmonic Magnitude	Δ	Distance Between Two Atoms
ω_x	Angular Frequency	E	Potential Well Depth
$\delta(t)$	Noise Signal	y_i	Position of i_{th} Atom
f_0	Fundamental Frequency	S_{ij}	Euclidean Distance Between y_i and y_j
ϕ_x	Parameter of harmonic Phase	F_{ij}^D	Interaction Force from i_{th} Atom to j_{th} Atom
T_r	Sampling Period	$\gamma(t)$	Depth Function
L	Objective Function of Harmonic Estimation Problem	β	Depth Weight
z_r	Original harmonic Signal	T	Minimum Number of Iterations
A_i	Acceleration of	Q_i^D	Constraint Force on i_{th} in D_{th} Dimension
F_i	Indication Force	A_i^D	Acceleration of i_{th} in D_{th} Dimension
G_i	Constraint Force for i_{th} atom	$M_i(t)$	Mass of i_{th} Atom

these techniques based on the mathematical modelling oriented solutions and sometimes also stuck into local minima and take more time to search for global minima. "Necessity is the mother of invention", this sentence is followed by researchers and they developed meta-heuristic techniques having capabilities to search global minima without stucking into local minimas. These techniques are classified into four categories:

- Nature Inspired (Population Based) Algorithm- These algorithms take inspiration from nature and implement the processes through mathematical translations. In these algorithms, all the participants of the algorithm share the data by their special activity. This shared data helps to find an optimal solution to the problem. Example of these techniques are Grey Wolf Optimizer (GWO) [5], Whale Optimization Algorithm (WOA) [6], and Ant Colony Optimization (ACO) [7], etc.
- Physics Based Algorithm- These algorithms have been developed by physics concepts such as Big Bang-Big Crunch (BB-BC) Algorithm [8], Quantum-Inspired Evolutionary Algorithm (QEA) [9], Atom Search Optimization (ASO) [10] etc.
- Human Behavior Based Algorithm- Human behavior-based algorithms are motivated by the activities and behavior of humans. These algorithms include the Teaching Learning Based Optimization Algorithm (TLBO) [11], Harmony Search (HS) [12], Exchange Market Algorithm (EMA) [13], etc.
- Hybrid Algorithm- Hybrid algorithms are made by the combination of two or more algorithms. This class of hybrid techniques is very wide and include algorithms like teaching-learning-based cuckoo search algorithm (TLCS) introduced by Huang et al. [14], Grey Wolf Optimizer (GWO) - Sine Cosine Algorithm (SCA) given by N. Singh et al. [15], Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) introduced by YT Kao et al. in [16] etc.

Recent trends of integrating metaheuristics with real life applications are increasing with every passing year. Harmonic estimation problem is not an exception. Recent application reported by Saxena shows the applicability of the advanced version of Crow Search Algorithm in the harmonic estimation [17]. Another interesting approach of integration of Firefly Algorithm with this problem has been reported in reference [18].

While dealing with optimization for real life, any algorithm is not found perfect to solve all types of problems. Majority of algorithms fail to solve the particular problem. Hence, the architecture of the algorithms is problem-oriented. This statement is theoretical explanation of no free lunch theorem. This theorem inspires researchers to develop new algorithms or suggest modifications in the algorithm. These modifications also verified through different methods. Atom Search Optimization is a newly developed algorithm that is invented by Zhao et al. in 2019.

This algorithm depends on the force motion of atom in the molecular dynamics. This algorithm has good exploration capability to solve optimization problems. Some problems solved by ASO are dispersion coefficient estimation [19] and hydro-geologic parameter estimation [10]. Furthermore, this algorithm has been applied to some of the major power system problems such as design of radial distribution system in reference [20]. Frequency stability has been evaluated by authors in reference [21]. In the work, PID controllers have been tuned for damping the oscillations. Design of Generator excitation system has been done in reference [22]. Optimal power flow calculation has been done with the help of the algorithm in reference [23]. These applications are proof that ASO is widely accepted algorithm for power system applications. Also it is worth to mention here that with every application, some modifications have been suggested to improve the performance. As, ASO algorithm has some shortcomings also like low search accuracy and slow convergence rate in comparison to another population-based algorithm. To solve this shortcoming we try to develop a new variant of ASO with the help of Laplace Transform and Opposition Based learning. This new variant improves the exploitation and exploration capabilities and convergence rate of ASO as showcased in discussion section.

While considering this discussion, following objectives have been formed for conducting this study:

1. To prepare and present a mathematical model by integration of Least Square algorithm and proposed Laplacian Atom Search Algorithm for carrying out the estimation of amplitude and phase components of Harmonics.
2. To propose a new version of ASO by integrating a Laplacian operator in position update phase. Also, to evaluate the impact of this modification on the optimization performance, of the ASO through conduction of various analyses and tests. Firstly, evaluation shall be conducted on the standard benchmark functions, through various analyses such as box plot, convergence and statistical attribute analyses.
3. To test the applicability of the proposed variant on the harmonic estimator design benches and compare the performance of L-ASO with other algorithms. also to integrate L-ASO for estimation of components of phase and amplitude from given standard electrical signals and compare the performance, with other existing approaches.

The organization of the paper is described as follows- section 2 describes the formulation of the problem, section 3 represents an overview of atom search algorithm. Framework of proposed algorithm is depicted in section 4, section 5 and 6 comprises statistical result analysis and stability analysis respectively. Section 7 represents harmonic estimator problem and in the last section, conclusion is given which concludes the

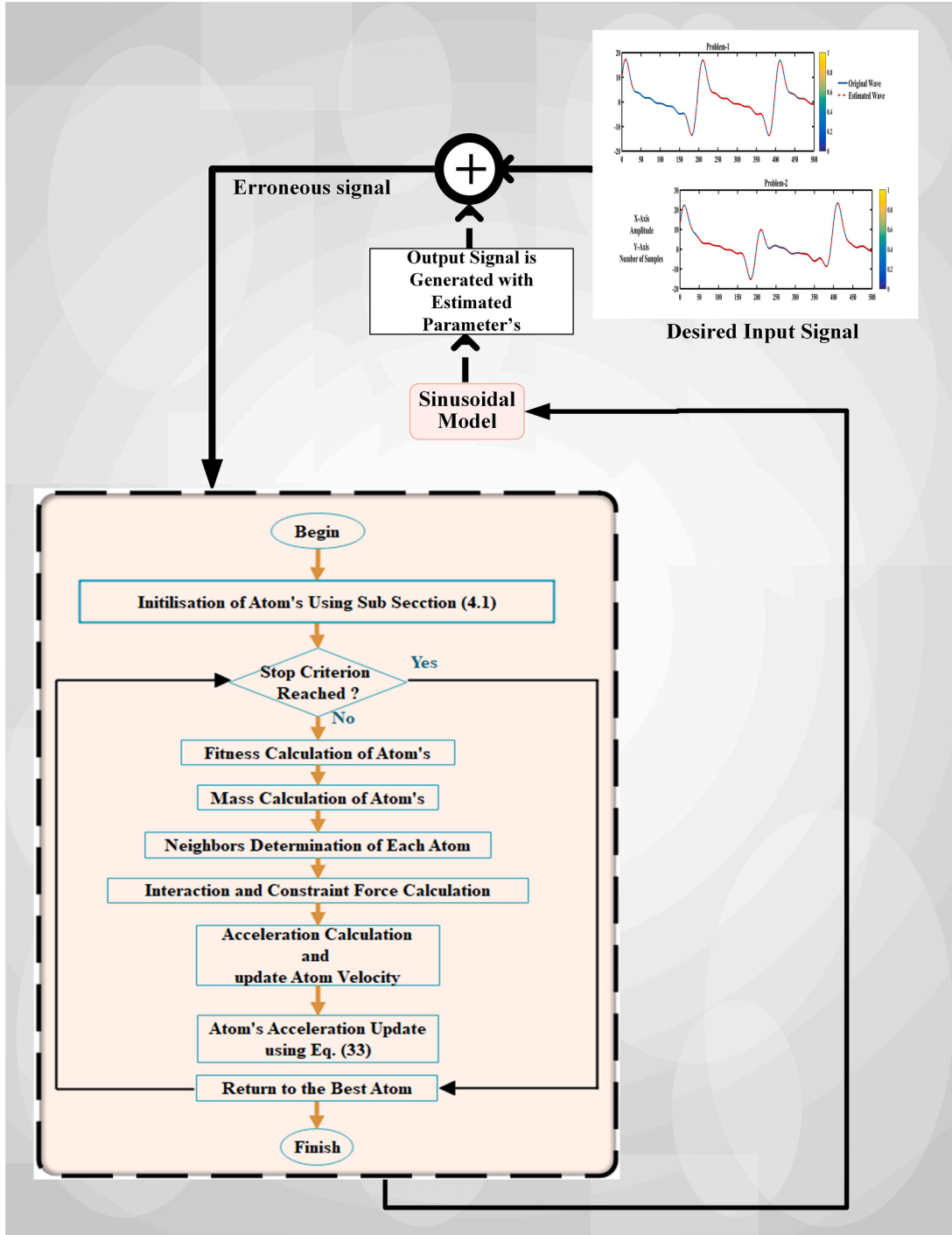


Fig. 1. Flowchart of LASO algorithm on Harmonics Estimation.

paper.

2. Problem formulation

Power harmonics, of voltage or current waveform is expressed as

$$Z(1) = \sum_{x=1}^X K_x \sin(w_x t + \phi_x) + K_{DC} \exp(-\mu_{DC} t) + \delta(t) \quad (1)$$

here, $w_x = x2\pi f_o$ denotes angular frequency, f_o is frequency (fundamental) X indicates order of harmonics, $\delta(t)$ shows noise signal (Additive white Gaussian noise) and $K_{DC} \exp(-\mu_{DC} t)$ represents decaying term. K_x

and ϕ_x are the unknown parameters of harmonics, magnitude and phase respectively. Now, $Z(1)$ is sampled with sampling period T_r , sampled signal written as

$$Z(c) = \sum_{x=1}^X K_x \sin(w_x c T_r + \phi_x) + K_{DC} \exp(-\mu_{DC} c T_r) + \delta(c) \quad (2)$$

In decaying term we applied Taylor series expansion [24], this series expansion is represented as

$$Z(c) = \sum_{x=1}^X \sin(w_x c T_r + \phi_x) + K_{DC} - K_{DC} \mu_{DC} c T_r + \delta(c) \quad (3)$$

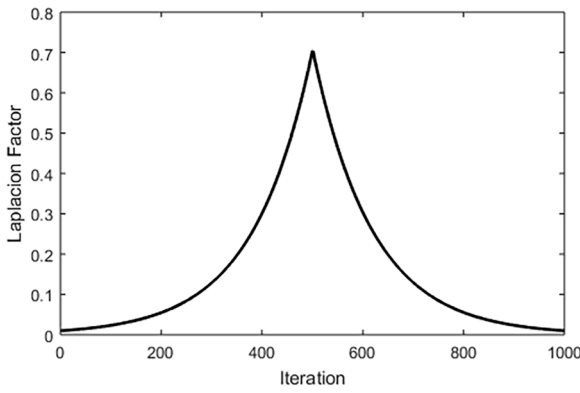


Fig. 2. Proposed Laplacian Factor.

At last, we get a simplified and generalized formula which helps to measure the unknown parameters amplitude and phase, for harmonics. Now we expand equation 3 in term of trigonometric sine and cosine function, which is expressed as

$$Z(c) = \sum_{x=1}^X [K_x \sin(w_x c T_r) \cos \phi_x + K_x \cos(w_x c T_r) \sin \phi_x] + K_{DC} - K_{DC} \mu_{DC} c T_r + \delta(c) \quad (4)$$

Then, the signal is transformed into parametric terms as

$$Z(c) = E(c) \phi(c) \quad (5)$$

Here $E(c)$ is defined as

$$E(c) = [\sin(w_1 c T_r) \cos(w_1 c T_r) \dots \sin(w_x c T_r) \cos(w_x c T_r) - B T_r]^T \quad (6)$$

Unknown parameters magnitude and phase component vectors are presented as

$$\phi(c) = [\phi_{1c} \phi_{2c} \dots \phi_{(2x-1)c} \phi_{2xc} \phi_{(2x+1)c} \phi_{(2x+2)c}]^T \quad (7)$$

$$\phi = [K_1 \cos(\phi_1) K_1 \sin(\phi_1) \dots K_x \cos(\phi_x) K_x \sin(\phi_x) \dots K_{DC} K_{DC} \mu_{DC}]^T \quad (8)$$

L is used as objective function of harmonic estimation problem which helps to measured optimal values of unknown magnitude and phase parameters denoted as

$$L = \min \left(\sum_{c=1}^C e_c^2(c) \right) = \min \left(\sum_{c=1}^C (Z_c - Z_{c_{est}})^2 \right) = MSE(Z_c - Z_{c_{est}}) \quad (9)$$

here Z_c denotes the original harmonic signal, $Z_{c_{est}}$ is measured harmonic signal. In the objective function, we get the difference of the original harmonic signal and measured harmonic signal which provides optimal value.

