



Simulation based evaluation of neutrosophic exponential imputations of population mean using neutrosophic ranked set sampling

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ABSTRACT

The prevalence of missing values and uncertainty in real-world dataset often hinders accurate analysis and reliable conclusions. Existing approaches often struggle to effectively address both aspects simultaneously. To handle both these aspects simultaneously, this paper proposes an approach that combines neutrosophic theory with exponential imputation and provides neutrosophic exponential imputations for estimating population mean using neutrosophic ranked set sampling (NRSS) framework. Neutrosophic imputation structures utilize the strengths of neutrosophic logic to manage uncertainty inherent in the data and incorporates NRSS to improve the efficiency of the imputation process. The mathematical properties such as bias and mean square error (MSE) of the proposed neutrosophic exponential estimators are obtained. The proposed neutrosophic exponential imputations are compared with some adapted prominent neutrosophic imputations. A thorough simulation study demonstrates that neutrosophic exponential imputations outperform the adapted prominent imputation structures in terms of both accuracy and robustness in handling datasets characterized by both missingness and uncertainty. A real data application of the proposed imputation structures is also presented in support of the mathematical results as well as simulation findings.

1. Introduction

Survey research often suffers from missing data due to factors such as equipment failure, data entry errors, or respondent non-response. This missing information can compromise the reliability and validity of research outcomes, leading to biased results and reduced efficiency. However, imputation techniques provide a valuable solution to address these issues and improve the quality of the analysis. Effective management of missing data is therefore essential for trustworthy survey research. Building upon the work of Rubin [1], which established the concepts of parameter distribution, observed at random, and missing at random (MAR). Heitjan and Basu [2] further clarified the distinction between missing completely at random (MCAR) and MAR. A thorough understanding of these missing data mechanisms is critical for selecting the most appropriate imputation structure. Adding auxiliary data significantly improves the accuracy and efficiency of data imputation, particularly when the missing data mechanism is MCAR. The literature addressing the estimation of population means in the presence of missing data is both vast and methodologically diverse. Many authors have explored a wide array of sampling frameworks, ranging from sophisticated and efficient designs such as robust extreme ranked set sampling as discussed by Al-Omari [3] to the more

traditional simple random sampling (SRS) methods. In many cases, these sampling techniques are supplemented with various imputation strategies aimed at handling missingness effectively. Notably, a considerable portion of this work emphasizes the use of auxiliary information, particularly correlation structures between variables, to enhance the accuracy of estimators. For instance, Al-Omari et al. [4] demonstrated how incorporating such information within the context of SRS can lead to substantial improvements in mean estimation. Bhushan and Pandey [5] focused on optimization with the study developing optimal imputation structures. Prasad [6] proposed innovative ratio exponential type estimators and imputation structures. Sohail et al. [7] used raw moments for imputation methods under SRS. Bhushan et al. [8] have investigated logarithmic imputations for population mean under SRS. Pandey et al. [9] suggested efficient and alternative population mean estimation approaches for imputing missing data. Yadav and Prasad [10] developed the generalized class of factor type exponential imputation techniques for population mean utilizing simulation approach. Rehman et al. [11] developed the survey methods for novel mean imputation and exemplified them with the abalone data. Recently, Triveni et al. [12] presented application of log-type estimators for addressing non-response using real data.

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Ranked set sampling (RSS) has emerged as a highly efficient and cost-effective alternative to SRS, especially in scenarios where actual measurements are expensive or time-consuming, but ranking units based on auxiliary information is relatively simple and inexpensive. The concept was first introduced by McIntyre [13], who demonstrated the superiority of RSS in estimating the population mean. Based on this foundation, Dell and Clutter [14] established the unbiasedness of the mean estimator derived from RSS, lending theoretical robustness to its application. Subsequent empirical and theoretical investigations have consistently highlighted the advantages of RSS over SRS, particularly in terms of variance or MSE reduction, which becomes even more pronounced in populations with high variability [15]. The utility of RSS has transcended traditional domains, finding meaningful applications in public health and medical research [16], where it has shown greater efficiency compared to not only SRS but also more complex designs such as stratified and cluster sampling [17]. Moreover, the scope of RSS has broadened beyond mean estimation. Chen et al. [18] explored its application in proportion estimation, reaffirming the versatility of the method. Recent developments have addressed practical challenges such as non-response [19], enhancing the applicability of the method in real-world surveys. Additionally, advanced methodological extensions have incorporated multi-auxiliary information through logarithmic imputation structures, further improving efficiency in estimation [20].

In recent years, considerable attention has been given to the estimation of population parameters under the neutrosophic data framework, particularly in the context of mean estimation. A significant contribution in this area was made by Tahir et al. [21], who developed a neutrosophic ratio-based estimator using SRS, effectively integrating uncertainty and indeterminacy into classical estimation procedures. Based upon this foundation, Yadav and Prasad [22] proposed the neutrosophic factor-type exponential estimators that aim to enhance efficiency under uncertainty. In a related development, Alqudah et al. [23] proposed a robust ratio-type estimator for mean estimation within the neutrosophic framework, emphasizing robustness against outliers and data irregularities. Complementing these efforts, Masood et al. [24] directed their focus toward the robust estimation of the neutrosophic finite median associated with an auxiliary variable, highlighting the versatility of neutrosophic approaches in handling imprecise and uncertain data. Singh and Kumari [25] explored the NRSS for estimating the population mean with an application to demographic data. Priya and Kumar [26] proposed robust neutrosophic exponential estimators for estimating the population mean. Kumar et al. [27] formulated an optimal neutrosophic framework for population mean estimation under SRS. Further, Kumar et al. [28] developed efficient classes of estimators for indeterminate population mean using NRSS. Fatima et al. [29] presented a comprehensive review of neutrosophic statistics for data analysis in the applied sciences. Shahzad et al. [30] examined neutrosophic extensions of Horvitz-Thompson type estimators. Singh and Tiwari [31] proposed an improved estimator for population mean utilizing known medians of two auxiliary variables within the neutrosophic framework. In addition, Singh et al. [32] developed generalized neutrosophic exponential estimators for efficient estimation of the population mean. Similarly, Singh and Gupta [33] proposed an improved estimation strategy by combining two auxiliary variables to enhance the estimation of finite population mean under the neutrosophic framework. These studies demonstrate the growing interest and methodological advancements in neutrosophic statistics, particularly in improving the reliability and robustness of the population parameter estimation under complex data environments.

While many imputation structures exist for handling MCAR data within SRS and RSS, they are insufficient for dealing with the uncertainty and indeterminacy inherent in imprecise or neutrosophic data. Current imputation structures, designed for well-defined missing values, fail to address the complexities introduced by uncertainty, hindering accurate population parameter estimation. No established

imputation structure or estimator currently exists to handle missingness in uncertain data within the NRSS framework. This gap necessitates the development of efficient neutrosophic imputation structures capable of incorporating uncertainty and indeterminacy to improve the accuracy of population mean estimation.

1.1. Construction of methodology and notation

The RSS provides significant advantages over SRS when precise measurements are difficult or costly, using ranking information for improved accuracy with fewer measurements. However, this efficiency diminishes when faced with uncertain or neutrosophic data, where inherent ambiguity and uncertainty exacerbate measurement challenges. Neutrosophic data, particularly quantitative types represented by intervals $[p, q]$ denoting lower and upper bounds, highlight the limitations of standard RSS in such scenarios. The unknown interval $[p, q]$ based on neutrosophic numbers can be represented in different ways. The neutrosophic interval values are computed as $W_{rssN} = W_{rssL} + W_{rssU} I_{rssN}$, where $I_{rssN} \in [I_{rssL}, I_{rssU}]$, N stands for neutrosophic number. This demonstrates that the notations utilized for neutrosophic data are in an interval form $W_{rssN} \in [p, q]$.

The NRSS approach utilizes bivariate random samples of size $m_N \in [m_L, m_U]$ units from a population of size N . Within each sample, the auxiliary variable $x_N \in [x_L, x_U]$ is ranked against the study variable $y_N \in [y_L, y_U]$. Ranking of neutrosophic numbers can be readily achieved using the structure proposed by Smarandache [34].

In the NRSS, select the smallest unit from the first set and use it as the starting measuring unit, discarding the other units in the set. Similarly, from the second set, select the second smallest unit and consider it the second measuring unit, discarding the other units in the set. To continue, choose the m_N^{th} smallest unit from the m_N^{th} collection and utilize it as the m_N^{th} measurement unit, discarding the remaining units. This procedure concludes a cycle. Repeating this cycle k times results in a neutrosophic ranked set sample of size $n_N = m_N k \in [n_L, n_U]$. The selection of NRSS data needs a total of $m_N^2 k$ units, although only $n_N = m_N k \in [n_L, n_U]$ units are required for computation. Let $X_{j(i)N} \in [X_{j(i)L}, X_{j(i)U}]$ and $Y_{j(i)N} \in [Y_{j(i)L}, Y_{j(i)U}]$, $j = 1, 2, \dots, k$; $i = 1, 2, \dots, m_N$ denote a pair of neutrosophic bivariate quantified sets of the i th units in the j th cycle. The symbols ‘ $[]$ ’ and ‘ $()$ ’ in the subscripts of variables y_N and x_N represent the imperfect and perfect rankings of units, respectively.

We consider a neutrosophic random sample of size n_N , selected from a finite neutrosophic population of size N_N units. Let $x_N(i) \in [x_L, x_U]$ and $y_N(i) \in [y_L, y_U]$ represent the data on i th sample observation of the neutrosophic auxiliary variable and neutrosophic study variable. The number of neutrosophic responding units from the retrieved n_N units is denoted by r_N . The set of neutrosophic responding units is R_N , while the set of non-responding units is $\bar{R}_N$. Each unit $i \in R_N$ yields the value $y_{N(i)}$. Nevertheless, the values for the units $i \in \bar{R}_N$ are absent and must be imputed in order to complete the sample dataset. Assuming that the imputation is carried out using additional neutrosophic auxiliary information $x_{N(i)}$, the data $x_{sN} = x_{iN}$ are available since the value of x_N for i th unit is known and positive for all $i \in s_N$. Let $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$ and $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$ be the population means corresponding to the sample means $\bar{y}_{nN, rss} \in [\bar{y}_{nL, rss}, \bar{y}_{nU, rss}]$ and $\bar{x}_{nN, rss} \in [\bar{x}_{nL, rss}, \bar{x}_{nU, rss}]$ of the neutrosophic study variable y_N and the neutrosophic auxiliary variable x_N , respectively. Let $\bar{y}_{rN, rss} \in [\bar{y}_{rL, rss}, \bar{y}_{rU, rss}]$ and $\bar{x}_{rN, rss} \in [\bar{x}_{rL, rss}, \bar{x}_{rU, rss}]$ denote the rank set sample mean for the responding units of variables y_N and x_N , respectively. Let $C_{y_N} \in [C_{yL}, C_{yU}]$ and $C_{x_N} \in [C_{xL}, C_{xU}]$ be the neutrosophic coefficients of variation of the neutrosophic variables y_N and x_N , respectively. The neutrosophic correlation coefficient between neutrosophic variables y_N and x_N is ρ_{xy_N} .