3. Overview of ATOM search optimization (ASO)

Molecular dynamics is an inspiration for researchers that are used in the atom search algorithm, introduced by Zhao et al. in [19]. This is the newest contribution in meta-heuristic algorithm class. The algorithm has gained popularity as a nature-inspired optimization algorithm and has been applied to solve various interesting optimization problems such as dispersion coefficient estimation [19] and hydro-geologic parameter estimation [10]. Molecular dynamics is used in different categories of science such as physics, material science, biology, and chemistry. The classical mechanism [25] is followed by the atoms during motion. Atoms have two properties: the first is repulsion between atoms and the second is an attraction between atoms. Acceleration of atom is evaluated by

Newton's second law which is represented as

$$A_i = \frac{F_i + G_i}{M_i} \quad (10)$$

Here A_i represents acceleration of atom [26], F_i denotes interaction force while G_i is constraint force for i^{th} atom, and M_i represents mass of the atom.

3.1. Numerical representation of interaction force

Potential of Lennard-Jones (LJ) [27] is a simplified mathematical concept that helps to approximate the force of interaction between groups of atoms. Numerical representation of L-J potential is written as follow

$$P(s_{ij}) = 4E \left[\left(\frac{\Delta}{s_{ij}} \right)^{12} - \left(\frac{\Delta}{s_{ij}} \right)^6 \right] \quad (11)$$

Here, Δ represents limited distance (when the potential of atom is zero), E shows the potential well depth. Repulsion and attraction of atoms are denoted as $\left(\frac{\Delta}{s_{ij}} \right)^6$ and $\left(\frac{\Delta}{s_{ij}} \right)^{12}$ respectively.

$$s_{ij} = \|y_j - y_i\| = \sqrt{(y_{i1} - y_{j1})^2 + (y_{i2} - y_{j2})^2 + \dots + (y_{in} - y_{jn})^2} \quad (12)$$

where, s_{ij} denotes the Euclidean distance between y_i and y_j , $y_i = (y_{i1}, y_{i2}, \dots, y_{in})$, $y_j = (y_{j1}, y_{j2}, \dots, y_{jn})$, are positions of atoms for i^{th} atom and j^{th} atom having n^{th} dimension.

$$F_{ij}^D(t) = -\nabla P(s_{ij}) = \frac{24E(t)}{\Delta(t)} \left[2 \left(\frac{\Delta(t)}{s_{ij}(t)} \right)^{13} - \left(\frac{\Delta(t)}{s_{ij}(t)} \right)^7 \right] \frac{s_{ij}}{s_{ij}^D} \quad (13)$$

Interaction force is applied on the atoms (from i^{th} atom to j^{th} atom), dimension 'D' and time taken is 't' which is represented as

$$F_{ij}^D(t) = -\nabla P(s_{ij}) = \frac{24E(t)}{\Delta(t)} \left[2 \left(\frac{\Delta(t)}{s_{ij}(t)} \right)^{13} - \left(\frac{\Delta(t)}{s_{ij}(t)} \right)^7 \right] \frac{s_{ij}}{S_{ij}^D} \quad (14)$$

$$F_{ij}'(t) = \frac{24E(t)}{\Delta(t)} \left[2 \left(\frac{\Delta(t)}{s_{ij}(t)} \right)^{13} - \left(\frac{\Delta(t)}{s_{ij}(t)} \right)^7 \right] \quad (15)$$

The atom tries to maintain a distance with one another, this distance is continuously changing in a fixed range during attraction or repulsion, and amplitude of repulsion is very large compared to the attraction. In this the direction of attraction and repulsion are negative and positive respectively, when the number of iterations increases that time this negative and positive effect is also increases and decreases during attraction and repulsion respectively. Equation 4 is not used to solve the optimization problem. So researchers provide an updated version of this to solve optimization as follow

$$F_{ij}'(t) = -\gamma(t) \left[2 \left(\frac{\Delta(t)}{s_{ij}(t)} \right)^{13} - \left(\frac{\Delta(t)}{s_{ij}(t)} \right)^7 \right] \quad (16)$$

Here $\gamma(t)$ is a depth function which helps to control the repulsion and attraction area of atoms. This depth function is written as

$$\gamma(t) = \beta \left(1 - \frac{t-1}{T} \right)^3 e^{\frac{-20t}{T}} \quad (17)$$

here, β denotes depth weight and T represents maximum number of iterations.

Table 1
Benchmark Functions Suit.

Benchmark Function Suit				
Function Name	Function Detail's	Dim	Limit	Minvalue
Sphere	$F_1(\psi) = \sum_{R=1}^S \psi_R^2$	30	[-100,100]	0
Schwefel 2.22	$F_2(\psi) = \sum_{R=1}^S \psi_R + \prod_{R=1}^S \psi_R $	30	[-10,10]	0
Schwefel 1.2	$F_3(\psi) = \sum_{R=1}^S \left(\sum_{j=1}^R \psi_j \right)$	30	[-100,100]	0
Schwefel 2.21	$F_4(\psi) = \max_R \{ \psi_R \} \quad 1 \leq R \leq S$	30	[-100,100]	0
Rosenbrock	$F_5(\psi) = \sum_{R=1}^{S-1} [100(\psi_{R+1} - \psi_R)^2 + (\psi_R - 1)^2]$	30	[-30,30]	0
Step	$F_6(\psi) = \sum_{R=1}^{S-1} (\psi_R + 0.5)^2$	30	[-100,100]	0
Quartic	$F_7(\psi) = \sum_{R=1}^{S-1} R \psi_R^4 + rand[0, 1]$	30	[-1.28,1.28]	0
Schwefel 2.26	$F_8(\psi) = \sum_{R=1}^S -\psi_R \sin \left(\sqrt{ \psi_R } \right)$	30	[-500,500]	-418.9829 ×5
Rastrigin	$F_9(\psi) = \sum_{R=1}^S \left[\psi_R^2 - 10 \cos \left(2\pi \psi_R + 10 \right) \right]$	30	[-5.12,5.12]	0
Ackley	$F_{10}(\psi) = -20 \exp \left(-0.2 \sqrt{\frac{1}{S} \sum_{R=1}^S \gamma_R^2} \right) - \exp \left(\frac{1}{S} \sum_{R=1}^S \cos(2\pi \psi_R) \right) + 20 + e$	30	[-32,32]	0
Griewank	$F_{11}(\psi) = \frac{1}{4000} \sum_{R=1}^S \psi_R^2 - \prod_{R=1}^S \cos \left(\frac{\psi_R}{\sqrt{R}} \right) + 1$	30	[-600,600]	0
Generalized Penalized	$F_{12}(\psi) = \frac{\pi}{a} \left\{ 10 \sin(\pi g_1) + \sum_{R=1}^{S-1} (g_R - 1)^2 [1 + 10 \sin^2(\pi g_{R+1})] + (g_S - 1)^2 \right\} + \sum_{R=1}^S b(\psi_R, 10, 100, 4)$	30	[-50,50]	0
	$c_e = 1 + \frac{a+1}{4} \quad g(\psi_R, h, o, l) = \begin{cases} f(\psi_R - k) & \psi_R > k \\ 0 & -k < \psi < k \\ f(-\psi_R - k)^l & \psi_R < -k \end{cases}$			
Penalty	$F_{13}(\psi) = 0.1 \{ \sin S^2(3\pi \psi_1) + \sum_{R=1}^S (\psi_R - 1)^2 [1 + \sin^2(3\pi \psi_R + 1)] + (\psi_S - 1)^2 [1 + \sin^2(2\pi \alpha_S)] \} + \sum_{R=1}^S q(\psi_R, 5, 100, 4)$	30	[-50,50]	0
Dejong (shekel's Foxholes)	$F_{14}(\psi) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{R=1}^j (\psi_R - h_R)^6} \right)^{-1}$	2	[-65,65]~	1
Kowalik	$F_{15}(\psi) = \sum_{j=1}^{11} \left[h_R - \frac{w_1(i_R^2 + j_R \psi_2)}{i_R^2 + R_R \psi_3 + \psi_4} \right]^2$	4	[-5,5]	0.00030
Camel Back-6 Hump	$F_{16}(\psi) = 4\psi_1^2 - 2.1\psi_1^4 + \frac{1}{3}\psi_1^6 + \psi_1\psi_2 - 4\psi_2^2 + 4\psi_2^4$	2	[-5,5]	-1.0316
Branin	$F_{17}(\psi) = \left(\psi_2 - \frac{5.1}{4\pi^2} \psi_1^2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) \cos \psi_1 + 10$	2	[-5,5]	0.398
Goldstein and Price	$F_{18}(\psi) = A(\psi) \times N(\psi)$ $(\psi) = 1 + (\psi_1 + \psi_2 + 1)^2(19 - 14\psi_1 + 3\psi_1^2 - 14\psi_2 + 6\psi_1\psi_2 + 3\psi_2^2)$	2	[-2,2]	3
Hartman~ 3-Dimensional	$F_{19}(\psi) = - \sum_{R=1}^4 x_R \exp \left(- \sum_{j=1}^3 h_{Rj}(\psi_j - e_{Rj})^2 \right)$	3	[1,3]	-3.86
Hartman 6-Dimensional	$F_{20}(\psi) = - \sum_{R=1}^4 x_R \exp \left(- \sum_{j=1}^6 h_{Rj}(\psi_j - e_{Rj})^2 \right)$	6	[0,1]	-3.32
Shekel 1	$F_{21}(\psi) = - \sum_{R=1}^5 \left[(Z - h_R)(Z - h_R)^b + z_R \right]^{-1}$	4	[0,10]	-10.1532
Shekel 2	$F_{22}(\psi) = - \sum_{R=1}^7 \left[(\psi - h_R)(\psi - h_R)^b + z_R \right]^{-1}$	4	[0,10]	-10.4028

Table 2

Comparison of L-ASO with ASO on Conventional Benchmark functions.

ASO Variant Analysis on Conventional Benchmark's								
Statistical Value's Algorithm's	F-1				F-2			
	Mean	Max	Min	SD	textbfMean	Max	Min	SD
ASO	-4.449E-12	3.212E-10	-3.384E-10	1.512E-10	-3.926E-11	4.689E-09	-5.637E-09	2.161E-09
L-ASO	-4.453E-12	1.157E-10	-2.508E-10	1.157E-10	6.394E-11	1.921E-09	-3.994E-09	1.921E-09
	F-3				F-4			
ASO	3.852E-02	2.195E+01	-2.235E+01	1.176E+01	3.041E-09	4.389E-07	-4.108E-07	1.845E-07
L-ASO	-2.921E-02	1.254E+01	-2.137E+01	1.254E+01	1.428E-08	5.676E-07	-1.341E-06	5.676E-07
	F-5				F-6			
ASO	8.816E-02	5.224E-01	-4.543E-02	1.501E-01	-9.373E-03	4.191E-01	-4.164E-01	2.383E-01
L-ASO	1.178E-01	2.192E-01	-5.491E-02	2.192E-01	2.652E-03	2.391E-01	-4.089E-01	2.391E-01
	F-7				F-8			
ASO	3.396E-03	1.586E-01	-1.434E-01	8.570E-02	4.882E+01	4.211E+02	-3.027E+02	2.751E+02
L-ASO	-5.235E-04	8.864E-02	-1.656E-01	8.864E-02	5.759E+01	2.770E+02	-3.030E+02	2.770E+02
	F-9				F-10			
ASO	-3.814E-02	1.658E+00	-1.526E+00	9.108E-01	5.375E-12	3.718E-10	-3.134E-10	1.560E-10
L-ASO	3.151E-02	9.045E-01	-1.625E+00	9.045E-01	-8.299E-12	1.409E-10	-3.284E-10	1.409E-10
	F-11				F-12			
ASO	3.875E-02	7.751E-01	-1.047E-01	1.912E-01	-9.934E-01	-8.680E-01	-1.000E+00	2.951E-02
L-ASO	-1.263E-02	5.649E-02	-2.526E-01	5.649E-02	-1.000E+00	1.767E-10	-1.000E+00	1.767E-10
	F-13				F-14			
ASO	1.001E+00	1.023E+00	9.811E-01	1.030E-02	-3.152E+01	-2.368E+01	-3.261E+01	1.928E+00
L-ASO	9.995E-01	2.457E-03	9.890E-01	2.457E-03	-2.991E+01	3.661E+00	-3.246E+01	3.661E+00
	F-15				F-16			
ASO	1.921E+00	2.490E+00	7.805E-01	4.271E-01	-3.114E-02	4.012E-01	-4.012E-01	4.096E-01
L-ASO	1.633E+00	1.156E+00	-2.961E+00	1.156E+00	-1.246E-01	3.773E-01	-4.012E-01	3.773E-01
	F-17				F-18			
ASO	4.000E+00	4.000E+00	4.000E+00	1.017E-09	-5.000E-01	-5.000E-01	-5.000E-01	2.516E-09
L-ASO	4.001E+00	4.210E-02	3.918E+00	4.210E-02	-5.000E-01	2.101E-09	-5.000E-01	2.101E-09
	F-19				F-20			
ASO	5.076E-01	5.076E-01	5.076E-01	2.062E-09	3.449E-01	3.449E-01	3.449E-01	1.247E-09
L-ASO	5.076E-01	2.259E-09	5.076E-01	2.259E-09	3.449E-01	1.315E-09	3.449E-01	1.315E-09
	F-21				F-22			
ASO	4.336E+00	6.000E+00	3.923E+00	7.247E-01	4.000E+00	4.000E+00	4.000E+00	9.947E-10
L-ASO	4.313E+00	7.299E-01	3.927E+00	7.299E-01	3.999E+00	3.841E-02	3.907E+00	3.841E-02