To establish the $Bias_N \in [Bias_L, Bias_U]$ and $MSE_N \in [MSE_L, MSE_U]$ of the neutrosophic estimators, we take the neutrosophic error

terms $e_{0N} \in [e_{0L}, e_{0U}]$, $e_{1N} \in [e_{1L}, e_{1U}]$, and $e_{2N} \in [e_{2L}, e_{2U}]$, together with the following expectations:

$$\left. \begin{aligned} e_{0N} &= \frac{(\bar{y}_{rN, rSS} - \bar{Y}_N)}{\bar{Y}_N}, \\ e_{1N} &= \frac{(\bar{x}_{rN, rSS} - \bar{X}_N)}{\bar{X}_N}, \\ e_{2N} &= \frac{(\bar{x}_{nN, rSS} - \bar{X}_N)}{\bar{X}_N}, \\ E(e_{0N}) &= E(e_{1N}) = E(e_{2N}) = 0, \\ E(e_{0N}^2) &= \frac{C_{yN}^2}{m_N k P} - \frac{1}{m_N^2 k P} \sum_{i=1}^{m_N} \frac{\tau_{yN[i]}^2}{\bar{Y}_N^2} = V_0^*, \\ E(e_{1N}^2) &= \frac{C_{xN}^2}{m_N k P} - \frac{1}{m_N^2 k P} \sum_{i=1}^{m_N} \frac{\tau_{xN(i)}^2}{\bar{X}_N^2} = V_1^*, \\ E(e_{2N}^2) &= \frac{C_{xN}^2}{m_N k} - \frac{1}{m_N^2 k} \sum_{i=1}^{m_N} \frac{\tau_{xN(i)}^2}{\bar{X}_N^2} = V_1, \\ E(e_{0N} e_{1N}) &= \frac{\rho_{xyN} C_{xN} C_{yN}}{m_N k P} - \frac{1}{m_N^2 k P} \sum_{i=1}^{m_N} \frac{\tau_{xyN[i]}}{\bar{X}_N \bar{Y}_N} = V_{01}^*, \\ E(e_{0N} e_{2N}) &= \frac{\rho_{xyN} C_{xN} C_{yN}}{m_N k} - \frac{1}{m_N^2 k} \sum_{i=1}^{m_N} \frac{\tau_{xyN[i]}}{\bar{X}_N \bar{Y}_N} = V_{01}, \\ E(e_{1N} e_{2N}) &= \frac{C_{xN}^2}{m_N k} - \frac{1}{m_N^2 k} \sum_{i=1}^{m_N} \frac{\tau_{xN(i)}^2}{\bar{X}_N^2} = V_1, \end{aligned} \right\} \quad (1.1)$$

where $\tau_{yN[i]} = (\mu_{yN[i]} - \bar{Y}_N)$, $\tau_{xN(i)} = (\mu_{xN(i)} - \bar{X}_N)$, $\tau_{xyN[i]} = (\mu_{xN(i)} - \bar{X}_N)(\mu_{yN[i]} - \bar{Y}_N)$, $C_{yN} = S_{yN}/\bar{Y}_N$, $C_{xN} = S_{xN}/\bar{X}_N$, $\mu_{yN[i]} = E(Y_{N[i]})$, and $\mu_{xN(i)} = E(X_{N(i)})$.

The article is divided into few sections. Section 2 reviews the adapted neutrosophic imputation structures along with the corresponding resulting estimators. Section 3 introduces neutrosophic exponential imputation structures, corresponding resulting estimators, and their properties. Section 4 theoretically compares the performance of the proposed estimators with the adapted ones. Section 5 and Section 6 evaluate the mathematical results through simulation studies using both symmetric and asymmetric data, and a real-world dataset analysis, respectively. Finally, Section 7 concludes the study.

2. Prominent neutrosophic imputations

There are no neutrosophic imputation structures in the literature to address missingness in the presence of uncertain data. To fill the gap of the literature and evaluate the efficiency of the proposed neutrosophic imputation structures and the corresponding resulting estimators, we adapt some prominent neutrosophic imputation structures and the corresponding neutrosophic estimators, with their specific characteristics.

2.1. Neutrosophic mean imputation

Traditional mean imputation structure usually fails with ambiguous datasets because it is based on certainty assumptions. The neutrosophic mean imputation structure provides a more complete and adaptable technique of estimating missing values by accounting for the degrees of truth, indeterminacy, and falsehood. Following Lee et al. [35], when a sample unit 'i' is missing and needs imputation, the neutrosophic mean imputation structure is as follows:

$$y_{im} = \begin{cases} y_{iN} & \text{for } i \in R_N \\ \bar{y}_{rN, rSS} & \text{for } i \in \bar{R}_N \end{cases}$$

The neutrosophic mean estimator is given as

$$T_m = \bar{y}_{rN, rSS}.$$

The variance of the neutrosophic mean estimator $T_{mN} = \bar{y}_{rN, rSS}$ is given by

$$V(T_m) = \bar{Y}_N^2 V_0^*.$$

When more neutrosophic auxiliary data is available, the neutrosophic imputation structure is categorized into following schemes:

Scheme I: When $\bar{X}_N$ is available, consider $\bar{x}_{nN, rSS}$.

Scheme II: When $\bar{X}_N$ is available, consider $\bar{x}_{rN, rSS}$.

Scheme III: When $\bar{X}_N$ is not available, consider $\bar{x}_{nN, rSS}$ and $\bar{x}_{rN, rSS}$.

2.2. Neutrosophic ratio imputations

When neutrosophic study and neutrosophic auxiliary variables are positively correlated, then the traditional neutrosophic ratio imputation structures produce efficient results. Following Singh and Deo [36], we describe the neutrosophic imputation methods under schemes I–III as follows:

Scheme I

$$y_{ir_1} = \begin{cases} y_{iN} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{rN, rSS} \left(\frac{\bar{X}_N}{\bar{x}_{nN, rSS}} \right) - r_N \bar{y}_{rN, rSS} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme II

$$y_{ir_2} = \begin{cases} y_{iN} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{rN, rSS} \left(\frac{\bar{X}_N}{\bar{x}_{rN, rSS}} \right) - r_N \bar{y}_{rN, rSS} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme III

$$y_{ir_3} = \begin{cases} y_{iN} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{rN, rSS} \left(\frac{\bar{x}_{nN, rSS}}{\bar{x}_{rN, rSS}} \right) - r_N \bar{y}_{rN, rSS} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

The neutrosophic ratio estimators under schemes I–III are given by

$$T_{r_1} = \bar{y}_{rN, rSS} \left(\frac{\bar{X}_N}{\bar{x}_{nN, rSS}} \right),$$

$$T_{r_2} = \bar{y}_{rN, rSS} \left(\frac{\bar{X}_N}{\bar{x}_{rN, rSS}} \right),$$

$$\text{and } T_{r_3} = \bar{y}_{rN, rSS} \left(\frac{\bar{x}_{nN, rSS}}{\bar{x}_{rN, rSS}} \right).$$

The approximate MSE of the neutrosophic ratio estimators is given by

$$MSE(T_{r_1}) \approx \bar{Y}_N^2 (V_0^* + V_1 - 2V_{01}),$$

$$MSE(T_{r_2}) \approx \bar{Y}_N^2 (V_0^* + V_1^* - 2V_{01}^*),$$

$$\text{and } MSE(T_{r_3}) \approx \bar{Y}_N^2 \{V_0^* + (V_1^* - V_1) - 2(V_{01}^* - V_{01})\}.$$

2.3. Neutrosophic regression imputations

When neutrosophic study and neutrosophic auxiliary variables are positively correlated, and regression line passes from the origin, then following Diana and Perri [37], the traditional neutrosophic regression imputation structures produce efficient results which are described under schemes I–III as follows:

Scheme I

$$y_{ir_1} = \begin{cases} y_{iN} & \text{for } i \in R_N \\ \bar{y}_{rN, rSS} + \frac{n_N \beta_1}{n_N - r_N} (\bar{X}_N - \bar{x}_{nN, rSS}) & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme II

$$y_{ir_2} = \begin{cases} y_{iN} & \text{for } i \in R_N \\ \bar{y}_{rN, rSS} + \frac{n_N \beta_2}{n_N - r_N} (\bar{X}_N - \bar{x}_{rN, rSS}) & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme III

$$y_{i_{lr3}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \bar{y}_{r_N, r_{ss}} + \frac{n_N \beta_3}{n_N - r_N} (\bar{x}_{n_N, r_{ss}} - \bar{x}_{r_N, r_{ss}}) & \text{for } i \in \bar{R}_N \end{cases}$$

The neutrosophic regression estimators are given as

$$T_{lr1} = \bar{y}_{r_N, r_{ss}} + \beta_1 (\bar{X}_N - \bar{x}_{n_N, r_{ss}}),$$

$$T_{lr2} = \bar{y}_{r_N, r_{ss}} + \beta_2 (\bar{X}_N - \bar{x}_{r_N, r_{ss}}),$$

$$\text{and } T_{lr3} = \bar{y}_{r_N, r_{ss}} + \beta_3 (\bar{x}_{n_N, r_{ss}} - \bar{x}_{r_N, r_{ss}}).$$

At the optimum values of scalars $\beta_{1(opt)} = RV_{01}/V_1$, $\beta_{2(opt)} = RV_{01}^*/V_1^*$, and $\beta_{3(opt)} = R(V_{01}^* - V_{01})/(V_1^* - V_1)$, where $R = \bar{Y}_N/\bar{X}_N$, the minimum MSE of the neutrosophic regression estimators is given by

$$\min.MSE(T_{lr1}) = \bar{Y}_N^2 \left(V_0^* - \frac{V_{01}^2}{V_1} \right),$$

$$\min.MSE(T_{lr2}) = \bar{Y}_N^2 \left(V_0^* - \frac{V_{01}^2}{V_1^*} \right),$$

$$\text{and } \min.MSE(T_{lr3}) = \bar{Y}_N^2 \left\{ V_0^* - \frac{(V_{01}^* - V_{01})^2}{(V_1^* - V_1)} \right\}.$$

2.4. Neutrosophic exponential ratio imputations

Neutrosophic exponential imputations use exponential functions within the neutrosophic framework to estimate missing data, incorporating truth, indeterminacy, and falsity components. These methods enhance imputation accuracy in datasets with uncertainty and incomplete information. Following Bahl and Tuteja [38], the neutrosophic exponential ratio imputations are provided under schemes I–III as follows:

Scheme I

$$y_{i_{t1}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{r_N, r_{ss}} \exp \left(\frac{\bar{X}_N - \bar{x}_{n_N, r_{ss}}}{\bar{X}_N + \bar{x}_{n_N, r_{ss}}} \right) - r_N \bar{y}_{r_N, r_{ss}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme II

$$y_{i_{t2}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{r_N, r_{ss}} \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{ss}}}{\bar{X}_N + \bar{x}_{r_N, r_{ss}}} \right) - r_N \bar{y}_{r_N, r_{ss}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme III

$$y_{i_{t3}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{r_N, r_{ss}} \exp \left(\frac{\bar{x}_{n_N, r_{ss}} - \bar{x}_{r_N, r_{ss}}}{\bar{x}_{n_N, r_{ss}} + \bar{x}_{r_N, r_{ss}}} \right) - r_N \bar{y}_{r_N, r_{ss}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

The neutrosophic exponential ratio estimators under schemes I–III are given by

$$T_{t1} = \bar{y}_{r_N, r_{ss}} \exp \left(\frac{\bar{X}_N - \bar{x}_{n_N, r_{ss}}}{\bar{X}_N + \bar{x}_{n_N, r_{ss}}} \right),$$

$$T_{t2} = \bar{y}_{r_N, r_{ss}} \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{ss}}}{\bar{X}_N + \bar{x}_{r_N, r_{ss}}} \right),$$

$$\text{and } T_{t3} = \bar{y}_{r_N, r_{ss}} \exp \left(\frac{\bar{x}_{n_N, r_{ss}} - \bar{x}_{r_N, r_{ss}}}{\bar{x}_{n_N, r_{ss}} + \bar{x}_{r_N, r_{ss}}} \right).$$