$$b_{ij}(t) = \begin{cases} b_{min} & \text{if } \frac{s_{ij}(t)}{\Delta(t)} < b_{min} \\ \frac{s_{ij}(t)}{\Delta(t)} & \text{if } b_{min} \leq \frac{s_{ij}(t)}{\Delta(t)} \leq b_{max} \\ b_{max} & \text{if } \frac{s_{ij}(t)}{\Delta(t)} > b_{max} \end{cases} \quad (18)$$

b_{max} and b_{min} represent upper limit and lower limit of b_{ij} . $\Delta(t)$ is known as length scale which is defined as follow

$$\Delta(t) = \left\| y_{ij}(t), \frac{\sum_{j \in H_{best}} y_{ij}(t)}{H(t)} \right\| \quad (19)$$

here, Hbest is the best function, which is a subset of H atoms.

$$\begin{cases} b_{min} = q_o + q(t) \\ b_{max} = p \end{cases} \quad (20)$$

p is the maximum bound and q is a drifting function which assists the algorithm shift from exploration to exploitation and is presented by

$$q(t) = 0.1 * \sin\left(\frac{\pi}{2} * \frac{t}{T}\right) \quad (21)$$

Total acted force on i_{th} atom is calculated as the addition of all the force components in dimension D, which is expressed as

$$F_i^D(t) = \sum_{j \in H_{best}} random_j F_{ij}^D(t) \quad (22)$$

here, $random_j$ is a random number which lies between 0 to 1.

3.2. Numerical representation of geometric constraints

The geometrical constraint in molecular mechanics plays a key role in the movements of atoms. For simplicity, it is considered that in ASO, an atom has a covalent bond with the perfect atom, so the perfect atom acts on every atom by a constraint force. In the D^{th} dimension this constraint force acting on the i^{th} atom is represented as:

$$Q_i^D = \lambda(t) (y_{best}^D(t) - y_i^D(t)) \quad (23)$$

here, $\lambda(t)$ is used as Lagrangian multiplier which is expressed as

$$\lambda(t) = \rho e^{-\frac{20t}{T}} \quad (24)$$

here, ρ is weight of Lagrangian multiplier. 0

3.3. Numerical representation of atomic motion

Acceleration of the i_{th} atom is evaluated by geometric constraints and interaction force for D^{th} dimension and time is t

Table 3
Result Analysis of ASO with Other Algorithm's on Conventional Benchmark's Function.

Analysis of ASO with Other Algorithm's on Conventional Benchmark's Function										
Algorithm	F-1					F-2				
	P-Value	Mean	Max	Min	SD	P-Value	Mean	Max	Min	SD
ASO	1.000E+00	-4.449E-12	3.212E-10	-3.384E-10	1.512E-10	6.952E-01	-3.926E-11	4.689E-09	-5.637E-09	2.161E-09
L-ASO	N/A	-4.453E-12	1.157E-10	-2.508E-10	1.157E-10	N/A	6.394E-11	1.921E-09	-3.994E-09	1.921E-09
BBO	3.011E-09	2.706E-05	7.408E-05	2.428E-06	2.382E-05	3.011E-09	6.397E-04	1.169E-03	3.087E-04	2.363E-04
MVO	3.011E-09	7.724E-03	1.458E-02	2.928E-03	3.176E-03	3.011E-09	2.564E-02	4.009E-02	1.266E-02	8.431E-03
SSA	3.011E-09	7.423E-10	1.289E-09	4.066E-10	2.425E-10	3.011E-09	2.088E-05	2.032E-04	5.291E-06	4.458E-05
F-3										
ASO	9.234E-01	3.852E-02	2.195E+01	-2.235E+01	1.176E+01	7.845E-01	3.041E-09	4.389E-07	-4.108E-07	1.845E-07
L-ASO	N/A	-2.921E-02	1.254E+01	-2.137E+01	1.254E+01	N/A	1.428E-08	5.676E-07	-1.341E-06	5.676E-07
BBO	4.341E-01	8.325E-02	1.797E-01	1.084E-02	4.331E-02	3.011E-09	2.287E-02	3.755E-02	7.711E-03	8.131E-03
MVO	4.341E-01	7.053E-02	1.815E-01	1.758E-02	3.846E-02	3.011E-09	7.160E-02	1.400E-01	3.665E-02	3.036E-02
SSA	4.341E-01	3.527E-09	1.269E-08	5.749E-10	2.643E-09	3.011E-09	1.670E-05	2.390E-05	1.032E-05	3.806E-06
F-5										
ASO	7.958E-01	8.816E-02	5.224E-01	-4.543E-02	1.501E-01	3.112E-01	-9.373E-03	4.191E-01	-4.164E-01	2.383E-01
L-ASO	N/A	1.178E-01	2.192E-01	-5.491E-02	2.192E-01	N/A	2.652E-03	2.391E-01	-4.089E-01	2.391E-01
BBO	6.167E-09	5.546E+00	1.345E+01	2.199E-01	3.103E+00	2.387E-01	3.439E-05	1.093E-04	3.765E-06	3.058E-05
MVO	3.011E-09	2.533E+02	1.872E+03	5.244E+00	4.746E+02	6.848E-01	9.329E-03	2.081E-02	3.939E-03	4.215E-03
SSA	3.011E-09	4.178E+01	3.753E+02	4.116E+00	8.872E+01	2.387E-01	8.140E-10	1.526E-09	4.055E-10	3.199E-10
F-7										
ASO	2.458E-01	3.396E-03	1.586E-01	-1.434E-01	8.570E-02	3.555E-01	4.882E+01	4.211E+02	-3.027E+02	2.751E+02
L-ASO	N/A	-5.235E-04	8.864E-02	-1.656E-01	8.864E-02	N/A	5.759E+01	2.770E+02	-3.030E+02	2.770E+02
BBO	2.985E-01	1.862E-03	4.432E-03	3.217E-04	1.149E-03	3.011E-09	-3.519E+03	-3.144E+03	-3.973E+03	2.318E+02
MVO	3.078E-01	1.752E-03	3.669E-03	4.426E-04	9.746E-04	3.011E-09	-3.057E+03	-2.374E+03	-3.834E+03	3.691E+02
SSA	2.105E-02	7.806E-03	1.725E-02	1.725E-03	4.942E-03	3.011E-09	-2.715E+03	-2.267E+03	-3.241E+03	2.749E+02
F-9										
ASO	2.905E-01	-3.814E-02	1.658E+00	-1.526E+00	9.108E-01	2.282E-01	5.375E-12	3.718E-10	-3.134E-10	1.560E-10
L-ASO	N/A	3.151E-02	9.045E-01	-1.625E+00	9.045E-01	N/A	-8.299E-12	1.409E-10	-3.284E-10	1.409E-10
BBO	3.011E-09	7.810E+00	1.691E+01	2.985E+00	3.699E+00	3.011E-09	1.907E-03	3.694E-03	8.160E-04	8.595E-04
MVO	3.011E-09	1.463E+01	3.085E+01	6.968E+00	6.656E+00	3.011E-09	2.494E-01	2.323E+00	1.964E-02	6.586E-01
SSA	3.011E-09	1.194E+01	1.791E+01	6.965E+00	3.386E+00	3.011E-09	4.427E-01	2.580E+00	8.500E-06	8.424E-01
F-11										
ASO	2.052E-03	3.875E-02	7.751E-01	-1.047E-01	1.912E-01	3.555E-01	-9.934E-01	-8.680E-01	-1.000E+00	2.951E-02
L-ASO	N/A	-1.263E-02	5.649E-02	-2.526E-01	5.649E-02	N/A	-1.000E+00	1.767E-10	-1.000E+00	1.767E-10
BBO	3.011E-09	4.404E-02	9.357E-02	7.607E-03	2.342E-02	3.011E-09	4.004E-07	1.271E-06	2.760E-08	3.991E-07
MVO	3.011E-09	3.457E-01	8.526E-01	1.184E-01	2.093E-01	3.011E-09	8.111E-02	6.283E-01	1.146E-04	1.764E-01
SSA	3.011E-09	2.027E-01	5.115E-01	8.861E-02	1.044E-01	3.011E-09	2.839E-01	1.929E+00	4.772E-12	5.031E-01
F-13										
ASO	8.418E-01	1.001E+00	1.023E+00	9.811E-01	1.030E-02	6.667E-01	-3.152E+01	-2.368E+01	-3.261E+01	1.928E+00
L-ASO	N/A	9.995E-01	2.457E-03	9.890E-01	2.457E-03	N/A	-2.991E+01	3.661E+00	-3.246E+01	3.661E+00
BBO	3.011E-09	1.263E-06	3.719E-06	6.054E-08	9.705E-07	2.552E-02	3.504E+00	1.362E+01	9.980E-01	3.264E+00
MVO	3.011E-09	2.969E-03	1.607E-02	3.828E-04	4.191E-03	2.594E-02	9.980E-01	9.980E-01	9.980E-01	1.927E-11
SSA	3.011E-09	2.747E-03	1.099E-02	2.863E-11	4.881E-03	8.830E-03	9.980E-01	9.980E-01	9.980E-01	1.906E-16
F-15										
ASO	8.857E-01	1.921E+00	2.490E+00	7.805E-01	4.271E-01	1.000E+00	-3.114E-02	4.012E-01	-4.012E-01	4.096E-01
L-ASO	N/A	1.633E+00	1.156E+00	-2.961E+00	1.156E+00	N/A	-1.246E-01	3.773E-01	-4.012E-01	3.773E-01
BBO	2.216E-03	1.929E-03	2.036E-02	3.196E-04	4.418E-03	2.544E-02	-1.032E+00	-1.032E+00	-1.032E+00	5.290E-14
MVO	2.216E-03	4.750E-03	2.036E-02	5.128E-04	8.015E-03	2.594E-02	-1.032E+00	-1.032E+00	-1.032E+00	3.439E-07
SSA	2.216E-03	8.903E-04	1.252E-03	3.555E-04	2.889E-04	2.594E-02	-1.032E+00	-1.032E+00	-1.032E+00	2.244E-14
F-17										
ASO	3.429E-01	4.000E+00	4.000E+00	4.000E+00	1.017E-09	6.667E-01	-5.000E-01	-5.000E-01	-5.000E-01	2.516E-09
L-ASO	N/A	4.001E+00	4.210E-02	3.918E+00	4.210E-02	N/A	-5.000E-01	2.101E-09	-5.000E-01	2.101E-09
BBO	2.095E-03	-6.406E+00	-1.859E+00	-1.054E+01	3.890E+00	2.594E-02	3.000E+00	3.000E+00	3.000E+00	9.375E-12
MVO	2.216E-03	-9.611E+00	-2.807E+00	-1.054E+01	2.301E+00	2.594E-02	7.050E+00	8.400E+01	3.000E+00	1.811E+01
SSA	2.216E-03	-8.624E+00	-2.427E+00	-1.054E+01	3.409E+00	2.594E-02	3.000E+00	3.000E+00	3.000E+00	1.910E-13
F-19										
ASO	7.000E-01	5.076E-01	5.076E-01	5.076E-01	2.062E-09	1.000E+00	3.449E-01	3.449E-01	3.449E-01	1.247E-09
L-ASO	N/A	5.076E-01	2.259E-09	5.076E-01	2.259E-09	N/A	3.449E-01	1.315E-09	3.449E-01	1.315E-09
BBO	4.880E-04	-3.863E+00	-3.863E+00	-3.863E+00	1.808E-15	2.934E-04	-3.292E+00	-3.203E+00	-3.322E+00	5.282E-02
MVO	7.082E-03	-3.863E+00	-3.863E+00	-3.863E+00	1.968E-06	2.934E-04	-3.226E+00	-3.198E+00	-3.322E+00	4.926E-02
SSA	7.068E-03	-3.863E+00	-3.863E+00	-3.863E+00	5.551E-14	2.934E-04	-3.221E+00	-3.187E+00	-3.322E+00	5.176E-02
F-21										
ASO	2.857E-02	4.336E+00	6.000E+00	3.923E+00	7.247E-01	1.000E+00	4.000E+00	4.000E+00	4.000E+00	9.947E-10
L-ASO	N/A	4.313E+00	7.299E-01	3.927E+00	7.299E-01	N/A	3.999E+00	3.841E-02	3.907E+00	3.841E-02
BBO	2.109E-03	-5.655E+00	-2.630E+00	-1.015E+01	3.768E+00	2.133E-03	-5.316E+00	-1.838E+00	-1.040E+01	3.480E+00
MVO	2.216E-03	-7.379E+00	-2.630E+00	-1.015E+01	3.256E+00	2.216E-03	-9.078E+00	-5.088E+00	-1.040E+01	2.354E+00
SSA	2.216E-03	-9.019E+00	-2.630E+00	-1.015E+01	2.378E+00	2.216E-03	-9.610E+00	-5.088E+00	-1.040E+01	1.937E+00
F-22										