The MSE of the neutrosophic exponential ratio estimators is given by

$$MSE(T_{t1}) \approx \bar{Y}_N^2 \left(V_0^* + \frac{V_1}{4} - V_{01} \right),$$

$$MSE(T_{t2}) \approx \bar{Y}_N^2 \left(V_0^* + \frac{V_1^*}{4} - V_{01}^* \right),$$

$$\text{and } MSE(T_{t3}) \approx \bar{Y}_N^2 \left\{ V_0^* + \frac{(V_1^* - V_1)}{4} - (V_{01}^* - V_{01}) \right\}.$$

2.5. Neutrosophic power ratio-cum exponential ratio imputations

Following Kadilar [39], we adapt the neutrosophic power ratio-cum exponential ratio imputation structures under schemes I–III as follows:

Scheme I

$$y_{i_{p1}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{r_N, r_{ss}} \left(\frac{\bar{X}_N}{\bar{x}_{n_N, r_{ss}}} \right)^{\kappa_1} \exp \left(\frac{\bar{X}_N - \bar{x}_{n_N, r_{ss}}}{\bar{X}_N + \bar{x}_{n_N, r_{ss}}} \right) - r_N \bar{y}_{r_N, r_{ss}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme II

$$y_{i_{p2}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{r_N, r_{ss}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{ss}}} \right)^{\kappa_2} \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{ss}}}{\bar{X}_N + \bar{x}_{r_N, r_{ss}}} \right) - r_N \bar{y}_{r_N, r_{ss}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme III

$$y_{i_{p3}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \bar{y}_{r_N, r_{ss}} \left(\frac{\bar{x}_{n_N, r_{ss}}}{\bar{x}_{r_N, r_{ss}}} \right)^{\kappa_3} \exp \left(\frac{\bar{x}_{n_N, r_{ss}} - \bar{x}_{r_N, r_{ss}}}{\bar{x}_{n_N, r_{ss}} + \bar{x}_{r_N, r_{ss}}} \right) - r_N \bar{y}_{r_N, r_{ss}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

The neutrosophic power ratio-cum exponential ratio estimators under schemes I–III are provided as

$$T_{p1} = \bar{y}_{r_N, r_{ss}} \left(\frac{\bar{X}_N}{\bar{x}_{n_N, r_{ss}}} \right)^{\kappa_1} \exp \left(\frac{\bar{X}_N - \bar{x}_{n_N, r_{ss}}}{\bar{X}_N + \bar{x}_{n_N, r_{ss}}} \right),$$

$$T_{p2} = \bar{y}_{r_N, r_{ss}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{ss}}} \right)^{\kappa_2} \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{ss}}}{\bar{X}_N + \bar{x}_{r_N, r_{ss}}} \right),$$

$$\text{and } T_{p3} = \bar{y}_{r_N, r_{ss}} \left(\frac{\bar{x}_{n_N, r_{ss}}}{\bar{x}_{r_N, r_{ss}}} \right)^{\kappa_3} \exp \left(\frac{\bar{x}_{n_N, r_{ss}} - \bar{x}_{r_N, r_{ss}}}{\bar{x}_{n_N, r_{ss}} + \bar{x}_{r_N, r_{ss}}} \right).$$

where κ_i , $i = 1, 2, 3$ are optimizing scalars. At the optimum values $\kappa_{1(opt)} = (V_{01}/V_1) - (1/2)$, $\kappa_{2(opt)} = (V_{01}^*/V_1^*) - (1/2)$, and $\kappa_{3(opt)} = \{(V_{01}^* - V_{01})/(V_1^* - V_1)\} - (1/2)$, the minimum MSE of the neutrosophic power ratio-cum exponential ratio estimators are provided by

$$\min.MSE(T_{p1}) = \bar{Y}_N^2 \left(V_0^* - \frac{V_{01}^2}{V_1} \right),$$

$$\min.MSE(T_{p2}) = \bar{Y}_N^2 \left(V_0^* - \frac{V_{01}^{*2}}{V_1^*} \right),$$

$$\text{and } \min.MSE(T_{p3}) = \bar{Y}_N^2 \left\{ V_0^* - \frac{(V_{01}^* - V_{01})^2}{(V_1^* - V_1)} \right\}.$$

3. Proposed neutrosophic exponential imputations

The importance of the proposed neutrosophic exponential imputations lies in its ability to effectively deal with the uncertainty, indeterminacy, and imprecision in missing data. Traditional imputations often fall short when data are incomplete and embedded with vague or uncertain information. By using the neutrosophic framework, this approach captures the degrees of truth, indeterminacy, and falsity associated with each observation. Incorporating NRSS further enhances the efficiency and accuracy of estimation by intelligently opting representative samples, thus reducing sampling variability and bias. When combined with exponential imputations, the NRSS-based neutrosophic approach not only preserves the underlying distributional characteristics of the data but also provides a robust mechanism for imputing missing values in a wide range of real-world applications, including environmental studies, medical diagnostics, and social sciences where uncertainty is inherent. With these advantages, we propose some neutrosophic exponential

imputations for the population mean using NRSS. These imputations are provided under schemes I–III as follows:

Scheme I

$$y_{i_{k_1}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \left\{ \Delta_1 \bar{y}_{r_N, r_{SS}} + \zeta_1 \bar{y}_{r_N, r_{SS}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{SS}}} \right)^g \right\} \right. \\ \quad \left. \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{SS}}}{\bar{X}_N + \bar{x}_{r_N, r_{SS}}} \right) - r_N \bar{y}_{r_N, r_{SS}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme II

$$y_{i_{k_2}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \left\{ \Delta_2 \bar{y}_{r_N, r_{SS}} + \zeta_2 \bar{y}_{r_N, r_{SS}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{SS}}} \right)^g \right\} \right. \\ \quad \left. \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{SS}}}{\bar{X}_N + \bar{x}_{r_N, r_{SS}}} \right) - r_N \bar{y}_{r_N, r_{SS}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

Scheme III

$$y_{i_{k_3}} = \begin{cases} y_{i_N} & \text{for } i \in R_N \\ \frac{1}{n_N - r_N} \left[n_N \left\{ \Delta_3 \bar{y}_{r_N, r_{SS}} + \zeta_3 \bar{y}_{r_N, r_{SS}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{SS}}} \right)^g \right\} \right. \\ \quad \left. \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{SS}}}{\bar{X}_N + \bar{x}_{r_N, r_{SS}}} \right) - r_N \bar{y}_{r_N, r_{SS}} \right] & \text{for } i \in \bar{R}_N \end{cases}$$

The proposed neutrosophic exponential estimators under schemes I–III are provided as

$$T_{k_1} = \left\{ \Delta_1 \bar{y}_{r_N, r_{SS}} + \zeta_1 \bar{y}_{r_N, r_{SS}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{SS}}} \right)^g \right\} \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{SS}}}{\bar{X}_N + \bar{x}_{r_N, r_{SS}}} \right),$$

$$T_{k_2} = \left\{ \Delta_2 \bar{y}_{r_N, r_{SS}} + \zeta_2 \bar{y}_{r_N, r_{SS}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{SS}}} \right)^g \right\} \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{SS}}}{\bar{X}_N + \bar{x}_{r_N, r_{SS}}} \right),$$

$$\text{and } T_{k_3} = \left\{ \Delta_3 \bar{y}_{r_N, r_{SS}} + \zeta_3 \bar{y}_{r_N, r_{SS}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{SS}}} \right)^g \right\} \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{SS}}}{\bar{X}_N + \bar{x}_{r_N, r_{SS}}} \right).$$

where Δ_i , $i = 1, 2, 3$ and ζ_i are optimizing scalars that are determined later. Also, g takes values 1 and -1 to generate different estimators.

Theorem 3.1. The first order approximated minimum MSE of the neutrosophic exponential estimators T_{k_i} , $i = 1, 2, 3$ are given by

$$\min. MSE(T_{k_i}) \approx \bar{Y}_N^2 \left(1 - \frac{I_i M_i^2 + J_i L_i^2 - 2K_i L_i M_i}{I_i J_i - K_i^2} \right). \quad (3.1)$$

Proof. Consider the proposed neutrosophic exponential estimator T_{k_1} under Scheme I as

$$T_{k_1} = \left\{ \Delta_1 \bar{y}_{r_N, r_{SS}} + \zeta_1 \bar{y}_{r_N, r_{SS}} \left(\frac{\bar{X}_N}{\bar{x}_{r_N, r_{SS}}} \right)^g \right\} \exp \left(\frac{\bar{X}_N - \bar{x}_{r_N, r_{SS}}}{\bar{X}_N + \bar{x}_{r_N, r_{SS}}} \right).$$

Utilizing the notations given in (1.1) and ignoring the error terms having power more than two, we can express the estimator T_{k_1} as

$$T_{k_1} \approx \bar{Y}_N (1 + e_{0N}) \left\{ \Delta_1 + \zeta_1 (1 + e_{2N})^{-g} \right\} \left(1 - \frac{e_{2N}}{2} + \frac{3}{8} e_{2N}^2 \right),$$

$$T_{k_1} \approx \bar{Y}_N (1 + e_{0N}) \left(\Delta_1 + \zeta_1 - \zeta_1 g e_{2N} + \frac{\zeta_1 g(g+1)}{2} e_{2N}^2 \right) \left(1 - \frac{e_{2N}}{2} + \frac{3}{8} e_{2N}^2 \right),$$

$$T_{k_1} - \bar{Y}_N \approx \bar{Y}_N \left[\Delta_1 \left\{ 1 + e_{0N} - \frac{e_{2N}}{2} - \frac{e_{0N} e_{2N}}{2} + \frac{3}{8} e_{2N}^2 \right\} \right. \\ \left. + \zeta_1 \left\{ 1 + e_{0N} - g e_{2N} - \frac{e_{2N}}{2} + \left(\frac{g(g+1)}{2} + \frac{g}{2} + \frac{3}{8} \right) e_{2N}^2 \right\} - 1 \right] \quad (3.2)$$

Taking expectation both sides to (3.2), we get the bias of the estimator T_{k_1} as

$$\text{Bias}(T_{k_1}) \approx \bar{Y}_N (\Delta_1 L_1 + \zeta_1 M_1 - 1),$$

where

$$L_1 = 1 + \frac{3}{8} V_1 - \frac{1}{2} V_{01} \text{ and } M_1 = 1 + \left\{ \frac{g(g+1)}{2} + \frac{g}{2} + \frac{3}{8} \right\} V_1 - \left(g + \frac{1}{2} \right) V_{01}.$$

Squaring and taking expectation both sides to (3.2), we can get the MSE of the estimator T_{k_1} up to first order approximation as

$$MSE(T_{k_1}) \approx \bar{Y}_N^2 \left[\begin{aligned} & 1 + \Delta_1^2 \left\{ 1 + V_0^* + V_1 - 2V_{01} \right\} \\ & + \zeta_1^2 \left\{ 1 + V_0^* + (2g^2 + 3g + 1)V_1 - (4g + 2)V_{01} \right\} \\ & + 2\Delta_1 \zeta_1 \left\{ 1 + V_0^* + \left(\frac{g(g+1)}{2} + g + 1 \right) V_1 - 2(g + 1)V_{01} \right\} \\ & - 2\Delta_1 \left\{ 1 + \frac{3}{8} V_1 - \frac{1}{2} V_{01} \right\} \\ & - 2\zeta_1 \left\{ 1 + \left(\frac{g(g+1)}{2} + \frac{g}{2} + \frac{3}{8} \right) V_1 - \left(g + \frac{1}{2} \right) V_{01} \right\} \end{aligned} \right].$$

The above expression can further be written as

$$MSE(T_{k_1}) \approx \bar{Y}_N^2 (1 + \Delta_1^2 I_1 + \zeta_1^2 J_1 + 2\Delta_1 \zeta_1 K_1 - 2\Delta_1 L_1 - 2\zeta_1 M_1), \quad (3.3)$$

where

$$I_1 = 1 + V_0^* + V_1 - 2V_{01},$$

$$J_1 = 1 + V_0^* + (2g^2 + 3g + 1)V_1 - (4g + 2)V_{01},$$

$$K_1 = 1 + V_0^* + \left\{ \frac{g(g+1)}{2} + g + 1 \right\} V_1 - 2(g + 1)V_{01},$$

$$L_1 = 1 + \frac{3}{8} V_1 - \frac{1}{2} V_{01},$$

$$\text{and } M_1 = 1 + \left\{ \frac{g(g+1)}{2} + \frac{g}{2} + \frac{3}{8} \right\} V_1 - \left(g + \frac{1}{2} \right) V_{01}.$$

Minimizing (3.3) with respect to Δ_1 and ζ_1 , we have

$$\Delta_{1(opt)} = \frac{J_1 L_1 - K_1 M_1}{I_1 J_1 - K_1^2} \text{ and } \zeta_{1(opt)} = \frac{I_1 M_1 - K_1 L_1}{I_1 J_1 - K_1^2}.$$

The minimum MSE can be obtained by putting optimum values of Δ_1 and ζ_1 in (3.3) as

$$\min. MSE(T_{k_1}) \approx \bar{Y}_N^2 \left(1 - \frac{I_1 M_1^2 + J_1 L_1^2 - 2K_1 L_1 M_1}{I_1 J_1 - K_1^2} \right).$$

Similarly, we can find out the bias, MSE, and minimum MSE of the estimators T_{k_2} and T_{k_3} under schemes II and III as

$$\text{Bias}(T_{k_i}) \approx \bar{Y}_N (\Delta_i L_i + \zeta_i M_i - 1), \quad i = 2, 3$$

$$\text{and } MSE(T_{k_i}) \approx \bar{Y}_N^2 (1 + \Delta_i^2 I_i + \zeta_i^2 J_i + 2\Delta_i \zeta_i K_i - 2\Delta_i L_i - 2\zeta_i M_i). \quad (3.4)$$

The optimum values of the scalars Δ_i , $i = 2, 3$ and ζ_i are given by

$$\Delta_{i(opt)} = \frac{J_i L_i - K_i M_i}{I_i J_i - K_i^2} \text{ and } \zeta_{i(opt)} = \frac{I_i M_i - K_i L_i}{I_i J_i - K_i^2}.$$

In general, the minimum MSE can be obtained by putting optimum values of Δ_i and ζ_i in (3.4) as

$$\min. MSE(T_{k_i}) \approx \bar{Y}_N^2 \left(1 - \frac{I_i M_i^2 + J_i L_i^2 - 2K_i L_i M_i}{I_i J_i - K_i^2} \right).$$

where

$$I_2 = 1 + V_0^* + V_1^* - 2V_{01}^*,$$

$$J_2 = 1 + V_0^* + (2g^2 + 3g + 1)V_1^* - (4g + 2)V_{01}^*,$$

$$K_2 = 1 + V_0^* + \left\{ \frac{g(g+1)}{2} + g + 1 \right\} V_1^* - 2(g + 1)V_{01}^*,$$

$$L_2 = 1 + \frac{3}{8} V_1^* - \frac{1}{2} V_{01}^*,$$

$$M_2 = 1 + \left\{ \frac{g(g+1)}{2} + \frac{g}{2} + \frac{3}{8} \right\} V_1^* - \left(g + \frac{1}{2} \right) V_{01}^*,$$

$$I_3 = 1 + V_0^* + (V_1^* - V_1) - 2(V_{01}^* - V_{01}),$$

$$J_3 = 1 + V_0^* + (2g^2 + 3g + 1)(V_1^* - V_1) - (4g + 2)(V_{01}^* - V_{01}),$$

$$K_3 = 1 + V_0^* + \left\{ \frac{g(g+1)}{2} + g + 1 \right\} (V_1^* - V_1) - 2(g + 1)(V_{01}^* - V_{01}),$$

$$L_3 = 1 + \frac{3}{8} (V_1^* - V_1) - \frac{1}{2} (V_{01}^* - V_{01}),$$

$$M_3 = 1 + \left\{ \frac{g(g+1)}{2} + \frac{g}{2} + \frac{3}{8} \right\} (V_1^* - V_1) - \left(g + \frac{1}{2} \right) (V_{01}^* - V_{01}). \quad \square$$

4. Theoretical comparisons

In survey sampling, theoretical comparisons of estimators are critical to assuring the quality, reliability, and efficiency of statistical conclusions generated from sample data. We can determine which estimator performs best under certain sampling situations by comparing them using MSE. This helps in opting an estimator that minimizes bias while generating more precise and valid results. Thus, we conduct a theoretical comparisons by comparing the MSE of the suggested and adapted neutrosophic estimators under the following lemmas.

Lemma 4.1. The proposed neutrosophic exponential estimator T_{k_i} , $i = 1, 2, 3$ dominate the neutrosophic mean estimator T_m under schemes I–III, if

$$\min.MSE(T_{k_i}) < V(T_m) \Rightarrow \frac{I_i M_i^2 + J_i L_i^2 - 2K_i L_i M_i}{I_i J_i - K_i^2} > 1 - V_0^*.$$

Lemma 4.2. (i). The proposed neutrosophic exponential estimator T_{k_1} dominates the neutrosophic ratio estimator T_{r_1} under Scheme I, if

$$\begin{aligned} \min.MSE(T_{k_1}) &< MSE(T_{r_1}) \\ \Rightarrow \frac{I_1 M_1^2 + J_1 L_1^2 - 2K_1 L_1 M_1}{I_1 J_1 - K_1^2} &> 1 - V_0^* - V_1 + 2V_{01}. \end{aligned}$$

(ii). The proposed neutrosophic exponential estimator T_{k_2} dominates the neutrosophic ratio estimator T_{r_2} under Scheme II, if

$$\begin{aligned} \min.MSE(T_{k_2}) &< MSE(T_{r_2}) \\ \Rightarrow \frac{I_2 M_2^2 + J_2 L_2^2 - 2K_2 L_2 M_2}{I_2 J_2 - K_2^2} &> 1 - V_0^* - V_1^* + 2V_{01}^*. \end{aligned}$$

(iii). The proposed neutrosophic exponential estimator T_{k_3} dominates the neutrosophic ratio estimator T_{r_3} under Scheme III, if

$$\begin{aligned} \min.MSE(T_{k_3}) &< MSE(T_{r_3}) \\ \Rightarrow \frac{I_3 M_3^2 + J_3 L_3^2 - 2K_3 L_3 M_3}{I_3 J_3 - K_3^2} &> 1 - V_0^* - (V_1^* - V_1) + 2(V_{01}^* - V_{01}). \end{aligned}$$

Lemma 4.3. (i). The proposed neutrosophic exponential estimator T_{k_1} dominates the neutrosophic regression estimator T_{lr_1} and neutrosophic power ratio-cum exponential ratio estimator T_{p_1} under Scheme I, if

$$\begin{aligned} \min.MSE(T_{p_1}) &< \min.MSE(T_1^*), \text{ where } T_1^* = T_{lr_1} \text{ and } T_{p_1} \\ \Rightarrow \frac{I_1 M_1^2 + J_1 L_1^2 - 2K_1 L_1 M_1}{I_1 J_1 - K_1^2} &> 1 - V_0^* + \frac{V_{01}^2}{V_1}. \end{aligned}$$

(ii). The proposed neutrosophic exponential estimator T_{k_2} dominates the neutrosophic regression estimator T_{lr_2} and neutrosophic power ratio-cum exponential ratio estimator T_{p_2} under Scheme II, if

$$\begin{aligned} \min.MSE(T_{k_2}) &< \min.MSE(T_2^*), \text{ where } T_2^* = T_{lr_2} \text{ and } T_{p_2} \\ \Rightarrow \frac{I_2 M_2^2 + J_2 L_2^2 - 2K_2 L_2 M_2}{I_2 J_2 - K_2^2} &> 1 - V_0^* + \frac{V_{01}^2}{V_1^*}. \end{aligned}$$

(iii). The proposed neutrosophic exponential estimator T_{k_3} dominates the neutrosophic regression estimator T_{lr_3} and neutrosophic power ratio-cum exponential ratio estimator T_{p_3} under Scheme III, if

$$\begin{aligned} \min.MSE(T_{k_3}) &< MSE(T_3^*), \text{ where } T_3^* = T_{lr_3} \text{ and } T_{p_3} \\ \Rightarrow \frac{I_3 M_3^2 + J_3 L_3^2 - 2K_3 L_3 M_3}{I_3 J_3 - K_3^2} &> 1 - V_0^* + \frac{(V_{01}^* - V_{01})^2}{(V_1^* - V_1)}. \end{aligned}$$

Lemma 4.4. (i). The proposed neutrosophic exponential estimator T_{k_1} dominates the neutrosophic exponential ratio estimator T_{t_1} under Scheme I, if

$$\min.MSE(T_{k_1}) < \min.MSE(T_{t_1})$$

$$\Rightarrow \frac{I_1 M_1^2 + J_1 L_1^2 - 2K_1 L_1 M_1}{I_1 J_1 - K_1^2} > 1 - V_0^* - \frac{V_1}{4} + V_{01}.$$

(ii). The proposed neutrosophic exponential estimator T_{k_2} dominates the neutrosophic exponential ratio estimator T_{t_2} under Scheme II, if

$$\begin{aligned} \min.MSE(T_{k_2}) &< \min.MSE(T_{t_2}) \\ \Rightarrow \frac{I_2 M_2^2 + J_2 L_2^2 - 2K_2 L_2 M_2}{I_2 J_2 - K_2^2} &> 1 - V_0^* - \frac{V_1^*}{4} + V_{01}^*. \end{aligned}$$

(iii). The proposed neutrosophic exponential estimator T_{k_3} dominates the neutrosophic exponential ratio estimator T_{t_3} under Scheme III, if

$$\begin{aligned} \min.MSE(T_{k_3}) &< \min.MSE(T_{t_3}) \\ \Rightarrow \frac{I_3 M_3^2 + J_3 L_3^2 - 2K_3 L_3 M_3}{I_3 J_3 - K_3^2} &> 1 - V_0^* - \frac{(V_1^* - V_1)}{4} + (V_{01}^* - V_{01}). \end{aligned}$$

5. Simulation study

Simulation study is important because it allows mathematical results to be tested under controlled settings. Following Singh and Horn [40] and Kumar and Kumar [41], a simulation analysis is performed on a symmetric population generated by the neutrosophic normal distribution (NND) and an asymmetric population generated by the neutrosophic gamma distribution (NGD). The models used to generate the neutrosophic data are mentioned below.

$$y_N = 10.9 + \sqrt{(1 - \rho_{xy_N}^2)} y_N^* + \rho_{xy_N} \left(\frac{S_{y_N}}{S_{x_N}} \right) x_N^* \quad (5.1)$$

and $x_N = 8.5 + x_N^*$.

The neutrosophic variables x_N^* and y_N^* follow distinct neutrosophic distributions. The simulation procedure is described in the following points.