$$A_i^D(t) = \frac{F_i^D(t) + G_i^D(t)}{M_i} \alpha \left(1 - \frac{-t-1}{T}\right)^3 e^{-\frac{20t}{T}} + \sum_j \frac{\text{random}_j [2(b_{ij}(t))^{13} - (b_{ij}(t))^7]}{M_i(t)} \frac{(y_j^D(t) - y_i^D(t))}{y_i(t), y_j(t)} + \rho e^{-\frac{20(y_{best}^D(t) - y_i^D(t))}{T}} \frac{1}{M_i(t)} \quad (25)$$

$M_i(t)$ is mass of the i^{th} atom for t^{th} iteration which is expressed as

$$m_i(t) = e - \frac{Fit_i(t) - Fit_{best}(t)}{Fit_{worst}(t) - Fit_{best}(t)} \quad (26)$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^N m_j(t)} \quad (27)$$

here, $Fit_{best}(t) = \min_{i=1,2,\dots,n} Fit_i(t)$ and $Fit_{worst}(t) = \max_{i=1,2,\dots,n} Fit_i(t)$. Velocity and position of the i^{th} atom at $(t+1)$ are written as

$$y_i^D(t+1) = y_i^D(t) + u_i^D(t+1) \quad (28)$$

$$u_i^D(t+1) = \text{random}_i^D u_i^D(t) + A_i^D(t) \quad (29)$$

In this, each atom needs to communicate with a large number of atoms with better fitness values, just like its neighbors, so H's value must be high. To increase exploitation towards the end of the algorithm, there must be a decrease in the number of neighbors H. So, H is computed as

$$H(t) = N - (N-2) \sqrt{\frac{t}{T}} \quad (30)$$

4. Framework of the proposed algorithm

4.1. Opposition based learning (OBL)

To find global optima it is necessary to proper use of search space of metaheuristic optimization algorithm. The meta-heuristic algorithms begin from initial points that are selected randomly and search global

optima by different control parameters and processing methods. Sometimes these randomly selected initial points stuck into local minima. To solve this problem H.R. Tizooosh invented a phenomenon which is known as opposition based learning. In this phenomenon, population divided into two parts: first one is a random population and the second one is opposite population of random population presented in [28]. This phenomenon helps to increase the search space of the algorithm to find global optima. The main concept of this phenomenon is to search for a better solution. A better solution can be achieved by exploring the search space and the opposite search space. To improve the convergence rate of the harmony search algorithm OBL phenomena is used in [29], Similarly to improve convergence characteristic OBL phenomena is used in the initialization of PSO in [30]. An OBL-KH algorithm is proposed for enhancing the Krill Herd (KH) algorithm in [31]. To prove the efficacy of OBL phenomena it is also used in [32–34].

These analyses have shown positive implications on the performance, of the algorithms. Hence, in initialization phase, whole population is divided into two folds the first phase, is random population part and remaining part is opposite population part as steps:

- Initialize the Random Atoms (A_T)
- $\bar{A}_T$ as : $\bar{A}_{ij} = ub_i + lb_i - A_{ij}$, $i = 1, 2, \dots, N/2$ and $j = 1, 2, \dots, N/2$ here : $\bar{A}_{ij}$ & A_{ij} denote the i^{th} point of the j^{th} solution of A_T and its corresponding $\bar{A}_T$.
- Select total N population from the set of these two populations eg. $\bar{A}_T \cup A$.

4.2. Position update using laplacian operator

In many approaches position update mechanism has been adopted by multiplying native update equations with some mathematical and chaotic operators [24,32] and [35]. Success of these approaches motivated authors to employ same philosophy in ASO. Here we define a laplacian operator first, that is given as:

Step 1: Define a number series of size $(1 \times \text{Atom No.})$ having laplacian distribution and take the mean (μ) and standard deviation (σ) of the series.

Step 2: Define a number series X_{prime} that is given as follows:

$$X_{prime} = \mu + \text{linspace}(-3 \times \sigma, 3 \times \sigma, T) \quad (31)$$

Further, using this number series a laplacian operator is employed here to adjust the acceleration factor of the ASO

$$F_x = \frac{1}{\sqrt{(2 * (\sigma)^2) * \exp\left(-\sqrt{\left(\frac{2}{\sigma^2}\right)}\right)}} |X_{prime} - \mu| \quad (32)$$

Step 3: The updated equation for acceleration can be written as per following:

Table 4
Rank Analysis of L-ASO with Other Algorithm's.

Functions	ASO	L-ASO	BBO	MVO	SSA
F1	2	1	4	5	3
F2	1	2	4	5	3
F3	3	1	5	4	2
F4	1	2	4	5	3
F5	1	2	3	5	4
F6	1	4	3	5	2
F7	4	1	3	2	5
F8	4	5	1	2	3
F9	1	2	3	5	4
F10	2	1	3	4	5
F11	2	1	3	5	4
F12	2	1	3	4	5
F13	5	4	1	3	2
F14	1	2	5	4	3
F15	5	4	2	3	1
F16	5	4	1	3	1
F17	4	5	3	1	2
F18	2	1	4	5	3
F19	5	4	1	3	2
F20	5	4	1	2	3
F21	5	4	3	2	1
F22	5	4	3	2	1
Rank (Avg.)	4 (3.00)	1 (2.68)	3 (2.86)	5 (3.59)	2 (2.82)

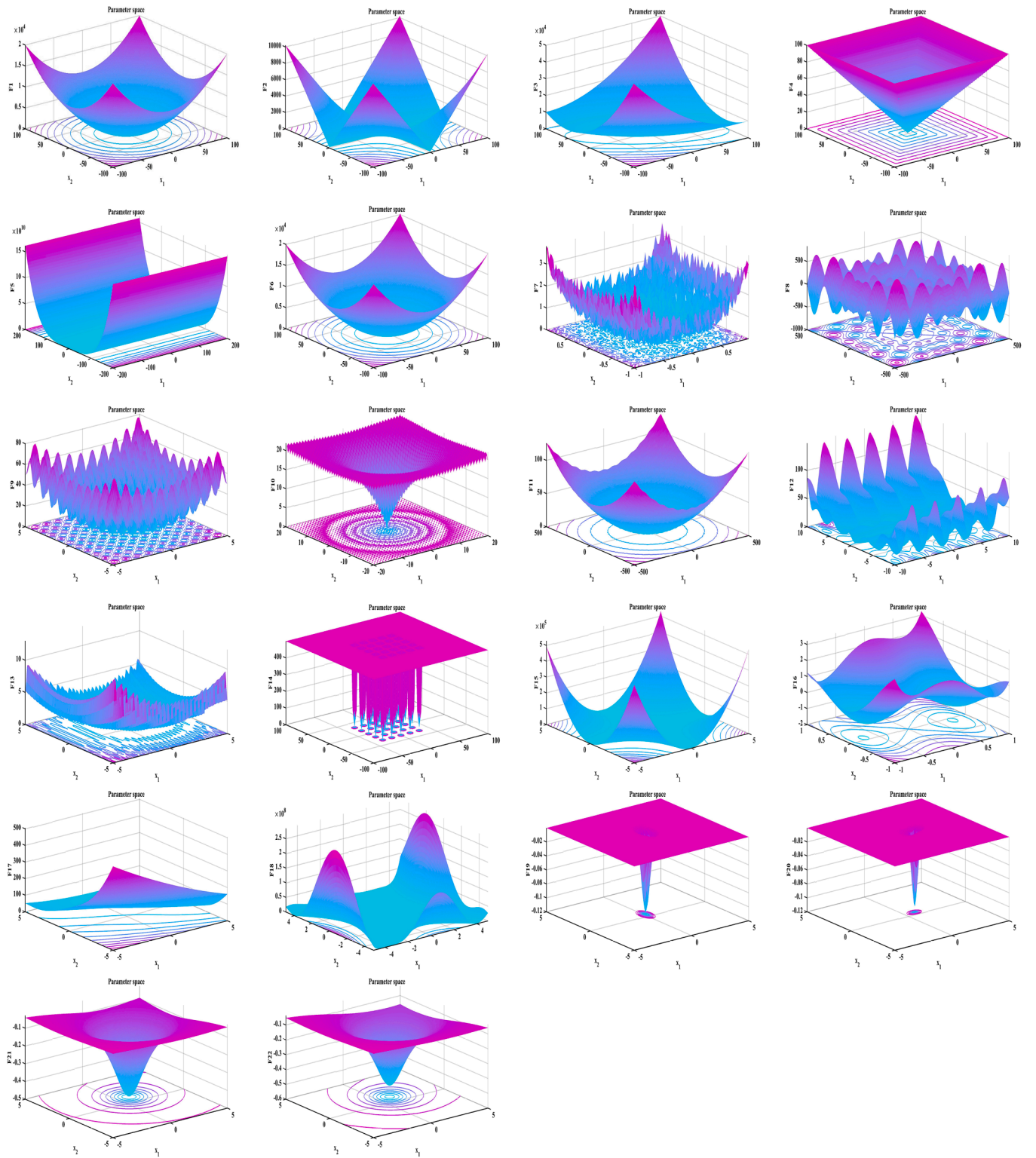


Fig. 3. Shape Curves of Benchmark Function's.

$$\lambda(t) = pe^{-\frac{20t}{T}} * Fx \quad (33)$$

4.3. Discussion

The proposed framework of algorithm implements two basic technologies in initialization and position update namely opposition based

learning and Laplacian factor based jump mechanism for avoiding the entrapment in locals. In past, it is witnessed in many research approaches that dividing population on the basis of opposite number generation is fruitful for exploring the search space. For a given search space agents starts search in diverse direction with the help of opposition based mechanism and yield favorable outcomes as far as accuracy of the fitness function values is concerned.