- (i). Adapting the models of (5.1) and following Patro and Smarandache [42], we generate a normal (symmetric) population from NND of size $N_N = [1000, 1000]$ with $x_N^* \sim NND([10, 18], [15, 20])$, $y_N^* \sim NND([14, 22], [18, 28])$, and different values of neutrosophic correlation coefficients $\rho_{xy_N} \in ([0.1, 0.2], [0.3, 0.4], [0.5, 0.6], [0.7, 0.8])$. Further, following Saleem and Aslam [43], we generate a gamma (asymmetric) population from NGD of size $N_N = [1000, 1000]$ with $x_N^* \sim NGD([0.20, 0.26], [0.22, 0.28])$, $y_N^* \sim NGD([0.16, 0.22], [0.18, 0.24])$, and different values of neutrosophic correlation coefficients $\rho_{xy_N} \in ([0.1, 0.2], [0.3, 0.4], [0.5, 0.7], [0.7, 0.8])$.
- (ii). Select a neutrosophic ranked set sample of size $n_N = [12, 12]$ with set size $m_N = [3, 3]$ and number of cycles $k = 4$ from the neutrosophic normal and neutrosophic gamma populations generated in previous point.
- (iv). Based on 10,000 iterations, compute the $MSE_N \in [MSE_L, MSE_U]$ and percent relative efficiency (PRE) $PRE_N \in [PRE_L, PRE_U]$ of the resulting neutrosophic estimators for the responding probability $P = 0.67$ and the ranked set sample n_N taken in the previous step utilizing the respective expression provided in (5.2).

$$\begin{aligned} MSE(T^*) &= \frac{1}{10,000} \sum_{j=1}^{10,000} (T_j^* - \bar{Y}_N)^2 \\ \text{and } PRE &= \frac{V(\bar{y}_{r_N, r_{SS}})}{MSE(T^*)} \times 100, \end{aligned} \quad (5.2)$$

where $T^* = T_m, T_{r_i}, i = 1, 2, 3, T_{lr_i}, T_{t_i}, i = 1, 2, 3, T_{p_i}, i = 1, 2, 3, T_{k_i}, i = 1, 2, 3$.

- (v). Tables 1 and 2 show the simulated $MSE_N \in [MSE_L, MSE_U]$ and simulated $PRE_N \in [PRE_L, PRE_U]$ respectively for the neutrosophic normal population, while Tables 3 and 4 show the

Table 1 $MSE_N \in [MSE_L, MSE_U]$ of resulting neutrosophic estimators using neutrosophic normal population.

Estimators	ρ_{xyN}			
	[0.1, 0.2]	[0.3, 0.4]	[0.5, 0.6]	[0.7, 0.8]
T_m	[36.72, 88.04]	[36.07, 86.62]	[35.58, 85.75]	[35.39, 85.90]
Scheme I				
T_{r_1}	[64.58, 121.80]	[54.54, 101.23]	[42.41, 75.75]	[28.59, 46.38]
T_{lr_1}	[35.88, 83.59]	[32.58, 72.67]	[26.90, 56.15]	[18.94, 34.09]
T_{i_1}	[42.14, 90.18]	[35.81, 76.60]	[28.90, 61.63]	[21.88, 46.73]
T_{p_1}	[35.88, 83.59]	[32.58, 72.67]	[26.90, 56.15]	[18.94, 34.09]
T_{k_1}	[33.73, 76.82]	[30.69, 67.78]	[25.32, 53.07]	[17.98, 32.46]
Scheme II				
T_{r_2}	[78.30, 138.43]	[63.64, 108.42]	[45.77, 70.82]	[25.24, 26.92]
T_{lr_2}	[35.47, 81.40]	[30.87, 65.80]	[22.62, 41.57]	[10.83, 8.57]
T_{i_2}	[44.81, 91.23]	[35.68, 71.67]	[25.61, 49.75]	[15.23, 27.44]
T_{p_2}	[35.47, 81.40]	[30.87, 65.80]	[22.62, 41.57]	[10.83, 8.57]
T_{k_2}	[33.43, 74.82]	[29.12, 61.36]	[20.46, 39.12]	[9.47, 6.89]
Scheme III				
T_{r_3}	[50.44, 104.67]	[45.17, 93.82]	[38.94, 80.82]	[32.04, 66.44]
T_{lr_3}	[36.31, 85.85]	[34.35, 79.75]	[31.30, 71.17]	[27.29, 60.38]
T_{i_3}	[39.39, 89.09]	[35.94, 81.69]	[32.29, 73.87]	[28.74, 66.61]
T_{p_3}	[36.31, 85.85]	[34.35, 79.75]	[31.30, 71.17]	[27.29, 60.38]
T_{k_3}	[33.90, 78.66]	[32.28, 74.29]	[29.57, 67.10]	[25.90, 57.45]

Table 2 $PRE_N \in [PRE_L, PRE_U]$ of resulting neutrosophic estimators using neutrosophic normal population.

Estimators	ρ_{xyN}			
	[0.1, 0.2]	[0.3, 0.4]	[0.5, 0.6]	[0.7, 0.8]
T_m	[100, 100]	[100, 100]	[100, 100]	[100, 100]
Scheme I				
T_{r_1}	[56.86, 72.27]	[66.13, 85.57]	[83.89, 113.20]	[123.79, 185.18]
T_{lr_1}	[102.33, 105.31]	[110.68, 119.19]	[132.26, 152.71]	[186.88, 251.97]
T_{i_1}	[87.13, 97.62]	[100.72, 113.07]	[123.11, 139.13]	[161.73, 183.80]
T_{p_1}	[102.33, 105.31]	[110.68, 119.19]	[132.26, 152.71]	[186.88, 251.97]
T_{k_1}	[108.87, 114.59]	[117.50, 127.80]	[140.48, 161.58]	[196.77, 264.58]
Scheme II				
T_{r_2}	[46.90, 63.59]	[56.67, 79.89]	[77.72, 121.08]	[140.22, 319.06]
T_{lr_2}	[103.52, 108.15]	[116.84, 131.64]	[157.24, 206.27]	[326.69, 1002.12]
T_{i_2}	[81.94, 96.49]	[101.08, 120.86]	[138.93, 172.35]	[232.38, 313.00]
T_{p_2}	[103.52, 108.15]	[116.84, 131.64]	[157.24, 206.27]	[326.69, 1002.12]
T_{k_2}	[109.84, 117.65]	[123.85, 141.17]	[173.89, 219.19]	[373.68, 1246.63]
Scheme III				
T_{r_3}	[72.80, 84.11]	[79.85, 92.33]	[91.36, 106.09]	[110.45, 129.29]
T_{lr_3}	[101.13, 102.55]	[104.99, 108.61]	[113.65, 120.48]	[129.69, 142.26]
T_{i_3}	[93.22, 98.81]	[100.35, 106.04]	[110.18, 116.08]	[123.15, 128.96]
T_{p_3}	[101.13, 102.55]	[104.99, 108.61]	[113.65, 120.48]	[129.69, 142.26]
T_{k_3}	[108.30, 111.91]	[111.74, 116.59]	[120.30, 127.79]	[136.65, 149.51]

simulated $MSE_N \in [MSE_L, MSE_U]$ and simulated $PRE_N \in [PRE_L, PRE_U]$ respectively for the neutrosophic gamma population.

5.1. Comparative analysis of simulation findings

A comparative analysis of the simulation findings is critical for understanding the performance of the neutrosophic estimators. It validates the efficiency conditions of the suggested estimators established through theoretical comparisons. The simulation results of Tables 1–4 are described below:

- Tables 1 and 3 present the MSE_N results for both the neutrosophic normal and neutrosophic gamma populations. The results indicate that, across schemes I–III, the proposed neutrosophic exponential estimators T_{k_i} , $i = 1, 2, 3$ consistently yield the lowest MSE_N values for all levels of ρ_{xyN} , outperforming the adapted neutrosophic mean estimator T_m , the neutrosophic ratio and regression estimators T_{r_i} , $i = 1, 2, 3$ and T_{lr_i} , the neutrosophic exponential ratio estimators T_{i_i} , $i = 1, 2, 3$, as well as the neutrosophic power ratio-cum exponential

ratio estimators T_{p_i} , $i = 1, 2, 3$. This can easily be observed from the multiple bar diagrams presented in Fig. 1 and Fig. 3 based on the results of Table 1 and Table 3, respectively. These findings highlight the enhanced efficiency and accuracy of the proposed estimators in handling missing values in uncertain data.

- Tables 2 and 4 present the PRE_N results for both the neutrosophic normal and neutrosophic gamma populations. The results reveal that, across schemes I–III, the proposed neutrosophic exponential estimators T_{k_i} , $i = 1, 2, 3$ consistently attain the highest PRE_N values for all values of ρ_{xyN} , outperforming the adapted neutrosophic mean estimator T_m , neutrosophic ratio and regression estimators T_{r_i} , $i = 1, 2, 3$ and T_{lr_i} , the neutrosophic exponential ratio estimators T_{i_i} , $i = 1, 2, 3$, and the neutrosophic power ratio-cum exponential ratio estimators T_{p_i} , $i = 1, 2, 3$. This can easily be observed from the multiple bar diagrams presented in Fig. 2 and Fig. 4 based on the results of Table 2 and Table 4, respectively. This demonstrates that the proposed estimators T_{k_i} are not only robust across different population types, but also deliver superior efficiency and reliability in neutrosophic population parameter estimation across varying values of ρ_{xyN} .

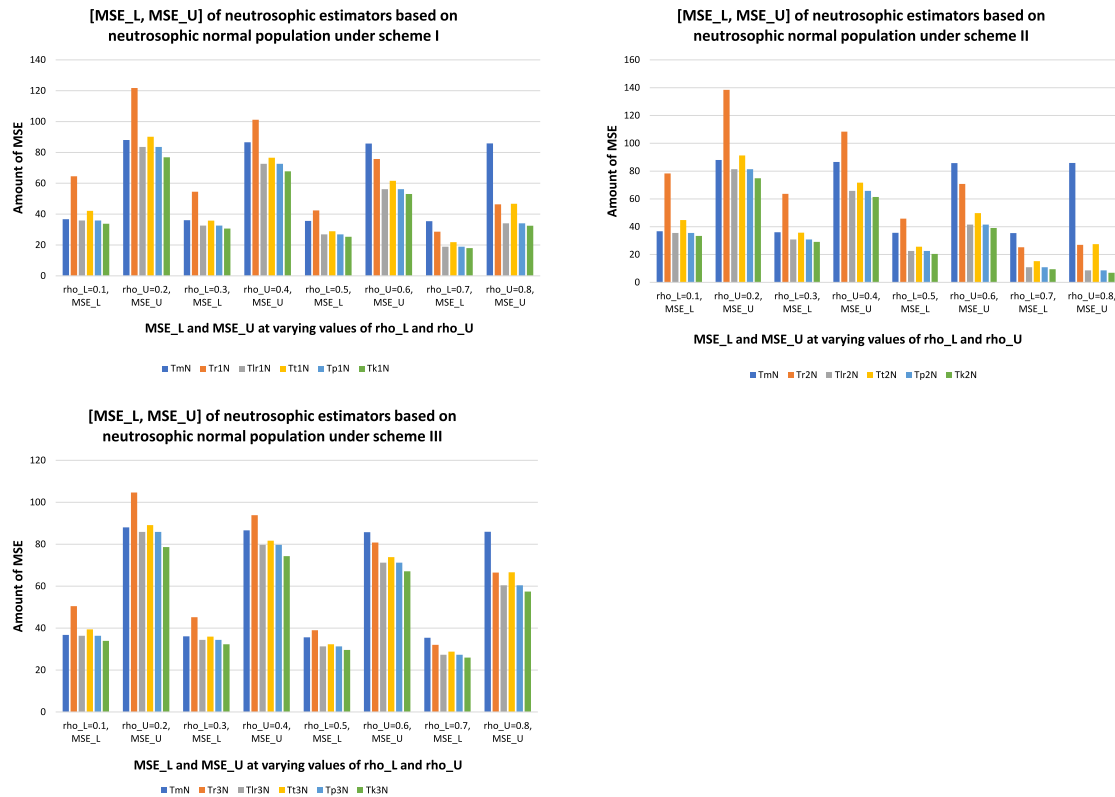


Fig. 1. MSE_N of the neutrosophic estimators under schemes I, II, and III reported in Table 1.

• Tables 1 and 3 indicate that the MSE_N of the neutrosophic mean estimator T_m remains nearly constant regardless of the value of ρ_{xy_N} (see, Figs. 1 and 3). Furthermore, under schemes I–III, the MSE_N of the adapted neutrosophic ratio and regression estimators T_{r_i} , $i = 1, 2, 3$ and T_{lr_i} , the neutrosophic exponential ratio estimators T_{t_i} , $i = 1, 2, 3$, the neutrosophic power ratio-cum-exponential ratio estimators T_{p_i} , $i = 1, 2, 3$, and the proposed neutrosophic exponential estimators T_{k_i} , $i = 1, 2, 3$ consistently decrease as ρ_{xy_N} increases from its minimum to maximum value (see, Figs. 1 and 3).

• Tables 2 and 4 reveal that the PRE_N of the resulting neutrosophic mean estimator T_m remains essentially stable across all values of ρ_{xy_N} (see, Figs. 2 and 4). Additionally, under schemes I–III, the PRE_N of the adapted neutrosophic ratio and regression estimators T_{r_i} , $i = 1, 2, 3$ and T_{lr_i} , as well as the neutrosophic exponential ratio estimators T_{t_i} , $i = 1, 2, 3$, power ratio-cum exponential ratio estimators T_{p_i} , $i = 1, 2, 3$, and the proposed neutrosophic exponential estimators T_{k_i} , $i = 1, 2, 3$, consistently increases as ρ_{xy_N} varies from its minimum to maximum value (see, Figs. 2 and 4).

• Moreover, across both populations, the proposed neutrosophic exponential estimators exhibit significantly better performance in scheme II compared to schemes I and III. This enhanced performance indicates that the estimators are particularly effective in scenarios where the underlying conditions align closely with the neutrosophic characteristics represented in scheme II (see, Figs. 1 to 4).

6. Real data application

Empirical data application plays a vital role in survey sampling as they provide a realistic representation of population characteristics, effectively bridging the gap between theoretical constructs and practical implementation. In this context, we consider a dataset comprising the daily stock prices of Moderna, which can be accessed from the open-source platform at <https://finance.yahoo.com/quote/MRNA/history/>. The neutrosophic survey variable $y_N \in [y_L, y_U]$ represents the stock's

daily price from September 1, 2020, to September 1, 2021, while the neutrosophic auxiliary variable $x_N \in [x_L, x_U]$ denotes the stock's daily price from September 1, 2019, to September 1, 2020. The descriptive statistics for the data are as follows: $N_N = [252, 252]$, $n_N = [12, 12]$, $m_N = [3, 3]$, $k = 4$, $P = 0.67$, $\bar{Y}_N = [162.973, 174.228]$, $\bar{X}_N = [35.94, 38.762]$, $C_{y_N} = [0.547, 0.559]$, $C_{x_N} = [0.603, 0.607]$, and $\rho_{xy_N} = [0.829, 0.804]$. Based on these descriptive values, we compute the $MSE_N \in [MSE_L, MSE_U]$ and the $PRE_N \in [PRE_L, PRE_U]$ for the derived neutrosophic estimators. The corresponding results are presented in Table 5. The calculation of PRE_N is performed using the formula provided below.

$$PRE = \frac{V(\bar{y}_{rN, rss})}{MSE(T^*)} \times 100, \quad (6.1)$$

The empirical results presented in Table 5 indicate that the proposed neutrosophic exponential estimators T_{k_i} , $i = 1, 2, 3$ achieve the lowest values of MSE_N when compared to those of the adapted neutrosophic estimators, including the neutrosophic sample mean estimator T_m , the neutrosophic ratio and regression estimators T_{r_i} , $i = 1, 2, 3$ and T_{lr_i} , the neutrosophic exponential ratio estimators T_{t_i} , and the neutrosophic power ratio-cum exponential ratio estimators T_{p_i} (see, Fig. 5). Additionally, the findings from Table 5 reveal that the proposed estimators T_{k_i} , $i = 1, 2, 3$ exhibit the highest PRE_N values in comparison to the adapted estimators (see, Fig. 6). Moreover, both the proposed and adapted neutrosophic estimators perform better under scheme II than under schemes I and III (see, Figs. 5 and 6).

7. Conclusions

Traditional imputation methods often fall short when execute to datasets characterized by uncertainty, frequently resulting in biased estimates and leading to unreliable statistical inferences. To address these shortcomings, this article introduced the neutrosophic exponential imputation structures that use the foundational principles of neutrosophic logic based on truth, indeterminacy, and falsity, to provide a more

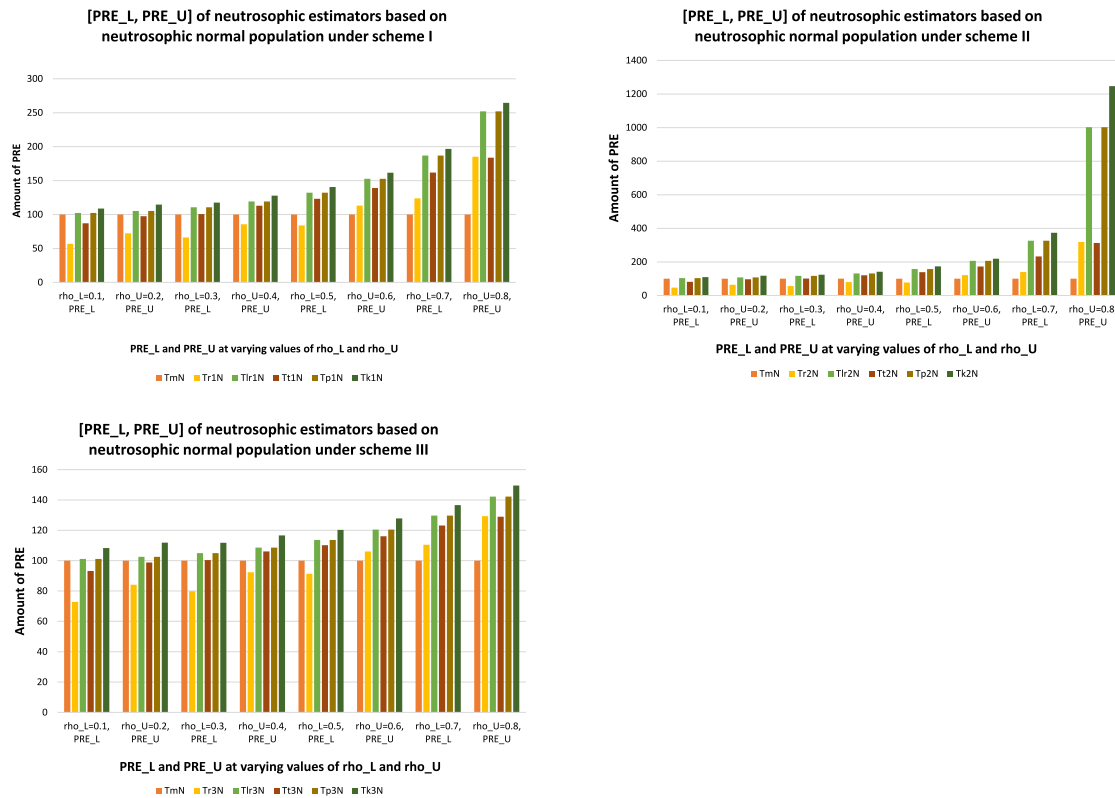


Fig. 2. PRE_N of the neutrosophic estimators under schemes I, II, and III reported in Table 2.

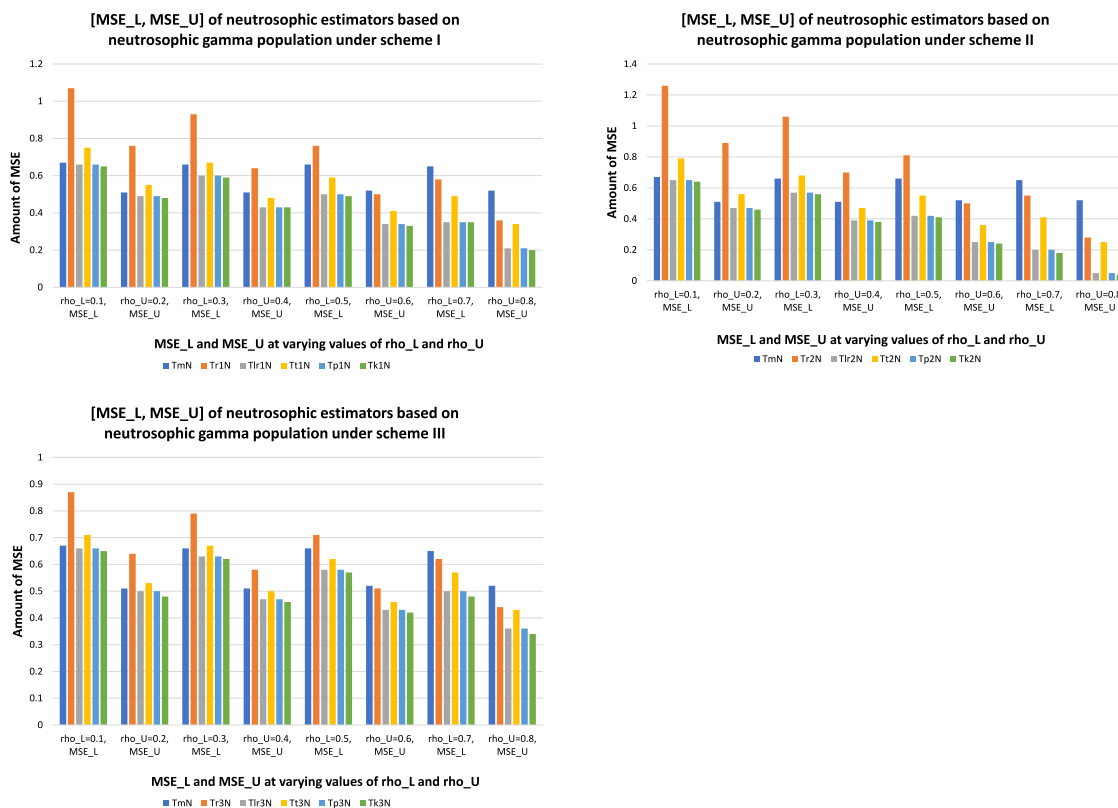


Fig. 3. MSE_N of the neutrosophic estimators under schemes I, II, and III reported in Table 3.