On the other hand, if this process were not controlled with the help of

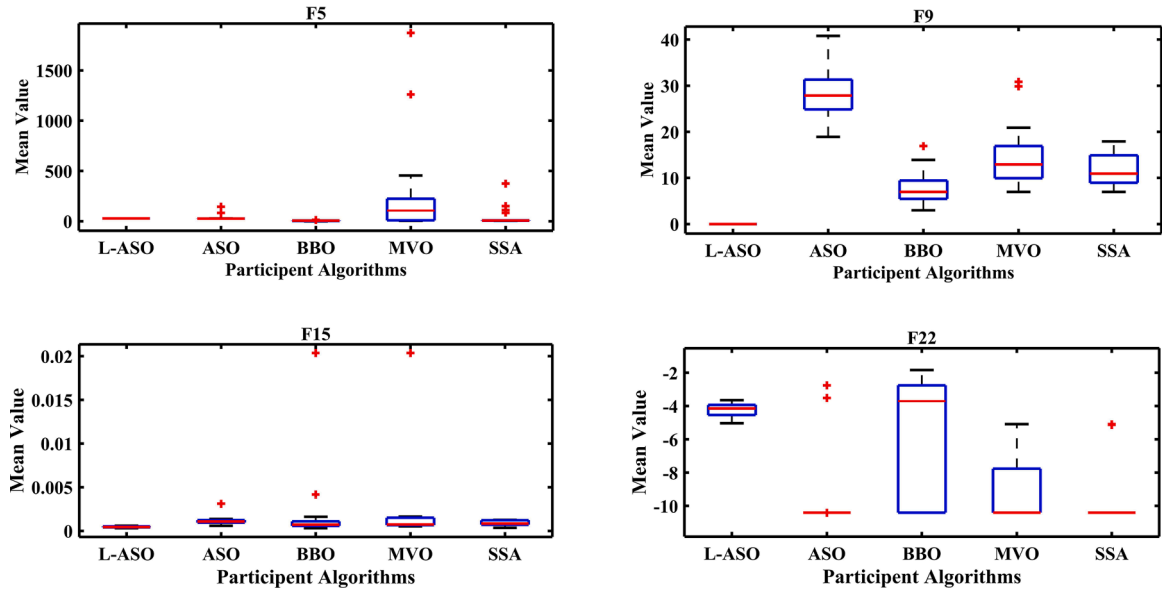


Fig. 4. Boxplot Analysis of Benchmark Function's.

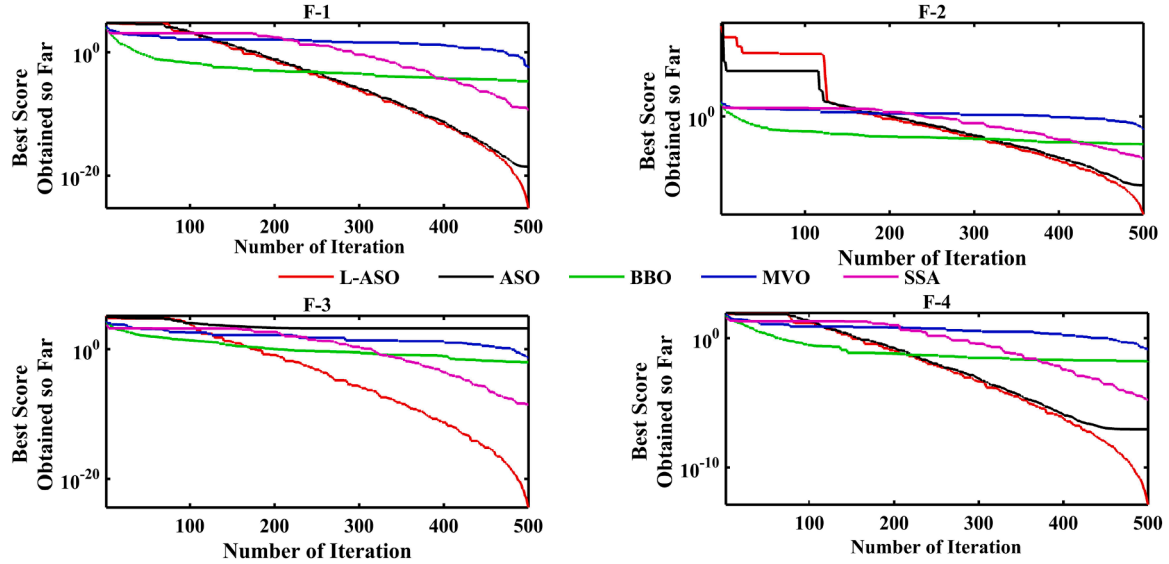


Fig. 5. Convergence Analysis of Uni-Modal Function's.

some bridging the convergence period of the algorithm would be very high. Hence, it is advisable that when an experiment is incorporated with the exploration phase, of the algorithm, simultaneously, another effort has to be made in terms of controlling exploration phase, of ignite the exploitation phase, in early iterations. Laplacian factor is multiplied with the position update expression hence, it provide an abrupt jump from the current position and try the solution at diverse location. In case of a linear gradient based mechanism this kind of jump is helpful when the algorithm is prone to trapped in local minima. Similar approaches have implemented named as craziness in particle swarm optimization and other algorithms.

On the basis of these two arguments, L-ASO possesses both virtues i. e. improved exploration capability with the incorporation of opposition based theory and improved exploitation (more control over the convergence) due to diverse gradient in the first and last half of the iterations. This diverse gradient of Laplacian factor provides high and low jump in the iterative process. This phenomenon provides a better control on the convergence of the algorithm as well as it provides redemption

from local minima entrapment. The same has been shown in Fig. 2. The Proposed factor helps the algorithm to get redemption from stagnation in local traps and enhance the convergence of the algorithm. In following section, we will present the results and discussion on the proposed L-ASO and we will exhibit how these modifications are proven boonful for the performance, of ASO.

5. Statistical result analysis

In this segment, two tests of assessment are considered to validate the effectiveness of the proposed technique, which can be described as follows

- In the first segment, a fair comparison of a newly developed variant of ASO is conducted. This is depicted in table 2. This comparison is conducted on standard benchmark test functions [36], which helps to prove the supremacy of a new variant. Standard benchmark test functions are further classified into three categories the first one is

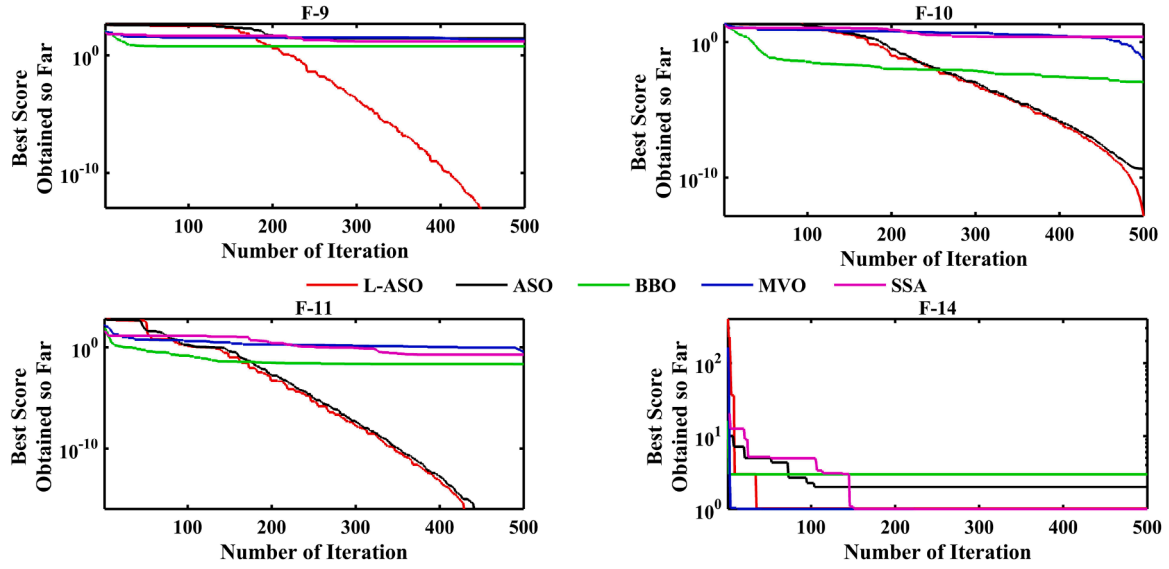


Fig. 6. Convergence Analysis of Multi-Modal Function's.

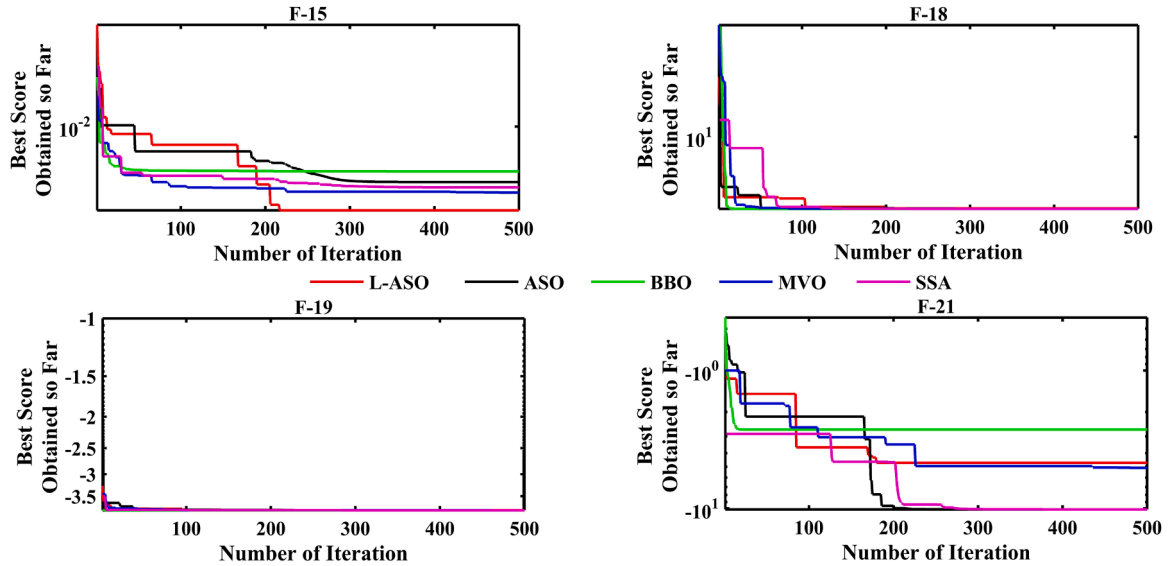


Fig. 7. Convergence Analysis of Fixed-Dimension Function's.

uni-modal test functions, second one is multi-modal test functions, and the third one is fixed dimension multi-modal test functions.

- In the second segment, This variant of ASO is compared with parent ASO, Bio-geography-based optimization (BBO) [37], Multi-Verse Optimizer (MVO) [38], and Salp swarm algorithm (SSA) [39] which is reported in table 3. This comparison also performed on the standard benchmark test functions, which helps to prove the virtue of a new variant of ASO. This comparison is proved by the ranking method, which is represented in table 4. These 3 tests are compiled and executed with the help of MATLAB R2015a and on a system with configuration of i5 8th generation processor, 1TB HDD, 8GB RAM.

5.1. Comparison of L-ASO with ATOM

In this section, to validate the performance analysis of new variant of ASO, it is compared with parent ASO. To make a trustworthy comparison during the execution time parameter setting is constant which includes the number of iterations=500, the number of search agents=30,

and the number of runs=20. In this validation dimension, minimum value, maximum value, and range of benchmark test functions are known. The validation process depends on the following numerical data.

1. Mean value (average value)
2. Max (maximum value)
3. Min (minimum value)
4. SD (standard deviation)

Standard benchmark test functions are divided into three types: uni-modal test functions, multi-modal test functions, and fixed dimension multi-modal test functions, which are reported in table 1. All 22 standard benchmark test function's shape curves are illustrated in Fig. 3 which describes about their search space and their minima's. Numerical results of standard benchmark test functions are reported in table 2.