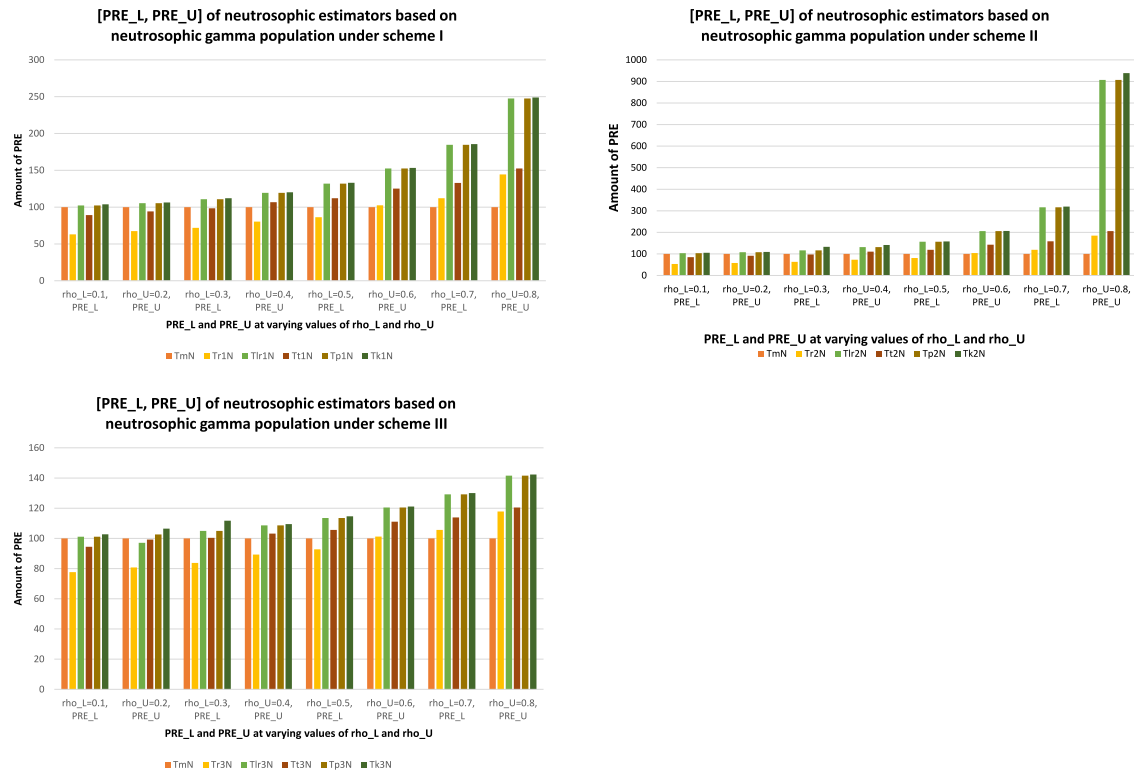


Fig. 4. PRE_N of the neutrosophic estimators under schemes I, II, and III reported in Table 4.

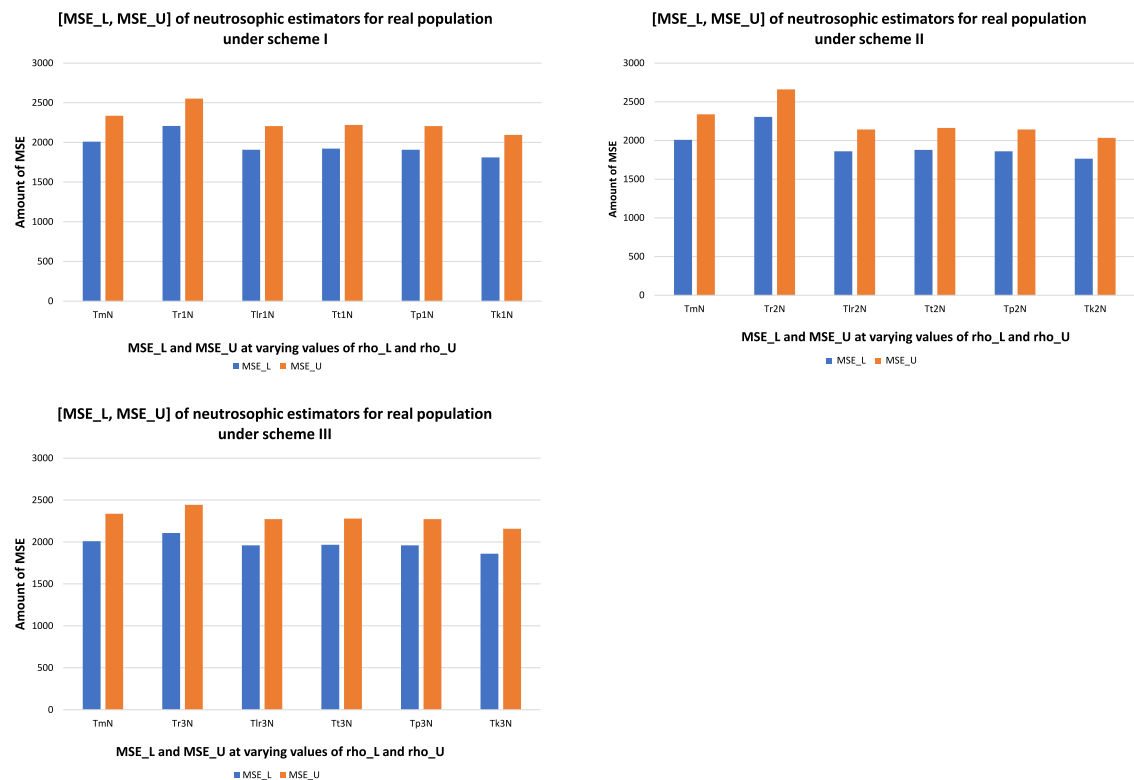


Fig. 5. MSE_N of the neutrosophic estimators for real population under schemes I, II, and III reported in Table 5.

Table 3 $MSE_N \in [MSE_L, MSE_U]$ of resulting neutrosophic estimators using neutrosophic gamma population.

Estimators	ρ_{xyN}			
	[0.1, 0.2]	[0.3, 0.4]	[0.5, 0.6]	[0.7, 0.8]
T_m	[0.67, 0.51]	[0.66, 0.51]	[0.66, 0.52]	[0.65, 0.52]
Scheme I				
T_{r_1}	[1.07, 0.76]	[0.93, 0.64]	[0.76, 0.50]	[0.58, 0.36]
T_{lr_1}	[0.66, 0.49]	[0.60, 0.43]	[0.50, 0.34]	[0.35, 0.21]
T_{t_1}	[0.75, 0.55]	[0.67, 0.48]	[0.59, 0.41]	[0.49, 0.34]
T_{p_1}	[0.66, 0.49]	[0.60, 0.43]	[0.50, 0.34]	[0.35, 0.21]
T_{k_1}	[0.65, 0.48]	[0.59, 0.43]	[0.49, 0.33]	[0.35, 0.20]
Scheme II				
T_{r_2}	[1.26, 0.89]	[1.06, 0.70]	[0.81, 0.50]	[0.55, 0.28]
T_{lr_2}	[0.65, 0.47]	[0.57, 0.39]	[0.42, 0.25]	[0.20, 0.05]
T_{t_2}	[0.79, 0.56]	[0.68, 0.47]	[0.55, 0.36]	[0.41, 0.25]
T_{p_2}	[0.65, 0.47]	[0.57, 0.39]	[0.42, 0.25]	[0.20, 0.05]
T_{k_2}	[0.64, 0.47]	[0.56, 0.39]	[0.41, 0.25]	[0.20, 0.05]
Scheme III				
T_{r_3}	[0.87, 0.64]	[0.79, 0.58]	[0.71, 0.51]	[0.62, 0.44]
T_{lr_3}	[0.66, 0.50]	[0.63, 0.47]	[0.58, 0.43]	[0.50, 0.36]
T_{t_3}	[0.71, 0.53]	[0.67, 0.50]	[0.62, 0.46]	[0.57, 0.43]
T_{p_3}	[0.66, 0.50]	[0.63, 0.47]	[0.58, 0.43]	[0.50, 0.36]
T_{k_3}	[0.65, 0.50]	[0.62, 0.47]	[0.57, 0.42]	[0.50, 0.36]

Table 4 $PRE_N \in [PRE_L, PRE_U]$ of resulting neutrosophic estimators using neutrosophic gamma population.

Estimators	ρ_{xyN}			
	[0.1, 0.2]	[0.3, 0.4]	[0.5, 0.6]	[0.7, 0.8]
T_m	[100, 100]	[100, 100]	[100, 100]	[100, 100]
Scheme I				
T_{r_1}	[63.05, 67.34]	[71.78, 80.33]	[86.24, 102.52]	[112.03, 144.38]
T_{lr_1}	[102.22, 105.37]	[110.64, 119.30]	[131.94, 152.45]	[184.56, 247.59]
T_{t_1}	[89.26, 94.15]	[98.46, 106.72]	[112.16, 125.23]	[132.83, 152.56]
T_{p_1}	[102.22, 105.37]	[110.64, 119.30]	[131.94, 152.45]	[184.56, 247.59]
T_{k_1}	[103.87, 106.31]	[112.06, 120.12]	[132.99, 153.19]	[185.56, 248.91]
Scheme II				
T_{r_2}	[53.35, 58.01]	[63.02, 73.24]	[80.76, 103.82]	[119.10, 184.76]
T_{lr_2}	[103.35, 108.24]	[116.75, 131.84]	[156.58, 205.56]	[316.31, 906.79]
T_{t_2}	[84.78, 91.51]	[97.73, 110.37]	[119.31, 143.00]	[158.45, 205.87]
T_{p_2}	[103.35, 108.24]	[116.75, 131.84]	[156.58, 205.56]	[316.31, 906.79]
T_{k_2}	[105.06, 109.20]	[118.21, 132.68]	[157.68, 206.57]	[319.43, 938.52]
Scheme III				
T_{r_3}	[77.60, 80.72]	[83.78, 89.24]	[92.71, 101.23]	[105.58, 117.84]
T_{lr_3}	[101.08, 102.57]	[104.97, 108.66]	[113.54, 120.40]	[129.14, 141.56]
T_{t_3}	[94.40, 97.03]	[99.24, 103.20]	[105.64, 111.01]	[113.86, 120.43]
T_{p_3}	[101.08, 102.57]	[104.97, 108.66]	[113.54, 120.40]	[129.14, 141.56]
T_{k_3}	[102.73, 103.52]	[106.41, 109.48]	[114.64, 121.12]	[130.03, 142.29]

Table 5 MSE_N and PRE_N of neutrosophic estimators for real data set.

Estimators	MSE_N	PRE_N
T_m	[2009.32, 2337.18]	[100.00, 100.00]
Scheme I		
T_{r_1}	[2207.91, 2553.52]	[91.00, 91.52]
T_{lr_1}	[1908.74, 2206.90]	[105.26, 105.90]
T_{t_1}	[1921.95, 2219.88]	[104.54, 105.28]
T_{p_1}	[1908.74, 2206.90]	[105.26, 105.90]
T_{k_1}	[1812.29, 2095.40]	[110.87, 111.53]
Scheme II		
T_{r_2}	[2305.71, 2660.08]	[87.14, 87.86]
T_{lr_2}	[1859.20, 2142.74]	[108.07, 109.07]
T_{t_2}	[1878.91, 2162.10]	[106.94, 108.09]
T_{p_2}	[1859.20, 2142.74]	[108.07, 109.07]
T_{k_2}	[1764.97, 2033.94]	[113.84, 114.90]
Scheme III		
T_{r_3}	[2107.13, 2443.74]	[95.35, 95.63]
T_{lr_3}	[1959.78, 2273.01]	[102.52, 102.82]
T_{t_3}	[1966.29, 2279.40]	[102.18, 102.53]
T_{p_3}	[1959.78, 2273.01]	[102.52, 102.82]
T_{k_3}	[1861.05, 2158.71]	[107.96, 108.26]

robust and nuanced solution of missing data. Specifically, we developed a new class of neutrosophic exponential estimators for the population mean, denoted by T_{k_i} , $i = 1, 2, 3$, and rigorously evaluated their performance using MSE and PRE.

Through both theoretical derivations and comprehensive simulation studies based on normally and gamma-distributed datasets, the proposed estimators consistently outperformed the adapted neutrosophic estimators, including the neutrosophic sample mean estimator (T_m), neutrosophic ratio and regression estimators (T_{r_i} , $i = 1, 2, 3$ and T_{lr_i}), neutrosophic exponential ratio estimators (T_{t_i}), and neutrosophic power ratio-cum-exponential ratio estimators (T_{p_i}). The superiority of the proposed estimators was evident across all imputation structures considered.

Furthermore, the practical applicability of the proposed methods is demonstrated through a case study involving real-world uncertain stock price data. The results highlighted the effectiveness of the neutrosophic exponential imputations in enhancing both the accuracy and reliability of the population mean estimation under uncertainty and indeterminacy.

Given the strong theoretical foundation, empirical performance, and practical relevance, the proposed neutrosophic exponential imputation structures are recommended as a valuable tool for handling missing

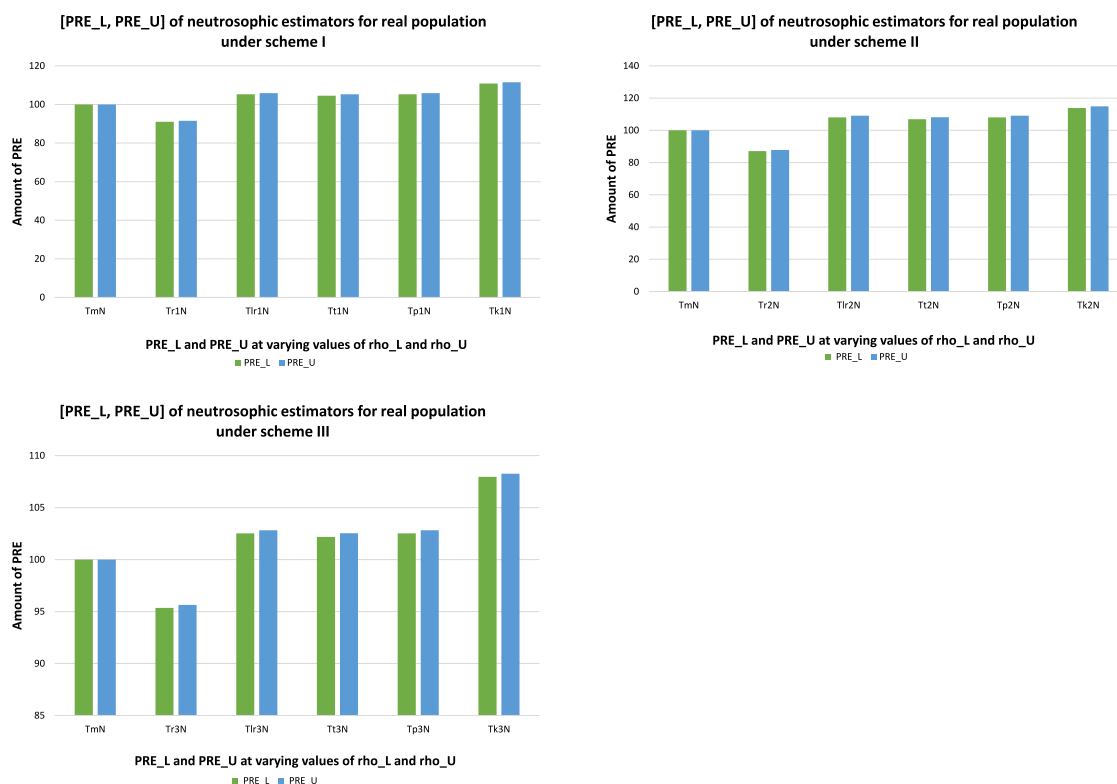


Fig. 6. PRE_N of the neutrosophic estimators for real population under schemes I, II, and III reported in Table 4.

values in uncertain datasets, particularly when accurate estimation of population parameters is critical.

CRediT authorship contribution statement

Anoop Kumar: Writing – review & editing, Supervision, Conceptualization. **Vishal Kumar:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Investigation, Formal analysis, Data curation.

Ethical statement

There are no human/animal subjects in this article therefore an ethics statement is not applicable because this study is applied on already published data.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

All data applied is included in the manuscript.

References

- [1] D.B. Rubin, Inference and missing data, *Biometrika* 63 (3) (1976) 581–592.
- [2] D.F. Heitjan, S. Basu, Distinguishing missing at random and missing completely at random, *Amer. Statist.* 50 (3) (1996) 207–213.
- [3] A.I. Al-Omari, Estimation of mean based on modified robust extreme ranked set sampling, *J. Stat. Comput. Simul.* 81 (8) (2011) 1055–1066.
- [4] A.I. Al-Omari, C.N. Bouza, C. Herrera, Imputation methods of missing data for estimating the population mean using simple random sampling with known correlation coefficient, *Qual. Quant.* 47 (1) (2013) 353–365.
- [5] S. Bhushan, A.P. Pandey, Optimal imputation of missing data for estimation of population mean, *J. Stat. Manag. Syst.* 19 (6) (2016) 755–769.
- [6] S. Prasad, A study on new methods of ratio exponential type imputation in sample surveys, *Hacet. J. Math. Stat.* 47 (5) (2018) 1281–1301.
- [7] M.U. Sohail, J. Shabbir, F. Sohil, Imputation of missing values by using raw moments, *Stat. Transit. New Ser.* 20 (1) (2019) 21–40.
- [8] S. Bhushan, A. Kumar, A.P. Pandey, S. Singh, Estimation of population mean in presence of missing data under simple random sampling, *Comm. Statist. Simulation Comput.* 52 (12) (2023) 6048–6069.
- [9] A.K. Pandey, G.N. Singh, D. Bhattacharyya, P.K. Singh, Efficient and alternative approaches for imputing missing data to estimate population mean, *Qual. Quant.* 58 (6) (2024) 5883–5897.
- [10] V.K. Yadav, S. Prasad, Generalized class of factor type exponential imputation techniques for population mean using simulation approach, *J. Stat. Comput. Simul.* 94 (9) (2024) 1997–2039.
- [11] S.A. Rehman, J. Shabbir, L.A. Al-Essa, On the development of survey methods for novel mean imputation and its application to abalone data, *Heliyon* 10 (11) (2024) e31423, <http://dx.doi.org/10.1016/j.heliyon.2024.e31423>.
- [12] G.R.V. Triveni, F. Danish, M. Alrasheedi, Application of log-type estimators for addressing non-response in survey sampling using real datasets, *Mathematics* 13 (7) (2025) 1089, <http://dx.doi.org/10.3390/math13071089>.
- [13] G.A. McIntyre, A method for unbiased selective sampling, using ranked sets, *Aust. J. Agric. Res.* 3 (4) (1952) 385–390.
- [14] T.R. Dell, J.L. Clutter, Ranked set sampling theory with order statistics background, *Biometrics* 54 (1972) 5–555.
- [15] S.L. Stokes, Ranked set sampling with concomitant variables, *Comm. Statist. Theory Methods* 6 (12) (1977) 1207–1211.
- [16] S.L. Stokes, T.W. Sager, Characterization of a ranked-set sample with application to estimating distribution functions, *J. Amer. Statist. Assoc.* 83 (402) (1988) 374–381.

- [17] G.P. Patil, A.K. Sinha, C. Taillie, Ranked set sampling for multiple characteristics, *Int. J. Ecol. Environ. Sci.* 20 (3) (1994) 357–373.
- [18] H. Chen, E.A. Stasny, D.A. Wolfe, Ranked set sampling for efficient estimation of a population proportion, *Stat. Med.* 24 (21) (2005) 3319–3329.
- [19] S.A. Rehman, J. Shabbir, An efficient class of estimators for finite population mean in the presence of non-response under ranked set sampling (RSS), *PLoS One* 17 (12) (2022) e0277232.
- [20] A. Kumar, S. Bhushan, W. Emam, Y. Tashkandy, M.J.S. Khan, Novel logarithmic imputation procedures using multi auxiliary information under ranked set sampling, *Sci. Rep.* 14 (1) (2024) 18027.
- [21] Z. Tahir, H. Khan, M. Aslam, J. Shabbir, Y. Mahmood, F. Smarandache, Neutrosophic ratio-type estimators for estimating the population mean, *Complex Intell. Syst.* 7 (6) (2021) 2991–3001.
- [22] V.K. Yadav, S. Prasad, Neutrosophic estimators for estimating the population mean in survey sampling, *Meas.: Interdiscip. Res. Perspect.* 22 (4) (2024) 373–397.
- [23] M.A. Alqudah, M. Zayed, M. Subzar, S.A. Wani, Neutrosophic robust ratio type estimator for estimating finite population mean, *Heliyon* 10 (8) (2024) 1–12.
- [24] S. Masood, B. Ibrar, J. Shabbir, A. Shokri, Z. Movahedeei, Estimating neutrosophic finite median employing robust measures of the auxiliary variable, *Sci. Rep.* 14 (1) (2024) 10255.
- [25] R. Singh, A. Kumari, Neutrosophic ranked set sampling scheme for estimating population mean: an application to demographic data, *Neutrosophic Sets Syst.* 68 (2024) 246–270.
- [26] Priya, A. Kumar, Robust neutrosophic exponential estimators of population mean in the presence of uncertainty, *Qual. Quant.* 59 (2025) 1–24, <http://dx.doi.org/10.1007/s11135-025-02150-6>.
- [27] A. Kumar, Priya, V. Tripathi, Optimal neutrosophic framework for population mean estimation under simple random sampling, *Neutrosophic Sets Syst.* 86 (2025) 689–709.
- [28] A. Kumar, Priya, A.M. Alomair, Efficient classes of estimators for estimating indeterminate population mean using neutrosophic ranked set sampling, *AIMS Math.* 10 (4) (2025) 8946–8964.
- [29] A. Fatima, U. Afzal, M. Aslam, F. Smarandache, A comprehensive review of neutrosophic statistics for data analysis in applied sciences, *J. Reliab. Stat. Stud.* 18 (1) (2025) 25–40.
- [30] U. Shahzad, H. Zhu, O. Albalawi, M. Arslan, Neutrosophic developments in horvitz-thompson type estimators, *Math. Popul. Stud.* 32 (2) (2025) 1–14, <http://dx.doi.org/10.1080/08898480.2025.2464020>.
- [31] R. Singh, S.N. Tiwari, Improved estimator for population mean utilizing known medians of two auxiliary variables under neutrosophic framework, *Neutrosophic Syst. Appl.* 25 (1) (2025) 38–52.
- [32] P. Singh, P. Sharma, A. Singh, Estimation of population mean using generalized neutrosophic exponential estimators, *Res. Math.* 12 (1) (2025) 2474774.
- [33] P. Singh, S. Gupta, Combining two auxiliary variables for elevated estimation of finite population mean under neutrosophic framework, *Neutrosophic Sets Syst.* 76 (2025) 275–287.
- [34] F. Smarandache, A unifying field in logics: neutrosophic logic. *Neutrosophy, neutrosophic set, neutrosophic probability: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability, Infin. Study* (2005).
- [35] H. Lee, E. Rancourt, C.E. Sarndal, Experiments with variance estimation from survey data with imputed values, *J. Off. Stat.* 10 (3) (1994) 231–243.
- [36] S. Singh, B. Deo, Imputation by power transformation, *Statist. Papers* 44 (4) (2003) 555–579.
- [37] G. Diana, P.F. Perri, Improved estimators of the population mean for missing data, *Comm. Statist. Theory Methods* 39 (18) (2010) 3245–3251.
- [38] S. Bahl, R. Tuteja, Ratio and product type exponential estimators, *J. Inf. Optim. Sci.* 12 (1) (1991) 159–164.
- [39] G.O. Kadilar, A new exponential type estimator for the population mean in simple random sampling, *J. Mod. Appl. Stat. Methods* 15 (2) (2016) 207–214.
- [40] S. Singh, S. Horn, An alternative estimator for multi-character surveys, *Metrika* 48 (2) (1998) 99–107.
- [41] A. Kumar, V. Kumar, Uncertainty based efficient neutrosophic imputation methods for population mean, *Nat. Acad. Sci. Lett.* (2025) 1–11, <http://dx.doi.org/10.1007/s40009-025-01660-z>.
- [42] S.K. Patro, F. Smarandache, The neutrosophic statistical distribution, more problems, more solutions, *Neutrosophic Sets Syst.* 12 (2016) 73–79.
- [43] M. Saleem, M. Aslam, Algorithms for generating neutrosophic gamma distributed data, *J. Reliab. Stat. Stud.* 18 (02) (2025) 289–312.