- Uni-modal test functions are F1-F7 functions, which are used to evaluate the strength of exploitation because these functions have only single global optima.
- Multi-modal test functions are F8-F13 functions, which help to validate the exploration ability of every algorithm. This type of test

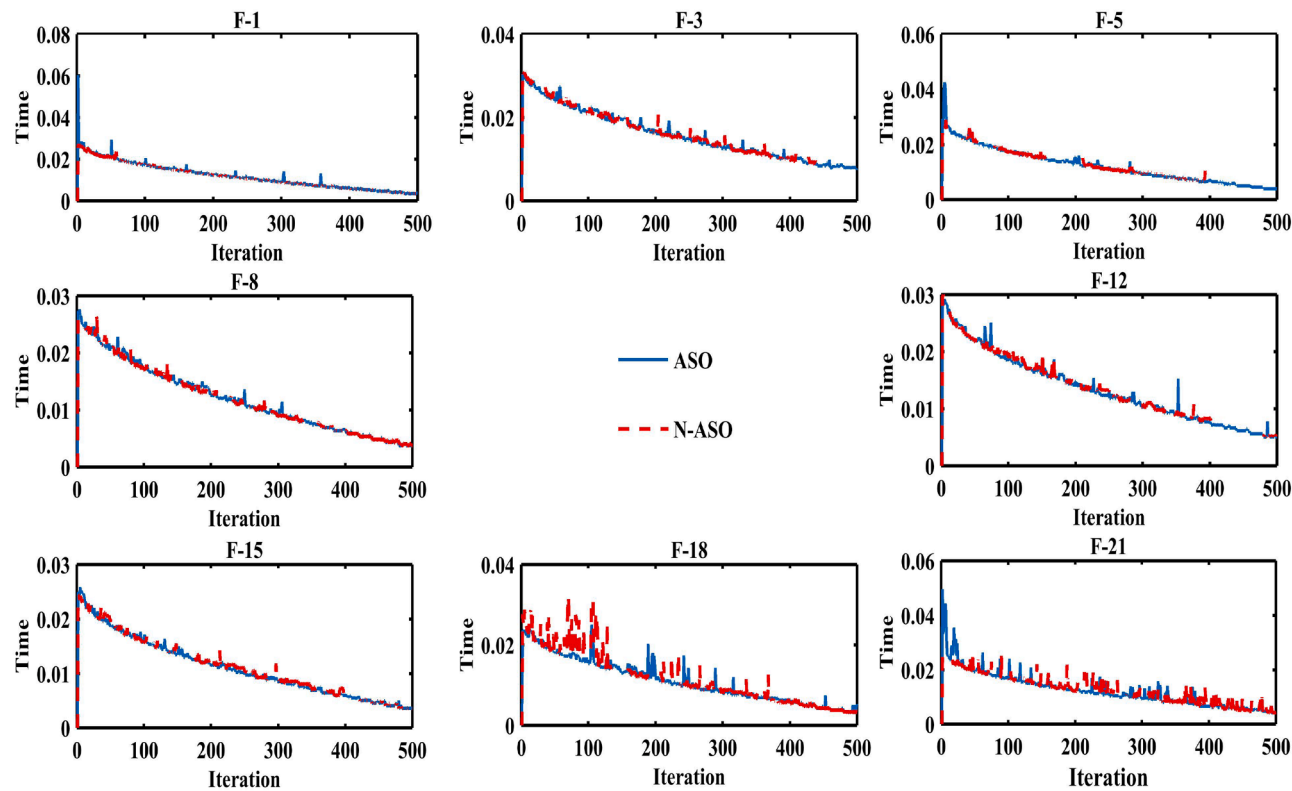


Fig. 8. Execution time analysis.

Table 5
Algorithms Comparison on Problem 1.

Algorithms Comparison on Problem 1						
Algorithms	Parameters	Fun	3rd	5th	7th	11th
Actual	F(Hz)	50	150	250	350	550
	A(V)	1.5	0.5	0.2	0.15	0.1
	P(deg.)	80	60	45	36	30
PSOPC-LS[42,43]	A(V)	1.4828	0.4886	0.1822	0.1561	0.0948
	P(deg.)	80.5423	62.2445	46.6343	34.6214	27.3154
MABC[44]	A(V)	1.5006	0.4997	0.1995	0.1498	0.1003
	P(deg.)	80.0187	60.0098	45.0905	36.0894	29.5511
GA-LS[42,43]	A(V)	1.48	0.485	0.18	0.158	0.0937
	P(deg.)	80.61	62.4	47.03	34.354	26.7
F-BFO[42,43]	A(V)	1.488	0.5103	0.198	0.1545	0.1028
	P(deg.)	80.42	58.1	45.75	34.73	29.358
BFO[42,43]	A(V)	1.48	0.5108	0.1945	0.1556	0.1034
	P(deg.)	80.4732	57.9005	45.8235	34.5606	29.358
Proposed	A(V)	1.5000	0.4999	0.1999	0.1500	0.1000
	P(deg.)	79.9951	60.0000	44.8876	35.5968	29.9498

Table 6
% Error Analysis of Harmonics Estimator Design Problem-1.

% Error Analysis of Harmonics Estimator Design Problem-1						
Algorithms	Parameters	Order of Harmonics				
		Fun	3rd	5th	7th	11th
BFO-RLS	% Phase Error	4.340E-01	5.660E-01	1.550E+00	3.310E+00	2.130E-01
PSOPC-LS[42,43]	% Amp. Error	3.840E-01	2.860E-01	9.020E-01	1.760E+00	1.750E+00
	% Phase Error	6.750E-01	3.670E+00	3.560E+00	3.820E+00	8.970E+00
	% Amp. Error	1.200E+00	2.400E+00	9.000E+00	4.060E+00	5.200E+00
GA-LS[42,43]	% Phase Error	7.630E-01	4.000E+00	4.510E+00	4.570E+00	1.100E+01
	% Amp. Error	1.330E+00	3.000E+00	1.000E+01	5.330E+00	6.300E+00
	% Phase Error	2.340E-02	1.640E-02	2.010E-01	2.490E-01	1.500E+00
MABC[44]	% Amp. Error	4.120E-02	5.530E-02	2.420E-01	1.550E-01	3.370E-01
	% Phase Error	3.640E-05	1.361E-04	2.034E-03	2.785E-04	3.056E-03
	% Amp. Error	6.153E-03	0.000E+00	2.498E-01	1.120E+00	1.674E-01

Table 7
Problem-1 Analysis with Noise Signals.

Problem-1 Analysis with Noise Signals						
Signal's	Parameters	Fun	3rd	5th	7th	11th
Actual Signal Parameter's	F(Hz)	50	150	250	350	550
	A(V)	1.5	0.5	0.2	0.15	0.1
	P(deg.)	80	60	45	36	30
Wave without Noise	A(V)	1.5000	0.4999	0.1999	0.1500	0.1000
	P(deg.)	79.9951	60.0000	44.8876	35.5968	29.9498
Original with 10dB Noise	A(V)	1.4954	0.4762	0.1948	0.1607	0.1223
	P(deg.)	79.9949	59.9946	44.9154	35.4433	29.9391
Original with 20dB Noise	A(V)	1.4979	0.4905	0.1955	0.1592	0.1144
	P(deg.)	79.9954	59.9948	44.9046	35.3332	29.9464
Original with 30dB Noise	A(V)	1.5003	0.5004	0.1960	0.1537	0.1036
	P(deg.)	79.9950	59.9955	44.8951	35.4776	29.9634
Original with 40dB Noise	A(V)	1.5001	0.4999	0.1985	0.1513	0.1005
	P(deg.)	79.9950	59.9952	44.8957	35.4991	29.9442

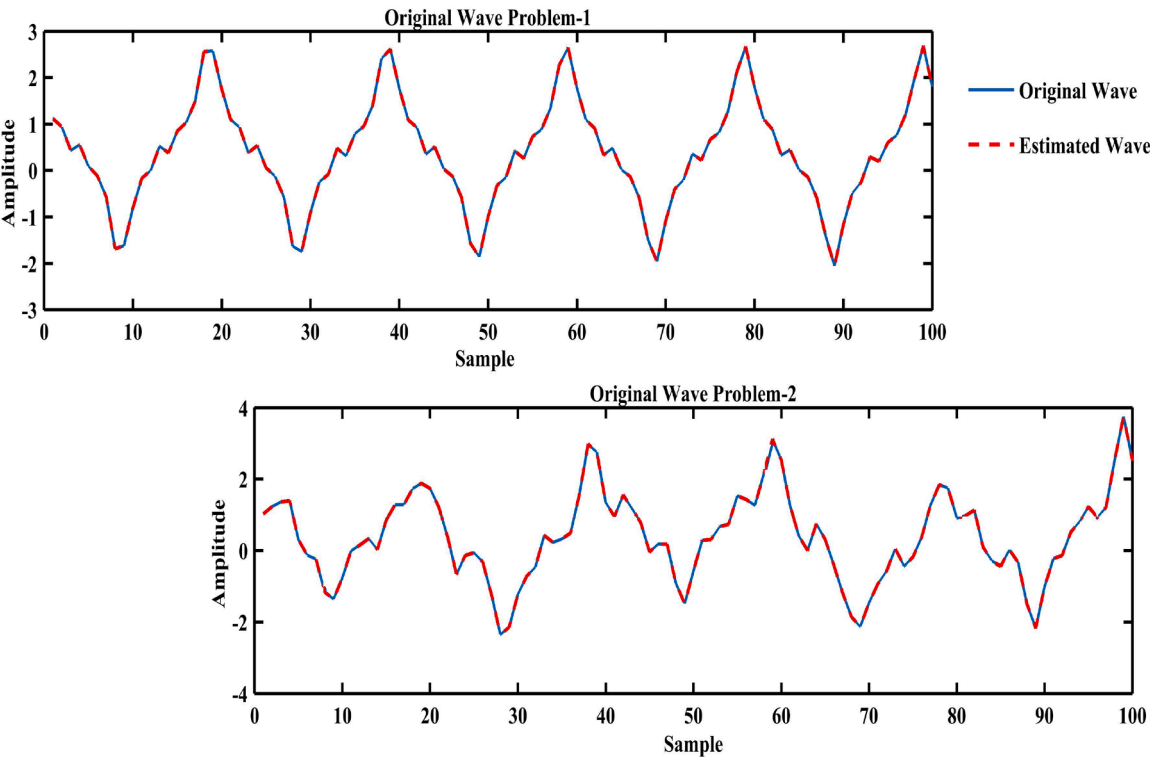


Fig. 9. Estimated Waves of Problem-1 and Problem-2 Under no Noise.

function has one global optima and a lot of local optima. The number of local optima varied according to the problem. Sometimes algorithms locked in local optimum and spend more time in finding for the global optimum. To escape from this problem L-ASO is used which uses Laplace Transform and opposition based learning. This modification in ASO helps to search global optima in minimum time.

- Fixed-dimension multi-modal test functions and multi-modal functions have an inverse relationship between them.

In this validation, it is concluded that L-ASO provides better results for 13 standard benchmark test functions, these results are highlighted in [table 2](#), which helps to prove the dominance of L-ASO.

5.2. Comparison of L-ASO with algorithms

To prove the superiority of L-ASO, it is compared with parent ASO, BBO, MVO, and SSA. This test also conducted on standard benchmark test functions. For this assessment parameter settings are also constant

which is the number of iterations=500, the number of search agents =30, and the number of runs =20. This assessment is done through the numerical values of the p-value, Mean, Max, Min, and SD values. These numerical values of algorithms are depicted in the [table 3](#). From this assessment, it is certified that L-ASO provides better Mean values for F1, F3, F7, F10, F11, F12, and F18, which proves the virtue of L-ASO. This certification is completed by the ranking method which is reported in [table 4](#). From this ranking [table 4](#) it is proved that L-ASO provides better results in comparison to parent ASO, BBO, MVO, and SSA because L-ASO gets the first rank.

6. Stability analysis

To check the stability of L-ASO we use diverse methods, these are as following

1. Box plot analysis
2. Wilcoxon rank sum test
3. Convergence behavior analysis

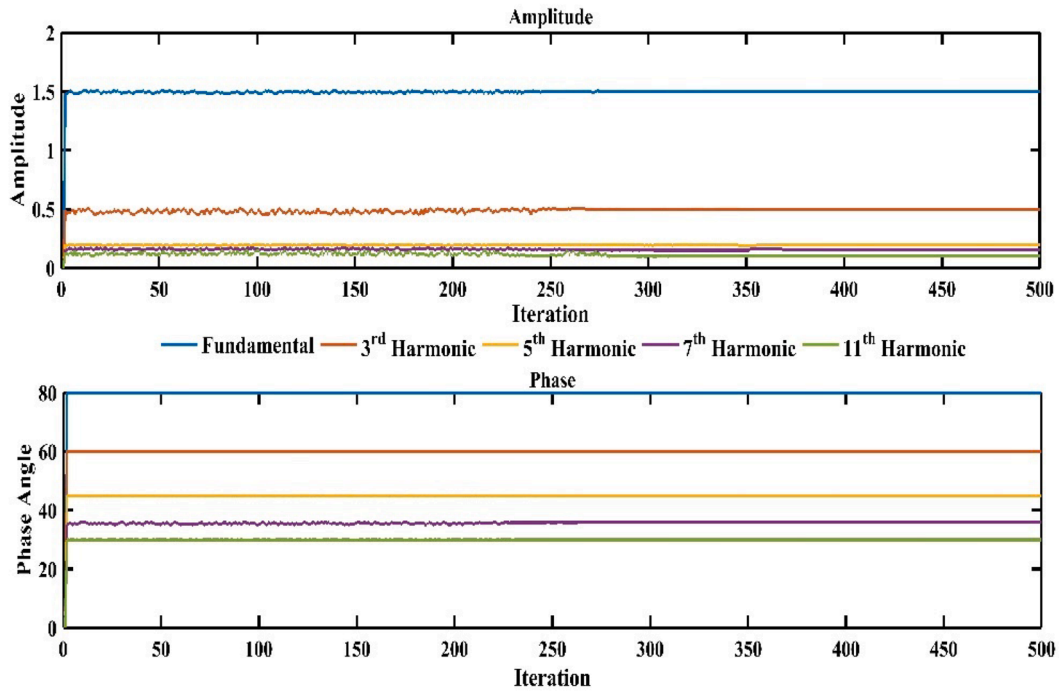


Fig. 10. Trajectory Analysis of Problem-1 Phase and Amplitude.

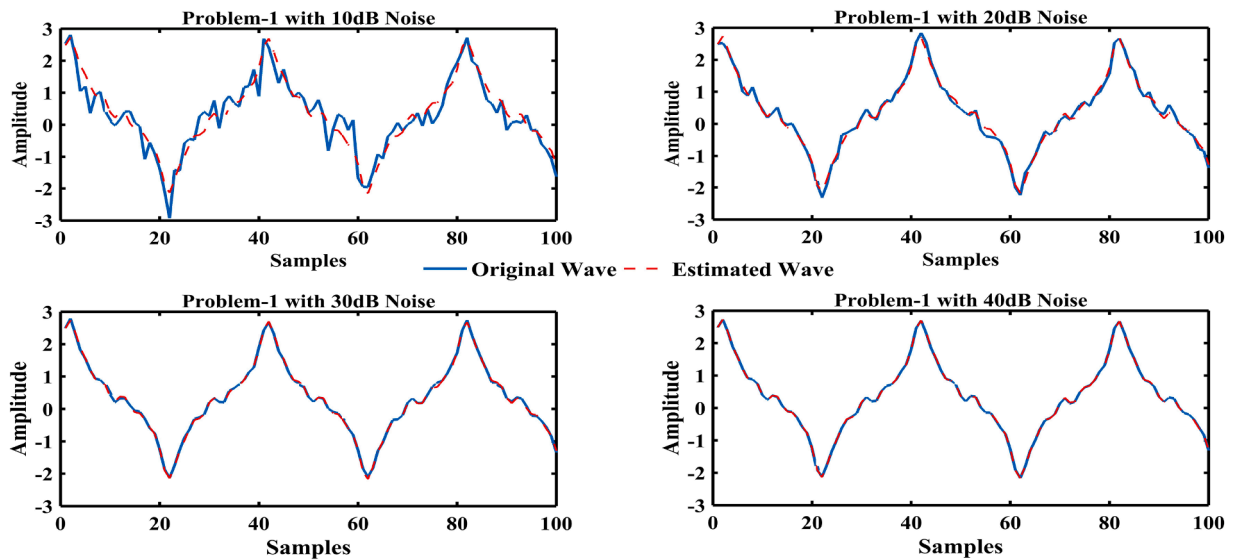


Fig. 11. Estimated Waves of Problem-1 Under Noise.

4. Execution time analysis

6.1. Box plot analysis

Box plot is a mechanism that helps to distribute the data for separate executions. In this analysis, L-ASO is compared with parent ASO, BBO, MVO, and SSA. Some box plots for Function (F5, F9, F15, F22) are illustrated in fig 4. The inter quartile range (IQR) decides the minimum and maximum values of box plots. Box plot analysis an indicator of a promising solution quality of L-ASO. It is observed from the plot that the box plots are super narrow and all the fitness values fall in a small range.

6.2. Wilcoxon rank sum test

"Wilcoxon rank-sum test" [40] is validated for statistical authentication of the proposed L-ASO. This test is used to evaluate the data set and its distribution in combinations from two algorithms, and the result of this test agrees whether the algorithm proposed, is statistically distinct or not. Wilcoxon rank-sum test conducted in this experiment, works on a 5% significant level. Whenever the p-value is less than 0.05, that means L-ASO and ASO behave differently. Otherwise it is assumed that both compared algorithms behave similar. Results of the Wilcoxon rank-sum test are reported in table 3. From this test, it is validated that the proposed modifications are trustworthy and gives better results. In front of L-ASO an entry mark N/A has been written that indicates that L-ASO has been compared with other algorithms. From the table, it is

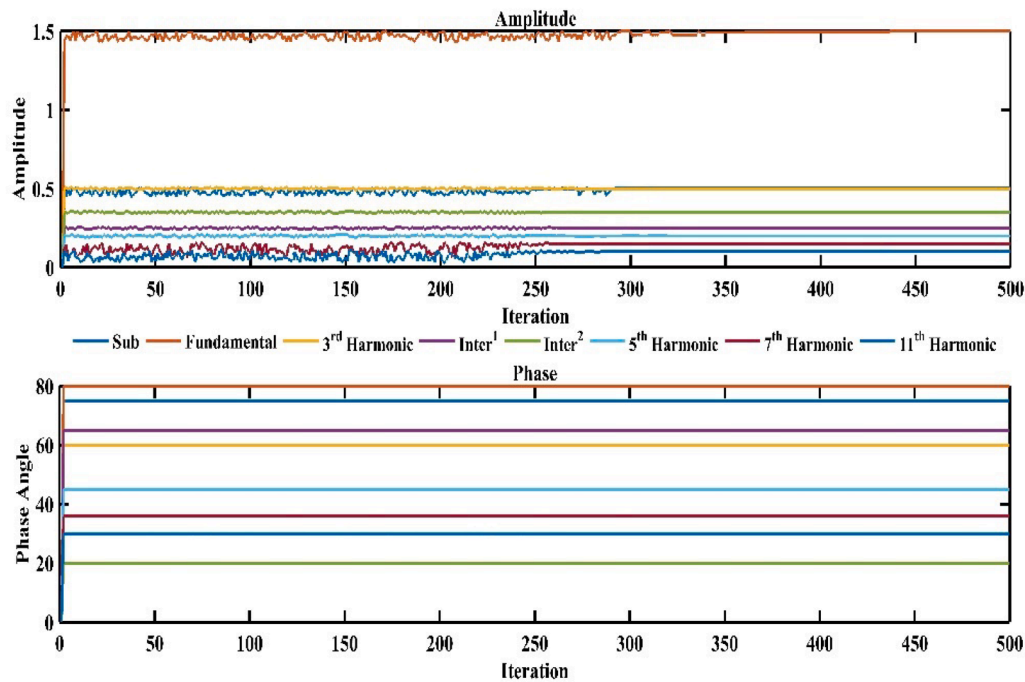


Fig. 12. Trajectory Analysis of Problem-2 Phase and Amplitude.

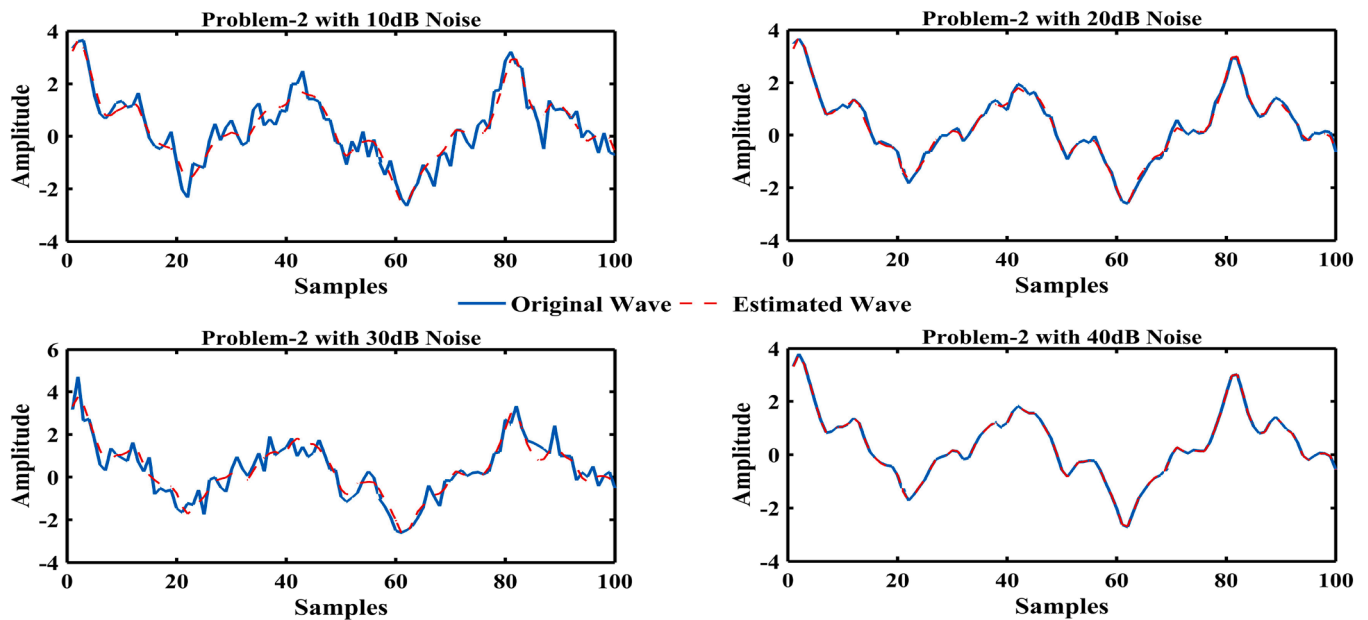


Fig. 13. Estimated Waves of Problem-2 Under Noise.

clearly visible that in almost cases the proposed algorithm is statistically distinct from other competitors.

6.3. Convergence behavior analysis

Assessment of the convergence of the proposed model L-ASO is conducted with the help of this analysis, especially in comparison with parent ASO, BBO, MVO, and SSA. Here, a different set of functions is chosen to take that is mentioned previously, and their convergence characteristics are depicted to examine the convergence property of the proposed variant, which means how quickly it reaches a global optimum position. In convergence plots, X axis represents the number of iterations and the Y-axis log scale demonstrates the best score for objective

function values. Some convergence curves are plotted in fig 5, 6, 7 for uni-modal, multi-modal and fixed-dimension multi-modal benchmark functions respectively.

6.4. Execution time analysis

For each iterative process, the execution time graph is plotted in fig 8. In this figure, execution of functions is shown. From this analysis, it is concluded that modify version of ASO provides optimal execution time for standard benchmark functions.

From the statistical result analysis and stability analysis, it is proved that modification in parent ASO is powerful and effective. Now, this new variant is used to design the harmonic estimator which evaluates the

Algorithm 1

Laplacian Atom Search Optimization (LASO).

```

1: Begin
2: Initialize the Atoms ( $A_T$ ) with Velocity as per subsection 4.1
3: For each atom  $A_{Ti}$  do
4: Fitness Calculation of Atom Fitness;
5:   if  $Fitness_i < Fitness_{Best}$  then
6:      $Fitness_{Best} = Fitness_i$ 
7:      $A_{T_{Best}} = A_{Ti}$ 
8:   End If
9: end
10: Massing calculation using equations{(25)} and (26)
11: Using equation {(29)} to evaluate the neighbors H
12: Calculate Fi (interaction force) and Qi (restriction force) using equations {(22)} and (23), respectively
13: Acceleration Calculation using Eq (25)
14: New update using Calculation using Eq (33)
15: End
16: Update Best solution Obtain so far  $A_{T_{Best}}$ 

```

Table 8

Results of Harmonics Estimator Design Problem-2.

Algorithm Comparison on Problem-2									
Algorithms	Parameters	Sub	Fun	3rd	Inter-1	Inter-2	5th	7th	11th
Actual	F(Hz)	20	50	150	180	230	250	350	550
	A(V)	0.505	1.5	0.5	0.25	0.35	0.2	0.15	0.1
	P(deg.)	75	80	60	65	20	45	36	30
PSOPC-LS [42,43]	A(V)	0.53	1.5049	0.281	0.24	0.377	0.211	0.165	0.11
	P(deg.)	73.51	79.45	58.12	63.28	18.23	48.1	37.109	31.87
MABC [44]	A(V)	0.5052	1.5008	0.4998	0.25	0.35	0.1997	0.15	0.1
	P(deg.)	74.9539	79.98	60.1254	64.9374	20.04	45.1636	35.9987	29.958
GA-LS [42,43]	A(V)	0.532	1.5083	0.472	0.238	0.381	0.251	0.172	0.117
	P(deg.)	73.02	79.23	57.55	62.41	17.64	48.33	38.78	32.56
F-BFO [42,43]	A(V)	0.521	1.489	0.489	0.261	0.371	0.28	0.1468	0.1019
	P(deg.)	74.61	79.86	61.16	64.33	19.729	47.22	36.658	30.52
BFO [42,43]	A(V)	0.525	1.4788	0.4877	0.2664	0.3729	0.2052	0.1464	0.1016
	P(deg.)	74.48	79.8361	61.2316	63.991	19.6887	47.698	36.7372	29.3928
Proposed Algo	A(V)	0.5049	1.4999	0.5001	0.2500	0.3499	0.1999	0.1500	0.1000
	P(deg.)	74.9947	79.9949	59.9950	64.9947	19.9948	44.9956	35.9949	29.9956

Table 9

% Error Analysis of Harmonics Estimator Design Problem-2.

% Error Analysis of Harmonics Estimator Design Problem-1									
Algorithms	Parameters	Order of Harmonics							
		Sub	Fun	3rd	Inter-1	Inter-2	5th	7th	11th
PSOPC-LS [42,43]	% Phase Error	1.99E+00	6.87E-01	3.13E+00	2.65E+00	8.85E+00	6.88E+00	3.08E+00	6.23E+00
	% Amp. Error	4.95E+00	3.26E-01	3.80E+00	4.00E+00	7.70E+00	5.50E+00	1.00E+01	1.10E+01
MABC [44]	% Phase Error	6.15E-02	2.50E-02	2.09E-01	9.64E-02	2.00E-01	3.64E-01	3.60E-03	1.40E-01
	% Amp. Error	4.68E-02	5.30E-02	4.41E-02	1.33E-02	1.10E-03	1.39E-01	3.19E-02	1.84E-02
GA-LS [42,43]	% Phase Error	2.64E+00	9.62E-01	4.08E+00	5.52E+00	1.18E+01	7.40E+00	7.72E+00	8.53E+00
	% Amp. Error	5.35E+00	5.53E-01	5.60E+00	4.80E+00	8.85E+00	7.50E+00	1.47E+01	1.70E+01
F-BFO [42,43]	% Phase Error	5.20E-01	1.75E-01	1.93E+00	1.03E+00	1.39E+00	4.93E+00	1.83E+00	1.73E+00
	% Amp. Error	3.25E+00	7.33E-01	4.89E-01	4.40E+00	6.00E+00	4.00E+00	2.13E+00	1.90E+00
BFO [42,43]	% Phase Error	2.53E-01	1.06E-01	1.54E+00	5.30E-01	6.61E-01	2.84E+00	1.24E+00	2.14E-01
	% Amp. Error	1.19E+00	1.95E-01	1.59E+00	3.24E+00	3.97E+00	4.54E-01	1.41E+00	1.48E+00
Proposed	% Phase Error	7.07E-03	6.37E-03	8.33E-03	8.15E-03	2.60E-02	9.56E-03	1.42E-02	1.47E-02
	% Amp. Error	1.98E-02	6.67E-03	1.28E-02	1.77E-02	2.86E-02	5.00E-02	2.32E-02	3.11E-02

exact amplitude and phase, of power harmonics. Design evaluation of the harmonic estimator is described in the next section.

7. Harmonic estimator design problem

In this segment, we have presented our upgraded version of ASO to develop a harmonic estimator. The outcomes estimates upgraded version ASO improves the exploitation and exploration criterion of ASO.

We considered two problems of harmonic estimator from [41] to validate the effectiveness of harmonic estimator.

7.1. Harmonic estimator design problem 1

To validate the performance analysis of harmonic estimator, design problem 1 is considered which is written as

$$C_1(q) = \frac{3}{2} \sin(2\pi f(\text{fundamental})q + 80) + \frac{1}{2} \sin(2\pi f(3)q + 60) + \frac{1}{5} \sin(2\pi f(5)q + 45) \\ + \frac{3}{20} \sin(2\pi f(7)q + 36) + \frac{1}{10} \sin(2\pi f(11)q + 30) + \frac{1}{2} \exp(-5q) + \sigma_j$$

This wave is made up of fundamental, 2nd, 3rd, 5th, 7th, and 11th harmonic components. In this wave, the fundamental frequency is considered as 50Hz. The main aim of problem-1 is to estimate the exact values of phase and amplitude for each harmonic.

The numerical results of Problem-1 are depicted in tables 5, 6 and 7. Numerical results of problem-1 help to verify the effectiveness and virtue of modification in ASO.

In table 5 phase, and amplitude values of the design problem 1, are reported, which is solved by L-ASO, PSOPC-LS[42,43], MABC [44], GA-LS [42,43], F-BFO [42,43], BFO [42,43]. We observed that L-ASO provides accurate values of phase and amplitude values of power harmonics, which means L-ASO is capable in solving harmonic estimation problem and helpful in design harmonic estimator. Original and estimated wave, under no noise for problem-1 is illustrated in fig 9.

Table 6 represents % error analysis of phase and amplitude values. In this analysis phase and amplitude, % error values are minimum for fundamental, 2nd, 3rd, 4th, 5th, and 6th harmonic components. From this % error analysis it is proved that L-ASO provides optimal values in comparison to ABC [45], BAT, and PSO.

Table 7 accumulated the phase and amplitude numerical results of noisy signals with the original signal. Noisy signals are used to prove the superiority of harmonic estimator. In this estimation some noise signals are taken, these are given as

1. 10dB noise signal
2. 20dB noise signal
3. 30dB noise signal
4. 40dB noise signal

These results represent that noisy signal affects the exact phase and amplitude estimation of the harmonic estimator. Original signal of problem-1 with noisy signals 10dB, 20dB, 30dB and 40dB are plotted in fig 11.

7.2. Harmonic estimator design problem 2

To validate the efficacy of harmonic estimator for sub-harmonic and inter-harmonics, components we take into account problem -2 which includes the sub-harmonic and inter-harmonic components with different harmonics. This consideration makes the difference between problem-1 and problem-2. For the performance analysis of harmonic

estimator, design problem 2 is considered which is represented as

$$C_2(q) = \frac{101}{200} \sin(2\pi f(\text{sub})q + 75) + \frac{3}{2} \sin(2\pi f(\text{fundamental})q + 80) + \\ \frac{1}{2} \sin(2\pi f(3)q + 60) + \frac{1}{4} \sin(2\pi f(\text{inter}^1)q + 80) + \\ \frac{7}{20} \sin(2\pi f(\text{inter}^2)q + 20) + \frac{1}{5} \sin(2\pi f(5)q + 45) + \\ \frac{3}{20} \sin(2\pi f(7)q + 36) + \frac{1}{10} \sin(2\pi f(11)q + 30) + \frac{1}{2} \exp(-5q) + \sigma_j$$

Sub-harmonics, are non-integer multiple of fundamental frequency but these harmonics, have less frequency in comparison to the fundamental frequency while inter-harmonics, are also non-integer multiple of fundamental frequency but these frequency are greater than the fundamental frequency. Sub-harmonic signals are the cause of the core saturation problem, voltage, amplification, current amplification, etc. Inter-harmonic signal also gives adverse effects like light flickering, interference in power line communication, core loss, iron loss, thermal effect, etc.

In problem-2 wave is made of 8 harmonic power harmonics, signals which include a sub, fundamental, 3rd, inter¹, inter², 5th, 7th, and 11th harmonic components. In this design problem, fundamental harmonic is considered 50 Hz. The main aim of design problem-2 is the exact estimation of phase and amplitude values of the harmonic estimator with the help of a modified version of ASO. Numerical results of design problem-2 are accumulated in table 8, 9 and 10. These results help to prove the dominance and effectiveness of modified version of ASO. Original and estimated wave under no noise for problem-2 are illustrated in fig 11.

Table 8 represents the phase and amplitude values of harmonic estimator, which are estimated by L-ASO, PSOPC-LS [42,43], MABC [44], GA-LS [42,43], F-BFO [42,43], BFO [42,43] algorithms. In this assessment modified version of ASO provides exact estimation of phase values of the sub, fundamental, 3rd, inter¹, inter², 5th, 7th, and 11th harmonics, while amplitude values of accurately measured by the modified version of ASO. This estimation is helpful to prove the supremacy of the modified version of ASO.

Table 9 shows the % error analysis of phase and amplitude values of the harmonic estimator, these values are measured by the help of phase and amplitude values which are stored in the table 8. These results

Table 10
Problem-2 Analysis with Noise Signals.

Problem-2 Analysis with Noise Signals									
Signal's	Parameters	Sub	Fun	3rd	Inter-1	Inter-2	5th	7th	11th
Actual Signal Parameter's	F(Hz)	20	50	150	180	230	250	350	550
	A(V)	0.505	1.5	0.5	0.25	0.35	0.2	0.15	0.1
	P(deg.)	75	80	60	65	20	45	36	30
Wave without Noise	A(V)	0.5049	1.4999	0.5001	0.2500	0.3499	0.1999	0.1500	0.1000
	P(deg.)	74.9947	79.9949	59.9950	64.9947	19.9948	44.9956	35.9949	29.9956
Original with 10dB Noise	A(V)	0.4789	1.4682	0.4989	0.2503	0.3508	0.1982	0.1187	0.0761
	P(deg.)	74.9947	79.9947	59.9951	64.9957	19.9945	44.9951	35.9955	29.9961
Original with 20dB Noise	A(V)	0.4904	1.4854	0.5008	0.2489	0.3506	0.2005	0.1430	0.0903
	P(deg.)	74.9946	79.9950	59.9943	64.9949	19.9953	44.9949	35.9958	29.9959
Original with 30dB Noise	A(V)	0.5008	1.4942	0.4999	0.2503	0.3500	0.1999	0.1486	0.1006
	P(deg.)	74.9946	79.9964	59.9948	64.9951	19.9952	44.9961	35.9957	29.9953
Original with 40dB Noise	A(V)	0.5034	1.4983	0.5003	0.2499	0.3499	0.1999	0.1503	0.1000
	P(deg.)	74.9944	79.9950	59.9951	64.9956	19.9952	44.9957	35.9947	29.9957

represent that phase and amplitude % error values are optimal, means modified version of ASO provides better design of harmonic estimator in PSOPC-LS [42,43], MABC [44], GA-LS [42,43], F-BFO [42,43], BFO [42, 43] algorithms.

Table 10 demonstrated the numerical values of phase and amplitude of original signal with noisy signals. Noisy signals are taken into account to analyze the performance, of the harmonic estimator. In this consequence 10dB, 20dB, 30dB and 40dB noise signals are used with original signals. From the numerical values of phase and amplitude, it is concluded that noisy signals disturb the performance, of the harmonic estimator.

8. Conclusion

Harmonic estimation is a prominent problem in the contemporary power system. For estimation of the harmonic content in a fundamental wave, magnitude and angle information of different harmonics are inevitable. Aiming this research issue, the work presented in the paper cites an application of a newly developed version of ASO. Modifications in ASO has been proposed in terms of a Laplacian Operator to balance exploration and exploitation virtues of ASO. Position updation has been done with the inculcation of this operator. Following are the major conclusions derived from the work.

- After a careful inspection of literature, it has been concluded that the application of optimization algorithms has been substantially increased for harmonic estimation in recent years. Hence, a newly proposed ASO and its application in this field are worthy of investigation. Considering the seriousness of this application, benchmarking of the proposed version has been done on the standard functions.
- Following NFL theorem and to make algorithm compatible for this application, a modification has been suggested in terms of Laplacian operator enabled exploration exploitation balanced mechanism. This operator has been embedded in position update equation. Implications of this modification has been evaluated with the help of various tests.
- The efficacy of the proposed modification has been tested on standard benchmark functions. Outcomes of various tests have shown effectiveness of modification and it has been concluded that performance, of the variant has been enhanced with the incorporation of the proposed modification.
- Finally L-ASO has been applied on standard wave forms and estimation of various components of the waves have been carried out. We observed that phase and amplitude components estimation have been done by L-ASO correctly. Further, as compared with the previously proposed methods, accuracy of the method is more significant.

Application of the proposed variant L-ASO on more challenging problems like regulator design, network expansion planning and forecasting of wind speed lays in future scope of this work. Also, authors will evaluate the performance of the algorithms on more challenging benchmarks.

Research involving human participants and/or animals

The research presented in this paper does not involve any Human Participants and/or Animals.

Informed consent

Consent is not required. All the relevant references have been cited.

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CRediT authorship contribution statement

Akash Saxena: Conceptualization, Project administration, Software, Writing – original draft, Writing – review & editing. **Shalini Shek-hawat:** Formal analysis, Methodology, Writing – original draft, Writing – review & editing. **Rajesh Kumar:** Project administration, Supervision, Visualization. **Aishwarya Mehta:** Writing – original draft. **Jitesh Jangid:** Writing – original draft.

Declaration of competing interest

The authors have no relevant financial or non-financial interests to disclose. The authors declare that they have no conflict of interest.

Data availability

No data was used for the research described in the article.

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